Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/93-4.2.4.1-a+b-cos-^m-A+B-cos+C-cos^2-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [393]. This is test number [93].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (393)	0.00 (0)
Mathematica	98.98 (389)	1.02 (4)
Fricas	60.56 (238)	39.44 (155)
Maple	60.05 (236)	39.95 (157)
Maxima	30.28 (119)	69.72 (274)
Mupad	19.08 (75)	80.92 (318)
Giac	4.58 (18)	95.42 (375)
Sympy	3.82 (15)	96.18 (378)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

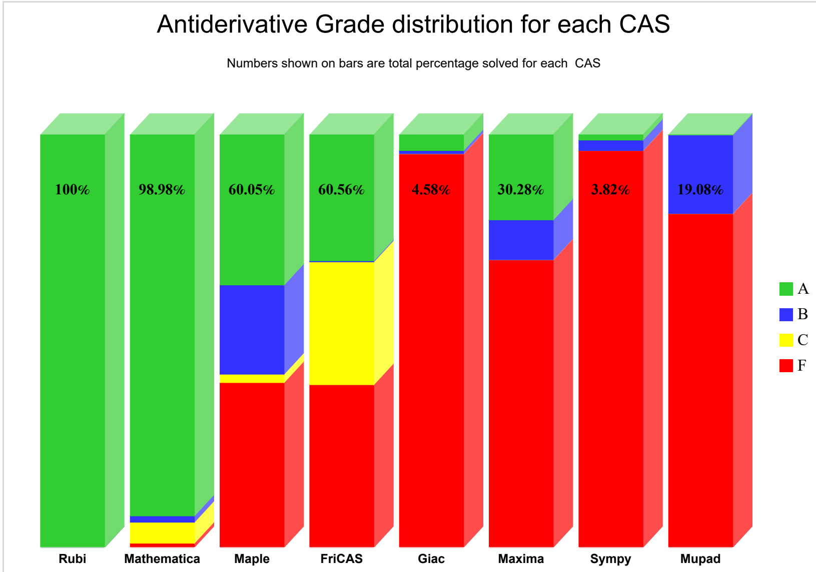
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

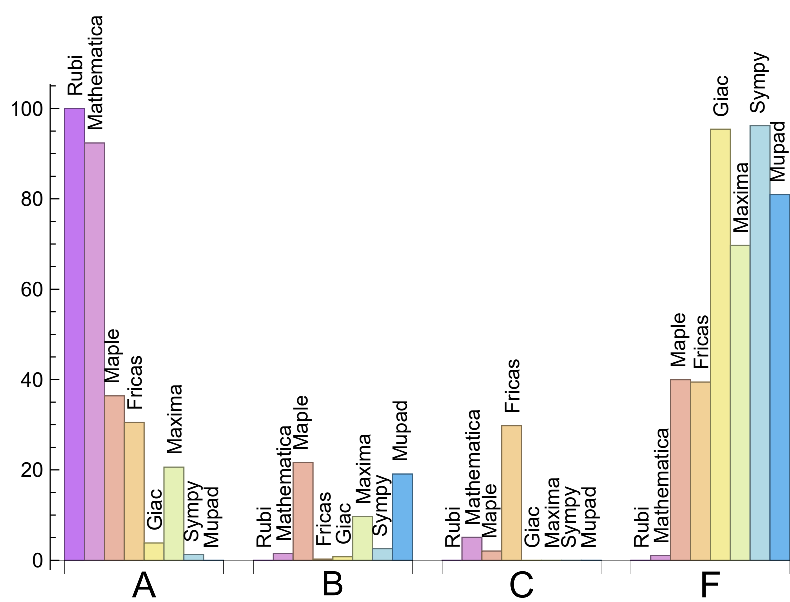
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	92.37	1.53	5.09	1.02
Maple	36.39	21.63	2.04	39.95
Fricas	30.53	0.25	29.77	39.44
Maxima	20.61	9.67	0.00	69.72
Giac	3.82	0.76	0.00	95.42
Sympy	1.27	2.54	0.00	96.18
Mupad	N/A	19.08	0.00	80.92

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	157	100.00 %	0.00 %	0.00 %
Fricas	155	100.00 %	0.00 %	0.00 %
Giac	375	97.07 %	0.80 %	2.13 %
Maxima	274	100.00 %	0.00 %	0.00 %
Sympy	378	20.63 %	55.03 %	24.34 %
Mupad	318	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

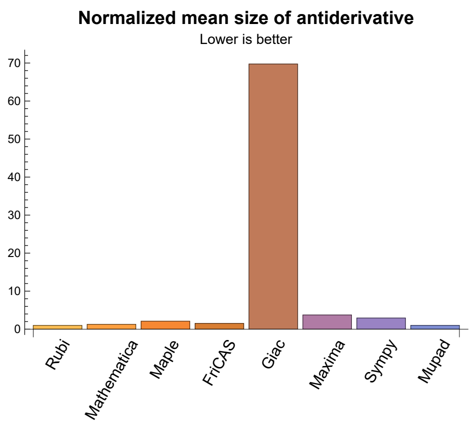
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

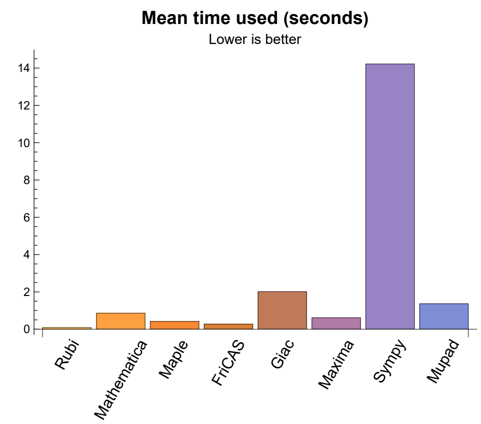
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	132.62	1.00	120.00	1.00
Mathematica	0.86	220.92	1.28	92.00	0.74
Maple	0.42	246.78	2.10	216.00	1.77
Maxima	0.61	473.18	3.74	107.00	1.18
Fricas	0.27	172.05	1.52	170.00	1.35
Sympy	14.22	191.47	2.94	184.00	2.10
Giac	2.01	4774.28	69.74	72.00	1.14
Mupad	1.36	94.19	1.00	84.00	0.86

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
70
60
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
14
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{163, 169, 208, 233, 267, 268, 276, 354, 355, 361, 393\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

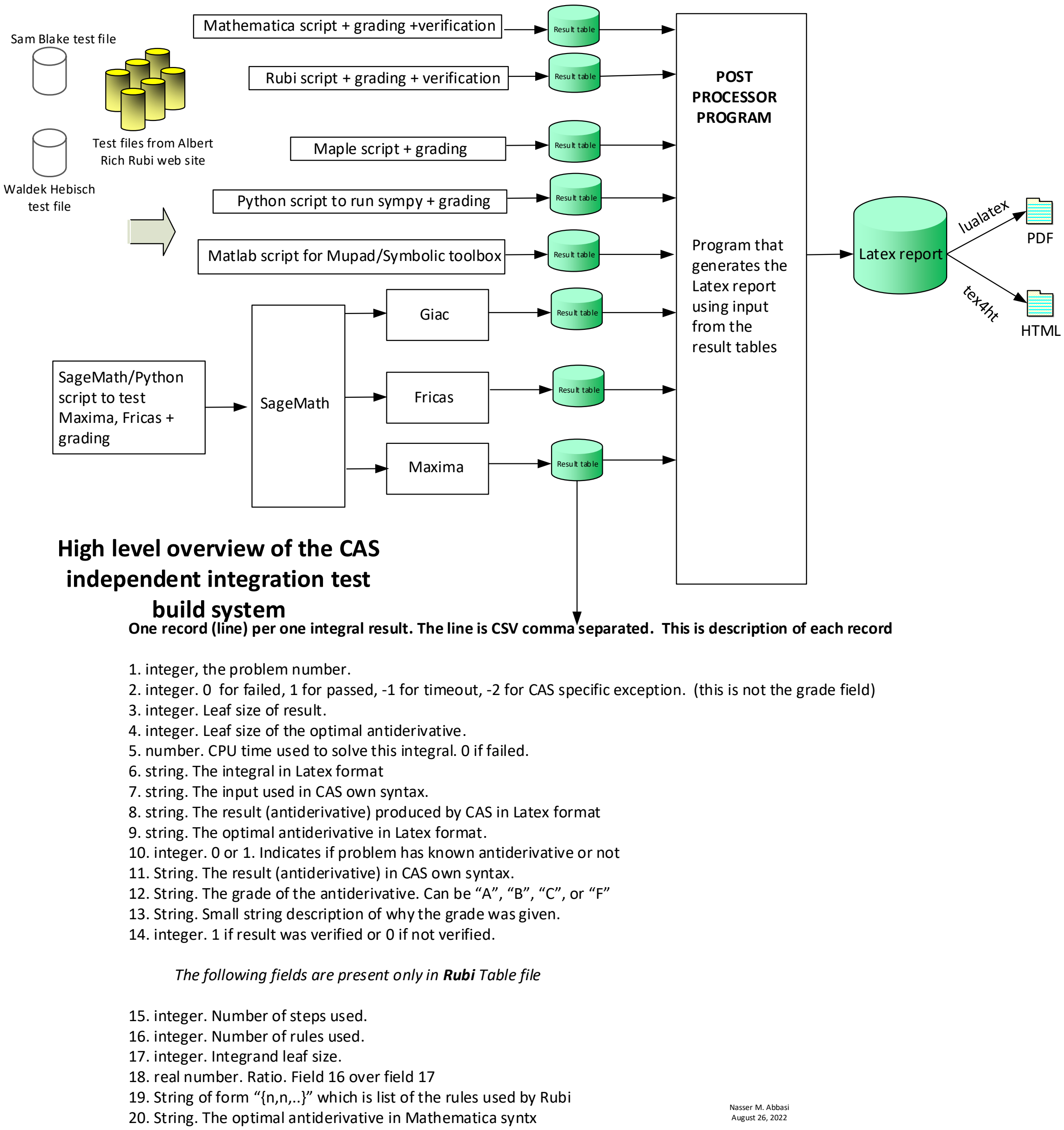
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 164, 165, 166, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391 }

B grade: { 208, 233, 354, 355, 361, 393 }

C grade: { 35, 36, 66, 67, 68, 76, 161, 163, 167, 169, 198, 200, 232, 267, 268, 276, 283, 383, 384, 386 }

F grade: { 202, 385, 387, 392 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 255, 256, 257, 258, 259, 263, 264, 265, 266, 267, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 95, 97, 121, 123, 244, 245, 246, 252, 253, 254, 260, 261, 262, 268, 269, 270, 276, 277, 278, 284, 285, 286, 287 }

C grade: { 26, 27, 28, 29, 30, 31, 32, 33 }

F grade: { 34, 35, 36, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335 }

B grade: { 35, 36, 95, 96, 97, 104, 105, 106, 113, 114, 115, 121, 122, 123, 129, 130, 131, 137, 138, 139, 294, 295, 296, 303, 304, 305, 312, 313, 314, 320, 321, 322, 328, 329, 330, 336, 337, 338 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 24, 25, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade: { 12 }

C grade: { 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287 }

F grade: { 34, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.6 Sympy

A grade: { 92, 118, 290, 291, 317 }

B grade: { 1, 2, 3, 4, 9, 10, 11, 35, 36, 91 }

C grade: { }

F grade: { 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 }

B grade: { 35, 36, 91 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 25, 35, 36, 65, 89, 90, 91, 92, 94, 96, 98, 99, 100, 101, 103, 105, 107, 108, 109, 110, 112, 114, 116, 117, 118, 120, 122, 124, 125, 126, 128, 130, 132, 133, 134, 136, 138, 266, 288, 289, 290, 291, 297, 298, 299, 300, 306, 307, 308, 309, 315, 316, 317, 323, 324, 325, 331, 332, 333 }

C grade: { }

F grade: { 16, 17, 18, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 93, 95, 97, 102, 104, 106, 111, 113, 115, 119, 121, 123, 127, 129, 131, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	141	94	75	80	199	93	74
N.S.	1	1.00	1.53	1.02	0.82	0.87	2.16	1.01	0.80
time (sec)	N/A	0.048	0.059	0.319	0.285	0.362	1.268	0.403	0.684

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	109	74	60	63	151	76	59
N.S.	1	1.00	1.51	1.03	0.83	0.88	2.10	1.06	0.82
time (sec)	N/A	0.040	0.038	0.185	0.284	0.363	0.584	0.417	0.664

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	54	43	45	105	57	43
N.S.	1	1.00	1.48	1.08	0.86	0.90	2.10	1.14	0.86
time (sec)	N/A	0.032	0.027	0.142	0.281	0.371	0.281	0.416	0.677

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	50	33	34	28	56	34	28
N.S.	1	1.00	1.67	1.10	1.13	0.93	1.87	1.13	0.93
time (sec)	N/A	0.016	0.022	0.098	0.282	0.373	0.119	0.393	0.043

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	30	38	40	0	40	22
N.S.	1	1.00	1.46	1.25	1.58	1.67	0.00	1.67	0.92
time (sec)	N/A	0.019	0.024	0.143	0.267	0.401	0.000	0.445	0.059

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	55	58	72	0	60	41
N.S.	1	1.00	1.20	1.38	1.45	1.80	0.00	1.50	1.02
time (sec)	N/A	0.023	0.036	0.169	0.286	0.380	0.000	0.451	0.100

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	85	97	95	0	98	77
N.S.	1	1.00	0.77	1.21	1.39	1.36	0.00	1.40	1.10
time (sec)	N/A	0.033	0.153	0.248	0.281	0.365	0.000	0.426	0.736

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	108	126	114	0	121	102
N.S.	1	1.00	0.77	1.10	1.29	1.16	0.00	1.23	1.04
time (sec)	N/A	0.044	0.358	0.298	0.288	0.383	0.000	0.435	0.774

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	93	106	130	85	354	87	119
N.S.	1	1.00	0.79	0.91	1.11	0.73	3.03	0.74	1.02
time (sec)	N/A	0.051	0.178	0.247	0.490	0.382	0.929	0.407	2.107

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	86	103	68	258	68	91
N.S.	1	1.00	0.76	0.97	1.16	0.76	2.90	0.76	1.02
time (sec)	N/A	0.040	0.115	0.171	0.496	0.384	0.415	0.411	1.225

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	73	49	158	43	67
N.S.	1	1.00	0.74	1.07	1.20	0.80	2.59	0.70	1.10
time (sec)	N/A	0.030	0.104	0.116	0.488	0.356	0.183	0.400	0.784

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	20	31	0	20	17
N.S.	1	1.00	1.00	1.40	1.33	2.07	0.00	1.33	1.13
time (sec)	N/A	0.016	0.012	0.141	0.488	0.360	0.000	0.415	0.642

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	27	37	0	34	28
N.S.	1	1.00	0.84	0.81	0.63	0.86	0.00	0.79	0.65
time (sec)	N/A	0.026	0.100	0.184	0.271	0.351	0.000	0.400	0.641

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	43	56	0	57	42
N.S.	1	1.00	0.94	0.89	0.66	0.86	0.00	0.88	0.65
time (sec)	N/A	0.033	0.226	0.235	0.268	0.348	0.000	0.423	0.663

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	60	74	0	79	56
N.S.	1	1.00	0.93	0.90	0.69	0.85	0.00	0.91	0.64
time (sec)	N/A	0.036	0.327	0.205	0.274	0.338	0.000	0.408	0.664

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	128	0	0	-1
N.S.	1	1.00	0.78	2.87	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.442	0.395	0.000	0.128	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	112	0	0	-1
N.S.	1	1.00	0.76	2.62	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.392	0.352	0.000	0.107	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	101	0	0	-1
N.S.	1	1.00	0.91	3.39	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.152	0.348	0.000	0.109	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	92	0	0	94
N.S.	1	1.00	0.77	3.15	0.00	1.23	0.00	0.00	1.25
time (sec)	N/A	0.040	0.163	0.343	0.000	0.115	0.000	0.000	0.324

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	-1
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.164	0.391	0.000	0.101	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	-1
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.235	0.349	0.000	0.102	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	601	0	139	0	0	-1
N.S.	1	1.00	0.70	5.23	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.293	0.730	0.000	0.106	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	77	413	0	135	0	0	-1
N.S.	1	1.00	0.67	3.59	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.420	0.653	0.000	0.110	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	99	0	19	0	0	-1
N.S.	1	1.00	1.10	4.71	0.00	0.90	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.063	0.223	0.000	0.358	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	0	19	0	0	19
N.S.	1	1.00	1.00	4.71	0.00	0.90	0.00	0.00	0.90
time (sec)	N/A	0.016	0.070	0.240	0.000	0.347	0.000	0.000	0.765

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	78	249	0	142	0	0	-1
N.S.	1	1.00	0.68	2.17	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.890	4.176	0.000	0.124	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	79	666	0	147	0	0	-1
N.S.	1	1.00	0.69	5.79	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.522	0.484	0.000	0.110	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	199	0	112	0	0	-1
N.S.	1	1.00	0.74	2.55	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.247	0.395	0.000	0.099	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	55	591	0	95	0	0	-1
N.S.	1	1.00	0.74	7.99	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.166	0.420	0.000	0.113	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	190	0	97	0	0	-1
N.S.	1	1.00	0.77	2.53	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.146	0.385	0.000	0.123	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	609	0	108	0	0	-1
N.S.	1	1.00	0.79	7.91	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.260	0.369	0.000	0.113	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	79	241	0	119	0	0	-1
N.S.	1	1.00	0.69	2.10	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.580	0.667	0.000	0.106	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	636	0	129	0	0	-1
N.S.	1	1.00	0.70	5.53	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.690	0.398	0.000	0.118	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.196	0.281	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	113	0	175	33	279	2494	30
N.S.	1	1.00	3.65	0.00	5.65	1.06	9.00	80.45	0.97
time (sec)	N/A	0.030	0.228	0.571	0.631	0.376	20.843	11.375	1.009

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	119	0	175	32	272	2489	30
N.S.	1	1.00	3.72	0.00	5.47	1.00	8.50	77.78	0.94
time (sec)	N/A	0.034	0.231	0.167	0.636	0.409	20.509	12.002	0.985

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	322	0	122	0	0	-1
N.S.	1	1.00	0.79	2.88	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.263	0.372	0.000	0.134	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	89	294	0	106	0	0	-1
N.S.	1	1.00	0.81	2.67	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.344	0.352	0.000	0.143	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	101	0	0	-1
N.S.	1	1.00	0.91	3.39	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.090	0.000	0.000	0.120	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	59	237	0	89	0	0	-1
N.S.	1	1.00	0.81	3.25	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.152	0.378	0.000	0.125	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	214	0	114	0	0	-1
N.S.	1	1.00	0.80	3.10	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.226	0.418	0.000	0.123	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	56	292	0	113	0	0	-1
N.S.	1	1.00	0.74	3.84	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.221	0.410	0.000	0.115	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	84	598	0	136	0	0	-1
N.S.	1	1.00	0.76	5.44	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.255	0.770	0.000	0.123	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	83	411	0	132	0	0	-1
N.S.	1	1.00	0.73	3.64	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.524	0.708	0.000	0.113	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	91	324	0	124	0	0	-1
N.S.	1	1.00	0.83	2.95	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.205	0.350	0.000	0.115	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	112	0	0	-1
N.S.	1	1.00	0.76	2.62	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.079	0.000	0.000	0.108	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	263	0	104	0	0	-1
N.S.	1	1.00	0.95	3.51	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.079	0.349	0.000	0.113	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	239	0	88	0	0	-1
N.S.	1	1.00	0.80	3.14	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.092	0.344	0.000	0.102	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	216	0	113	0	0	-1
N.S.	1	1.00	0.79	3.00	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.163	0.409	0.000	0.117	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	112	0	0	-1
N.S.	1	1.00	0.74	3.77	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.198	0.464	0.000	0.100	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	84	599	0	141	0	0	-1
N.S.	1	1.00	0.74	5.30	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.230	0.740	0.000	0.115	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	413	0	136	0	0	-1
N.S.	1	1.00	0.72	3.59	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.339	0.718	0.000	0.107	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	128	0	0	-1
N.S.	1	1.00	0.78	2.87	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.078	0.000	0.000	0.116	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	296	0	116	0	0	-1
N.S.	1	1.00	0.78	2.64	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.092	0.384	0.000	0.121	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	263	0	106	0	0	-1
N.S.	1	1.00	0.94	3.37	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.066	0.385	0.000	0.133	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	239	0	90	0	0	-1
N.S.	1	1.00	0.83	3.06	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.178	0.369	0.000	0.108	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	115	0	0	-1
N.S.	1	1.00	0.77	2.92	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.156	0.397	0.000	0.113	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	114	0	0	-1
N.S.	1	1.00	0.74	3.77	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.197	0.390	0.000	0.122	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	601	0	145	0	0	-1
N.S.	1	1.00	0.70	5.23	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.277	0.781	0.000	0.106	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	413	0	140	0	0	-1
N.S.	1	1.00	0.72	3.59	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.438	0.738	0.000	0.115	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	94	349	0	128	0	0	-1
N.S.	1	1.00	0.64	2.37	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.408	0.365	0.000	0.133	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	83	321	0	125	0	0	-1
N.S.	1	1.00	0.72	2.79	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.447	0.407	0.000	0.152	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	77	293	0	109	0	0	-1
N.S.	1	1.00	0.69	2.62	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.253	0.353	0.000	0.107	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	73	260	0	104	0	0	-1
N.S.	1	1.00	0.91	3.25	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.079	0.385	0.000	0.107	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	92	0	0	94
N.S.	1	1.00	0.77	3.15	0.00	1.23	0.00	0.00	1.25
time (sec)	N/A	0.039	0.133	0.000	0.000	0.142	0.000	0.000	0.818

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	200	213	0	117	0	0	-1
N.S.	1	1.00	2.82	3.00	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.347	0.400	0.000	0.109	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	141	291	0	116	0	0	-1
N.S.	1	1.00	1.93	3.99	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.061	1.505	0.398	0.000	0.128	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	522	601	0	139	0	0	-1
N.S.	1	1.00	4.66	5.37	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.080	6.324	0.836	0.000	0.110	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	412	0	135	0	0	-1
N.S.	1	1.00	0.67	3.75	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.415	0.703	0.000	0.103	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	97	729	0	160	0	0	-1
N.S.	1	1.00	0.66	4.96	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.834	1.192	0.000	0.121	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	-1
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.448	0.391	0.000	0.143	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	-1
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.284	0.395	0.000	0.119	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	104	0	0	-1
N.S.	1	1.00	0.86	3.29	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.178	0.378	0.000	0.102	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	92	0	0	-1
N.S.	1	1.00	0.78	3.06	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.144	0.343	0.000	0.098	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	-1
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.146	0.000	0.000	0.129	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	140	294	0	116	0	0	-1
N.S.	1	1.00	1.87	3.92	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.055	1.436	0.398	0.000	0.102	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	81	601	0	139	0	0	-1
N.S.	1	1.00	0.72	5.32	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.324	0.770	0.000	0.102	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	77	413	0	135	0	0	-1
N.S.	1	1.00	0.69	3.69	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.410	0.717	0.000	0.104	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	125	0	0	-1
N.S.	1	1.00	0.75	2.82	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.440	0.405	0.000	0.113	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	109	0	0	-1
N.S.	1	1.00	0.70	2.57	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.271	0.388	0.000	0.104	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	104	0	0	-1
N.S.	1	1.00	0.86	3.29	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.183	0.385	0.000	0.118	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	92	0	0	-1
N.S.	1	1.00	0.78	3.06	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.154	0.344	0.000	0.103	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	117	0	0	-1
N.S.	1	1.00	0.77	2.92	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.151	0.416	0.000	0.097	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	116	0	0	-1
N.S.	1	1.00	0.74	3.77	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.224	0.000	0.000	0.134	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	81	601	0	139	0	0	-1
N.S.	1	1.00	0.72	5.37	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.110	0.762	0.000	0.123	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	77	413	0	135	0	0	-1
N.S.	1	1.00	0.68	3.65	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.343	0.721	0.000	0.114	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	601	0	139	0	0	-1
N.S.	1	1.00	0.70	5.23	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.096	0.000	0.000	0.124	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	77	413	0	135	0	0	-1
N.S.	1	1.00	0.67	3.59	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.107	0.000	0.000	0.111	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	70	70	111	63	0	0	97
N.S.	1	1.00	0.60	0.60	0.96	0.54	0.00	0.00	0.84
time (sec)	N/A	0.039	0.245	2.381	0.628	0.382	0.000	0.000	2.473

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	75	200	0	0	112
N.S.	1	1.00	0.59	0.78	0.66	1.77	0.00	0.00	0.99
time (sec)	N/A	0.037	0.195	0.497	0.620	0.456	0.000	0.000	2.248

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	57	46	139	79987	72
N.S.	1	1.00	0.70	0.64	0.77	0.62	1.88	1080.91	0.97
time (sec)	N/A	0.021	0.099	0.304	0.619	0.360	40.808	6.582	0.952

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	52	162	146	0	45
N.S.	1	1.00	0.58	0.60	0.58	1.80	1.62	0.00	0.50
time (sec)	N/A	0.017	0.084	0.315	0.581	0.461	19.126	0.000	0.434

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	44	55	80	201	0	0	-1
N.S.	1	1.00	0.65	0.81	1.18	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.054	0.296	0.638	0.408	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	80	185	0	0	81
N.S.	1	1.00	0.76	0.76	1.36	3.14	0.00	0.00	1.37
time (sec)	N/A	0.021	0.075	0.271	0.593	0.425	0.000	0.000	1.394

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	59	134	728	213	0	0	-1
N.S.	1	1.00	0.76	1.72	9.33	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.108	0.323	0.649	0.425	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	350	47	0	0	217
N.S.	1	1.00	0.65	0.68	4.43	0.59	0.00	0.00	2.75
time (sec)	N/A	0.032	0.228	0.299	0.630	0.371	0.000	0.000	3.316

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	214	2318	255	0	0	-1
N.S.	1	1.00	0.66	1.75	19.00	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.286	0.320	0.714	0.458	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	70	70	117	69	0	0	98
N.S.	1	1.00	0.59	0.59	0.98	0.58	0.00	0.00	0.82
time (sec)	N/A	0.038	0.254	0.338	0.644	0.392	0.000	0.000	2.240

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	67	88	82	209	0	0	113
N.S.	1	1.00	0.58	0.76	0.71	1.80	0.00	0.00	0.97
time (sec)	N/A	0.035	0.203	0.451	0.627	0.474	0.000	0.000	1.941

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	53	47	60	50	0	0	54
N.S.	1	1.00	0.70	0.62	0.79	0.66	0.00	0.00	0.71
time (sec)	N/A	0.021	0.034	0.291	0.659	0.357	0.000	0.000	0.604

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	52	54	55	165	0	0	46
N.S.	1	1.00	0.56	0.58	0.59	1.77	0.00	0.00	0.49
time (sec)	N/A	0.016	0.102	0.266	0.587	0.412	0.000	0.000	1.043

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	55	83	204	0	0	-1
N.S.	1	1.00	0.63	0.79	1.19	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.063	0.270	0.659	0.429	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	45	80	188	0	0	82
N.S.	1	1.00	0.74	0.74	1.31	3.08	0.00	0.00	1.34
time (sec)	N/A	0.019	0.063	0.240	0.611	0.408	0.000	0.000	1.229

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	59	134	761	216	0	0	-1
N.S.	1	1.00	0.74	1.68	9.51	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.121	0.280	0.656	0.439	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	52	54	355	50	0	0	218
N.S.	1	1.00	0.64	0.67	4.38	0.62	0.00	0.00	2.69
time (sec)	N/A	0.034	0.158	0.253	0.606	0.348	0.000	0.000	2.546

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	81	214	2434	260	0	0	-1
N.S.	1	1.00	0.65	1.71	19.47	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.216	0.298	0.707	0.434	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	70	70	127	75	0	0	100
N.S.	1	1.00	0.56	0.56	1.02	0.60	0.00	0.00	0.80
time (sec)	N/A	0.047	0.286	0.356	0.620	0.374	0.000	0.000	2.160

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	67	88	92	219	0	0	72
N.S.	1	1.00	0.55	0.72	0.75	1.80	0.00	0.00	0.59
time (sec)	N/A	0.038	0.198	0.466	0.619	0.461	0.000	0.000	0.804

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	52	47	64	54	0	0	56
N.S.	1	1.00	0.65	0.59	0.80	0.68	0.00	0.00	0.70
time (sec)	N/A	0.023	0.146	0.265	0.602	0.370	0.000	0.000	0.560

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	52	54	59	171	0	0	48
N.S.	1	1.00	0.53	0.55	0.60	1.73	0.00	0.00	0.48
time (sec)	N/A	0.017	0.116	0.268	0.583	0.400	0.000	0.000	1.035

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	55	87	210	0	0	-1
N.S.	1	1.00	0.59	0.74	1.18	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.086	0.268	0.609	0.409	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	80	194	0	0	84
N.S.	1	1.00	0.69	0.69	1.23	2.98	0.00	0.00	1.29
time (sec)	N/A	0.021	0.086	0.245	0.583	0.441	0.000	0.000	1.186

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	59	134	821	222	0	0	-1
N.S.	1	1.00	0.70	1.60	9.77	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.110	0.289	0.654	0.417	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	367	54	0	0	220
N.S.	1	1.00	0.60	0.64	4.32	0.64	0.00	0.00	2.59
time (sec)	N/A	0.032	0.259	0.254	0.635	0.372	0.000	0.000	2.362

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	2662	270	0	0	-1
N.S.	1	1.00	0.61	1.63	20.32	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.193	0.286	0.697	0.465	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	75	207	0	0	115
N.S.	1	1.00	0.59	0.78	0.66	1.83	0.00	0.00	1.02
time (sec)	N/A	0.035	0.151	0.488	0.620	0.428	0.000	0.000	1.932

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	57	49	0	0	75
N.S.	1	1.00	0.70	0.64	0.77	0.66	0.00	0.00	1.01
time (sec)	N/A	0.021	0.094	0.325	0.620	0.368	0.000	0.000	0.950

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	52	169	146	0	81
N.S.	1	1.00	0.58	0.60	0.58	1.88	1.62	0.00	0.90
time (sec)	N/A	0.017	0.071	0.325	0.588	0.416	23.828	0.000	1.486

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	44	55	80	207	0	0	-1
N.S.	1	1.00	0.65	0.81	1.18	3.04	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.051	0.320	0.601	0.440	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	85	191	0	0	84
N.S.	1	1.00	0.76	0.76	1.44	3.24	0.00	0.00	1.42
time (sec)	N/A	0.020	0.054	0.286	0.574	0.426	0.000	0.000	1.254

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	59	134	728	219	0	0	-1
N.S.	1	1.00	0.76	1.72	9.33	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.085	0.317	0.619	0.420	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	355	50	0	0	220
N.S.	1	1.00	0.65	0.68	4.49	0.63	0.00	0.00	2.78
time (sec)	N/A	0.031	0.139	0.303	0.623	0.364	0.000	0.000	2.827

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	80	214	2318	261	0	0	-1
N.S.	1	1.00	0.66	1.75	19.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.194	0.330	0.669	0.407	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	67	88	75	207	0	0	115
N.S.	1	1.00	0.55	0.72	0.61	1.70	0.00	0.00	0.94
time (sec)	N/A	0.037	0.154	0.452	0.646	0.422	0.000	0.000	1.961

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	52	47	57	49	0	0	75
N.S.	1	1.00	0.65	0.59	0.71	0.61	0.00	0.00	0.94
time (sec)	N/A	0.022	0.105	0.281	0.616	0.367	0.000	0.000	0.843

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	52	54	52	169	0	0	81
N.S.	1	1.00	0.53	0.55	0.53	1.71	0.00	0.00	0.82
time (sec)	N/A	0.017	0.083	0.284	0.579	0.503	0.000	0.000	0.704

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	55	80	207	0	0	-1
N.S.	1	1.00	0.59	0.74	1.08	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.055	0.289	0.598	0.432	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	93	191	0	0	84
N.S.	1	1.00	0.69	0.69	1.43	2.94	0.00	0.00	1.29
time (sec)	N/A	0.020	0.063	0.266	0.572	0.450	0.000	0.000	1.240

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	59	134	736	219	0	0	-1
N.S.	1	1.00	0.70	1.60	8.76	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.081	0.283	0.625	0.410	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	380	50	0	0	220
N.S.	1	1.00	0.60	0.64	4.47	0.59	0.00	0.00	2.59
time (sec)	N/A	0.033	0.153	0.262	0.623	0.369	0.000	0.000	2.421

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	2350	261	0	0	-1
N.S.	1	1.00	0.61	1.63	17.94	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.131	0.282	0.648	0.440	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	70	88	75	207	0	0	115
N.S.	1	1.00	0.57	0.72	0.61	1.70	0.00	0.00	0.94
time (sec)	N/A	0.037	0.156	0.447	0.601	0.414	0.000	0.000	1.971

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	55	47	57	49	0	0	75
N.S.	1	1.00	0.69	0.59	0.71	0.61	0.00	0.00	0.94
time (sec)	N/A	0.022	0.064	0.275	0.622	0.395	0.000	0.000	0.839

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	55	54	52	169	0	0	81
N.S.	1	1.00	0.56	0.55	0.53	1.71	0.00	0.00	0.82
time (sec)	N/A	0.017	0.067	0.289	0.597	0.465	0.000	0.000	0.722

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	47	55	80	207	0	0	-1
N.S.	1	1.00	0.64	0.74	1.08	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.053	0.291	0.599	0.433	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	93	191	0	0	117
N.S.	1	1.00	0.69	0.69	1.43	2.94	0.00	0.00	1.80
time (sec)	N/A	0.020	0.059	0.271	0.583	0.397	0.000	0.000	1.900

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	59	135	754	219	0	0	-1
N.S.	1	1.00	0.70	1.61	8.98	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.080	0.309	0.625	0.411	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	412	50	0	0	220
N.S.	1	1.00	0.60	0.64	4.85	0.59	0.00	0.00	2.59
time (sec)	N/A	0.032	0.142	0.258	0.619	0.403	0.000	0.000	2.493

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	2418	261	0	0	-1
N.S.	1	1.00	0.61	1.63	18.46	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.166	0.286	0.648	0.448	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.133	0.316	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.183	0.293	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.118	0.240	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.137	0.296	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.181	0.313	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	96	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.131	0.326	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.125	0.318	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.177	0.301	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.122	0.229	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	88	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.133	0.287	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.181	0.303	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	96	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.137	0.330	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.272	0.326	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.170	0.306	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.115	0.248	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.030	0.303	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.044	0.320	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.158	0.332	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.116	0.228	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.038	0.378	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	87	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.032	0.219	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	283	0	0	0	0	0	-1
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	3.831	0.263	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	101	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.840	0.356	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	481	0	0	0	0	0	-1
N.S.	1	1.00	5.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	6.308	0.361	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.137	0.233	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.132	0.367	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.117	0.212	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	277	0	0	0	0	0	-1
N.S.	1	1.00	3.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	3.978	0.269	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	103	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.797	0.362	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	473	0	0	0	0	0	-1
N.S.	1	1.00	5.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	6.298	0.293	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.209	0.235	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.120	0.362	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.151	0.210	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	104	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.651	0.270	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.226	0.359	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.226	0.299	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	138	142	0	0	0	0	0	-1
N.S.	1	0.93	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.312	0.285	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	-1
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.240	0.277	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	-1
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.334	0.257	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0	-1
N.S.	1	0.93	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.313	0.263	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	142	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.274	0.262	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	139	142	0	0	0	0	0	-1
N.S.	1	0.93	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.286	0.257	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.270	0.607	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.218	0.208	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	120	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.204	0.214	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.182	0.152	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	111	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.250	0.214	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	117	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.222	0.197	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	114	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.182	0.200	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	122	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.189	0.181	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	-1
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.245	0.227	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	-1
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.222	0.242	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	-1
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.198	0.217	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.197	0.270	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	140	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.190	0.255	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	132	140	0	0	0	0	0	-1
N.S.	1	0.94	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.191	0.213	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	-1
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.205	0.214	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0	-1
N.S.	1	0.93	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.199	0.217	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	238	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	1.654	0.218	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	175	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.899	0.197	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	240	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.930	0.191	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	144	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.482	0.176	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.160	0.182	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	279	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	2.783	0.164	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	276	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	2.746	0.156	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	256	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	1.817	0.147	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	256	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	2.028	0.140	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	119	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	0.393	0.211	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	10805	0	0	0	0	0	-1
N.S.	1	1.00	37.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	26.517	0.204	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.285	0.252	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.414	0.255	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.402	0.211	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	140	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.574	0.229	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.467	0.212	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.451	0.217	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.387	0.211	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	136	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.371	0.458	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.492	0.263	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.298	0.233	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.251	0.252	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.179	0.273	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	109	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.206	0.283	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	109	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.190	0.262	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	118	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.183	0.257	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.290	0.276	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.430	0.287	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.354	0.279	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.262	0.267	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.250	0.296	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.274	0.263	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.232	0.262	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.236	0.285	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	356	0	0	0	0	0	-1
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	44.931	0.263	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	13441	0	0	0	0	0	-1
N.S.	1	1.00	45.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	26.661	0.257	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	290	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	3.039	0.194	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	289	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	2.983	0.188	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	263	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	2.201	0.146	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	261	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	2.179	0.142	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	142	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.313	0.241	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	125	382	0	191	0	0	-1
N.S.	1	1.00	0.60	1.83	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.161	1.119	0.417	0.000	0.146	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	111	351	0	177	0	0	-1
N.S.	1	1.00	0.62	1.95	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.838	0.411	0.000	0.131	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	94	317	0	163	0	0	-1
N.S.	1	1.00	0.65	2.19	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.382	0.400	0.000	0.115	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	83	283	0	149	0	0	-1
N.S.	1	1.00	0.74	2.53	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.242	0.389	0.000	0.132	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	260	0	180	0	0	-1
N.S.	1	1.00	0.72	2.39	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.315	0.402	0.000	0.125	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	90	505	0	199	0	0	-1
N.S.	1	1.00	0.64	3.61	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.423	0.709	0.000	0.114	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	122	804	0	220	0	0	-1
N.S.	1	1.00	0.67	4.44	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.643	0.926	0.000	0.117	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	143	725	0	231	0	0	-1
N.S.	1	1.00	0.68	3.45	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.185	1.040	1.194	0.000	0.124	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	128	384	0	195	0	0	-1
N.S.	1	1.00	0.61	1.83	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.153	0.707	0.387	0.000	0.149	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	108	353	0	183	0	0	-1
N.S.	1	1.00	0.60	1.95	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.107	0.358	0.000	0.129	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	95	319	0	165	0	0	-1
N.S.	1	1.00	0.65	2.18	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.268	0.351	0.000	0.112	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	85	285	0	148	0	0	-1
N.S.	1	1.00	0.73	2.46	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.184	0.349	0.000	0.117	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	80	262	0	179	0	0	-1
N.S.	1	1.00	0.70	2.30	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.291	0.439	0.000	0.123	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	92	506	0	200	0	0	-1
N.S.	1	1.00	0.63	3.49	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.346	0.653	0.000	0.111	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	122	805	0	223	0	0	-1
N.S.	1	1.00	0.66	4.33	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.490	0.938	0.000	0.121	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	134	727	0	235	0	0	-1
N.S.	1	1.00	0.62	3.38	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.180	1.830	1.246	0.000	0.131	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	125	384	0	203	0	0	-1
N.S.	1	1.00	0.59	1.81	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.141	0.478	0.369	0.000	0.159	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	109	353	0	189	0	0	-1
N.S.	1	1.00	0.60	1.93	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.123	0.370	0.000	0.128	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	319	0	169	0	0	-1
N.S.	1	1.00	0.64	2.11	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.203	0.395	0.000	0.148	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	79	285	0	150	0	0	-1
N.S.	1	1.00	0.66	2.38	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.219	0.351	0.000	0.126	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	262	0	181	0	0	-1
N.S.	1	1.00	0.69	2.26	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.311	0.448	0.000	0.110	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	92	508	0	204	0	0	-1
N.S.	1	1.00	0.63	3.46	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.332	0.655	0.000	0.124	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	121	807	0	229	0	0	-1
N.S.	1	1.00	0.64	4.29	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.442	1.008	0.000	0.125	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	134	727	0	243	0	0	-1
N.S.	1	1.00	0.62	3.35	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.824	1.152	0.000	0.122	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	127	381	0	194	0	0	-1
N.S.	1	1.00	0.59	1.78	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.710	0.422	0.000	0.144	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	108	350	0	180	0	0	-1
N.S.	1	1.00	0.58	1.89	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.686	0.404	0.000	0.127	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	97	316	0	166	0	0	-1
N.S.	1	1.00	0.65	2.11	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.324	0.366	0.000	0.109	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	282	0	152	0	0	128
N.S.	1	1.00	0.70	2.41	0.00	1.30	0.00	0.00	1.09
time (sec)	N/A	0.088	0.099	0.332	0.000	0.109	0.000	0.000	0.386

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	803	259	0	183	0	0	-1
N.S.	1	1.00	7.30	2.35	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.109	6.330	0.411	0.000	0.112	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	757	508	0	202	0	0	-1
N.S.	1	1.00	5.45	3.65	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.138	6.355	0.730	0.000	0.111	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	116	807	0	223	0	0	-1
N.S.	1	1.00	0.64	4.48	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.487	1.058	0.000	0.114	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	133	726	0	234	0	0	-1
N.S.	1	1.00	0.64	3.47	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.176	0.750	1.147	0.000	0.123	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	194	0	0	-1
N.S.	1	1.00	0.60	1.77	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.760	0.369	0.000	0.129	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	108	353	0	180	0	0	-1
N.S.	1	1.00	0.57	1.88	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.793	0.385	0.000	0.122	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	94	319	0	166	0	0	-1
N.S.	1	1.00	0.61	2.08	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.397	0.368	0.000	0.115	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	152	0	0	-1
N.S.	1	1.00	0.71	2.38	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.205	0.354	0.000	0.116	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	80	262	0	183	0	0	-1
N.S.	1	1.00	0.69	2.26	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.200	0.381	0.000	0.129	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	761	508	0	202	0	0	-1
N.S.	1	1.00	5.28	3.53	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.134	6.309	0.721	0.000	0.122	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	119	807	0	223	0	0	-1
N.S.	1	1.00	0.65	4.41	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.503	1.022	0.000	0.119	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	136	729	0	234	0	0	-1
N.S.	1	1.00	0.64	3.44	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.768	1.187	0.000	0.130	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	194	0	0	-1
N.S.	1	1.00	0.60	1.77	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.746	0.426	0.000	0.147	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	111	353	0	180	0	0	-1
N.S.	1	1.00	0.59	1.88	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.640	0.408	0.000	0.124	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	97	319	0	166	0	0	-1
N.S.	1	1.00	0.63	2.08	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.328	0.399	0.000	0.121	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	152	0	0	-1
N.S.	1	1.00	0.71	2.38	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.196	0.354	0.000	0.104	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	807	262	0	183	0	0	-1
N.S.	1	1.00	6.96	2.26	0.00	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.106	6.235	0.393	0.000	0.121	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	92	508	0	202	0	0	-1
N.S.	1	1.00	0.63	3.46	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.415	0.700	0.000	0.117	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	119	807	0	223	0	0	-1
N.S.	1	1.00	0.64	4.36	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.459	0.972	0.000	0.133	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	136	729	0	234	0	0	-1
N.S.	1	1.00	0.64	3.44	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.523	1.149	0.000	0.151	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	119	807	0	223	0	0	-1
N.S.	1	1.00	0.63	4.29	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.132	1.008	0.000	0.129	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	109	134	159	292	0	0	141
N.S.	1	1.00	0.49	0.60	0.71	1.31	0.00	0.00	0.63
time (sec)	N/A	0.093	0.313	3.019	0.669	0.423	0.000	0.000	3.640

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	116	276	0	0	137
N.S.	1	1.00	0.50	0.62	0.63	1.50	0.00	0.00	0.74
time (sec)	N/A	0.075	0.293	0.353	0.675	0.420	0.000	0.000	2.806

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	80	236	241	0	104
N.S.	1	1.00	0.52	0.58	0.56	1.65	1.69	0.00	0.73
time (sec)	N/A	0.041	0.205	0.261	0.661	0.404	41.496	0.000	1.399

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	64	212	184	0	54
N.S.	1	1.00	0.50	0.51	0.52	1.72	1.50	0.00	0.44
time (sec)	N/A	0.024	0.116	0.235	0.637	0.412	19.571	0.000	0.564

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	63	104	304	0	0	-1
N.S.	1	1.00	1.00	0.68	1.12	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.114	0.211	0.630	0.472	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	60	72	144	312	0	0	-1
N.S.	1	1.00	0.65	0.77	1.55	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.082	0.201	0.638	0.453	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	69	151	780	233	0	0	-1
N.S.	1	1.00	0.62	1.36	7.03	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.158	0.224	0.687	0.423	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	87	157	1009	265	0	0	-1
N.S.	1	1.00	0.57	1.03	6.64	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.469	0.240	0.728	0.426	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	110	248	2611	299	0	0	-1
N.S.	1	1.00	0.57	1.28	13.53	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.373	0.214	0.776	0.459	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	109	134	169	309	0	0	142
N.S.	1	1.00	0.48	0.59	0.74	1.35	0.00	0.00	0.62
time (sec)	N/A	0.090	0.346	0.195	0.709	0.429	0.000	0.000	3.151

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	92	114	126	285	0	0	138
N.S.	1	1.00	0.49	0.60	0.67	1.51	0.00	0.00	0.73
time (sec)	N/A	0.075	0.243	0.342	0.732	0.412	0.000	0.000	1.768

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	76	83	86	249	0	0	71
N.S.	1	1.00	0.52	0.56	0.59	1.69	0.00	0.00	0.48
time (sec)	N/A	0.042	0.085	0.259	0.664	0.418	0.000	0.000	0.812

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	61	63	67	217	0	0	55
N.S.	1	1.00	0.48	0.50	0.53	1.71	0.00	0.00	0.43
time (sec)	N/A	0.023	0.127	0.214	0.652	0.435	0.000	0.000	1.169

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	93	63	107	308	0	0	-1
N.S.	1	1.00	0.97	0.66	1.11	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.144	0.197	0.660	0.461	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	60	72	147	316	0	0	-1
N.S.	1	1.00	0.62	0.75	1.53	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.081	0.184	0.659	0.462	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	69	151	813	240	0	0	-1
N.S.	1	1.00	0.61	1.32	7.13	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.145	0.201	0.719	0.402	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	88	157	1044	272	0	0	-1
N.S.	1	1.00	0.56	1.01	6.69	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.060	0.221	0.737	0.434	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	111	248	2732	308	0	0	-1
N.S.	1	1.00	0.56	1.25	13.80	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.270	0.211	0.800	0.480	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	109	134	185	331	0	0	144
N.S.	1	1.00	0.45	0.56	0.77	1.37	0.00	0.00	0.60
time (sec)	N/A	0.090	0.299	0.203	0.710	0.444	0.000	0.000	3.039

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	92	114	140	303	0	0	94
N.S.	1	1.00	0.46	0.57	0.70	1.52	0.00	0.00	0.47
time (sec)	N/A	0.078	0.353	0.336	0.677	0.422	0.000	0.000	1.066

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	75	83	94	263	0	0	73
N.S.	1	1.00	0.48	0.54	0.61	1.70	0.00	0.00	0.47
time (sec)	N/A	0.042	0.274	0.231	0.668	0.420	0.000	0.000	0.719

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	61	63	71	227	0	0	57
N.S.	1	1.00	0.45	0.47	0.53	1.68	0.00	0.00	0.42
time (sec)	N/A	0.025	0.158	0.201	0.642	0.399	0.000	0.000	1.228

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	93	63	111	316	0	0	-1
N.S.	1	1.00	0.91	0.62	1.09	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.165	0.186	0.633	0.443	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	151	324	0	0	-1
N.S.	1	1.00	0.59	0.71	1.48	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.130	0.181	0.619	0.479	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	69	151	873	250	0	0	-1
N.S.	1	1.00	0.58	1.26	7.28	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.137	0.201	0.694	0.464	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1112	286	0	0	-1
N.S.	1	1.00	0.53	0.96	6.78	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.391	0.232	0.684	0.458	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	2972	326	0	0	-1
N.S.	1	1.00	0.53	1.19	14.29	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.084	0.376	0.205	0.770	0.425	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	116	282	0	0	140
N.S.	1	1.00	0.50	0.62	0.63	1.53	0.00	0.00	0.76
time (sec)	N/A	0.075	0.253	0.362	0.706	0.441	0.000	0.000	2.774

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	80	242	0	0	107
N.S.	1	1.00	0.52	0.58	0.56	1.69	0.00	0.00	0.75
time (sec)	N/A	0.040	0.204	0.322	0.679	0.420	0.000	0.000	1.413

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	64	218	184	0	93
N.S.	1	1.00	0.50	0.51	0.52	1.77	1.50	0.00	0.76
time (sec)	N/A	0.023	0.088	0.249	0.675	0.420	23.322	0.000	1.012

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	63	104	309	0	0	-1
N.S.	1	1.00	1.00	0.68	1.12	3.32	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.112	0.234	0.680	0.450	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	60	72	149	317	0	0	-1
N.S.	1	1.00	0.65	0.77	1.60	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.071	0.222	0.668	0.439	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	69	150	785	239	0	0	-1
N.S.	1	1.00	0.62	1.35	7.07	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.112	0.244	0.709	0.432	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	87	157	1014	271	0	0	-1
N.S.	1	1.00	0.57	1.03	6.67	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.189	0.272	0.715	0.415	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	110	248	2611	305	0	0	-1
N.S.	1	1.00	0.57	1.28	13.53	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.238	0.250	0.726	0.452	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	92	114	116	282	0	0	140
N.S.	1	1.00	0.46	0.57	0.58	1.42	0.00	0.00	0.70
time (sec)	N/A	0.081	0.187	0.318	0.693	0.456	0.000	0.000	2.603

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	75	83	80	242	0	0	107
N.S.	1	1.00	0.48	0.54	0.52	1.56	0.00	0.00	0.69
time (sec)	N/A	0.043	0.190	0.243	0.710	0.470	0.000	0.000	1.148

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	61	63	64	218	0	0	93
N.S.	1	1.00	0.45	0.47	0.47	1.61	0.00	0.00	0.69
time (sec)	N/A	0.024	0.110	0.224	0.668	0.460	0.000	0.000	0.830

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	93	63	104	309	0	0	-1
N.S.	1	1.00	0.91	0.62	1.02	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.121	0.225	0.649	0.549	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	157	317	0	0	-1
N.S.	1	1.00	0.59	0.71	1.54	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.083	0.221	0.659	0.445	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	69	151	802	239	0	0	-1
N.S.	1	1.00	0.58	1.26	6.68	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.099	0.209	0.727	0.438	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1048	271	0	0	-1
N.S.	1	1.00	0.53	0.96	6.39	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.173	0.229	0.701	0.445	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	2660	305	0	0	-1
N.S.	1	1.00	0.53	1.19	12.79	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.085	0.231	0.220	0.738	0.424	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	95	114	116	282	0	0	140
N.S.	1	1.00	0.48	0.57	0.58	1.42	0.00	0.00	0.70
time (sec)	N/A	0.076	0.180	0.327	0.707	0.426	0.000	0.000	2.337

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	78	83	80	242	0	0	107
N.S.	1	1.00	0.50	0.54	0.52	1.56	0.00	0.00	0.69
time (sec)	N/A	0.044	0.141	0.245	0.683	0.441	0.000	0.000	1.160

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	64	63	64	218	0	0	93
N.S.	1	1.00	0.47	0.47	0.47	1.61	0.00	0.00	0.69
time (sec)	N/A	0.025	0.088	0.219	0.622	0.443	0.000	0.000	0.853

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	63	104	309	0	0	-1
N.S.	1	1.00	0.94	0.62	1.02	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.108	0.208	0.627	0.448	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	157	317	0	0	-1
N.S.	1	1.00	0.59	0.71	1.54	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.073	0.211	0.633	0.463	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	69	151	820	239	0	0	-1
N.S.	1	1.00	0.58	1.26	6.83	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.100	0.212	0.699	0.419	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1098	271	0	0	-1
N.S.	1	1.00	0.53	0.96	6.70	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.243	0.228	0.712	0.427	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	2760	305	0	0	-1
N.S.	1	1.00	0.53	1.19	13.27	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.086	0.237	0.208	0.721	0.445	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.283	0.240	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.260	0.206	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.252	0.266	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	116	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.390	0.302	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	123	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.261	0.283	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.221	0.288	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	111	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.388	0.237	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.248	0.207	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	109	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.255	0.257	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	108	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.232	0.275	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.248	0.307	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.223	0.304	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.292	0.171	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.193	0.171	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	108	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.142	0.171	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	779	0	0	0	0	0	-1
N.S.	1	1.00	5.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	6.318	0.278	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	699	0	0	0	0	0	-1
N.S.	1	1.00	4.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	6.361	0.306	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.324	0.253	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.380	0.204	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.246	0.179	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	111	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.239	0.207	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.247	0.161	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	703	0	0	0	0	0	-1
N.S.	1	1.00	4.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	6.326	0.286	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.375	0.353	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	222	169	0	0	0	0	0	-1
N.S.	1	0.96	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.715	0.260	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0	-1
N.S.	1	0.96	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.464	0.229	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0	-1
N.S.	1	0.96	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.442	0.225	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0	-1
N.S.	1	0.96	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	0.467	0.217	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	164	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.444	0.280	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	225	166	0	0	0	0	0	-1
N.S.	1	0.96	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.586	0.224	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	161	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	0.280	0.287	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.523	0.338	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.363	0.263	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	142	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.262	0.219	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	127	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.225	0.322	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	131	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.323	0.266	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	137	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.507	0.269	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	142	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.356	0.271	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0	-1
N.S.	1	0.96	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.566	0.304	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0	-1
N.S.	1	0.96	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.465	0.287	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	162	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.416	0.331	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	157	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	0.486	0.324	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	163	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	0.448	0.286	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0	-1
N.S.	1	0.96	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.460	0.284	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	376	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	2.799	0.267	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	137	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.954	0.245	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.132	0.230	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	105	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.660	0.224	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.218	0.243	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	296	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	3.634	0.199	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	294	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	3.764	0.202	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	268	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	2.612	0.203	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	266	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	2.608	0.200	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	3.671	0.466	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	16142	0	0	0	0	0	-1
N.S.	1	1.00	53.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	27.177	0.232	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [288] had the largest ratio of [43]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	21	0.095
2	A	3	2	1.00	21	0.095
3	A	3	2	1.00	21	0.095
4	A	2	1	1.00	19	0.053
5	A	2	2	1.00	19	0.105
6	A	2	2	1.00	21	0.095
7	A	3	3	1.00	21	0.143
8	A	4	3	1.00	21	0.143
9	A	5	3	1.00	21	0.143
10	A	4	3	1.00	21	0.143
11	A	3	3	1.00	21	0.143
12	A	2	2	1.00	21	0.095
13	A	3	3	1.00	21	0.143
14	A	3	2	1.00	21	0.095
15	A	3	2	1.00	21	0.095
16	A	4	4	1.00	25	0.160
17	A	4	4	1.00	25	0.160
18	A	3	3	1.00	25	0.120
19	A	3	3	1.00	25	0.120
20	A	3	3	1.00	25	0.120
21	A	3	3	1.00	25	0.120
22	A	4	4	1.00	25	0.160
23	A	4	4	1.00	25	0.160
24	A	1	1	1.00	23	0.043
25	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	5	1.00	25	0.200
27	A	5	5	1.00	25	0.200
28	A	4	4	1.00	25	0.160
29	A	4	4	1.00	25	0.160
30	A	4	4	1.00	25	0.160
31	A	4	4	1.00	25	0.160
32	A	5	5	1.00	25	0.200
33	A	5	5	1.00	25	0.200
34	A	2	2	1.00	23	0.087
35	A	1	1	1.00	33	0.030
36	A	1	1	1.00	32	0.031
37	A	5	5	1.00	33	0.152
38	A	5	5	1.00	31	0.161
39	A	3	3	1.00	25	0.120
40	A	4	4	1.00	31	0.129
41	A	4	4	1.00	33	0.121
42	A	4	4	1.00	33	0.121
43	A	5	5	1.00	33	0.152
44	A	5	5	1.00	33	0.152
45	A	5	5	1.00	31	0.161
46	A	4	4	1.00	25	0.160
47	A	4	4	1.00	31	0.129
48	A	4	4	1.00	33	0.121
49	A	4	4	1.00	33	0.121
50	A	4	4	1.00	33	0.121
51	A	5	5	1.00	33	0.152
52	A	5	5	1.00	33	0.152
53	A	4	4	1.00	25	0.160
54	A	5	5	1.00	31	0.161
55	A	4	4	1.00	33	0.121
56	A	4	4	1.00	33	0.121
57	A	4	4	1.00	33	0.121
58	A	4	4	1.00	33	0.121
59	A	5	5	1.00	33	0.152
60	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	6	5	1.00	33	0.152
62	A	5	5	1.00	33	0.152
63	A	5	5	1.00	33	0.152
64	A	4	4	1.00	31	0.129
65	A	3	3	1.00	25	0.120
66	A	4	4	1.00	31	0.129
67	A	4	4	1.00	33	0.121
68	A	5	5	1.00	33	0.152
69	A	5	5	1.00	33	0.152
70	A	6	5	1.00	33	0.152
71	A	5	5	1.00	33	0.152
72	A	5	5	1.00	33	0.152
73	A	4	4	1.00	33	0.121
74	A	4	4	1.00	31	0.129
75	A	3	3	1.00	25	0.120
76	A	4	4	1.00	31	0.129
77	A	5	5	1.00	33	0.152
78	A	5	5	1.00	33	0.152
79	A	5	5	1.00	33	0.152
80	A	5	5	1.00	33	0.152
81	A	4	4	1.00	33	0.121
82	A	4	4	1.00	33	0.121
83	A	4	4	1.00	31	0.129
84	A	3	3	1.00	25	0.120
85	A	5	5	1.00	31	0.161
86	A	5	5	1.00	33	0.152
87	A	4	4	1.00	25	0.160
88	A	4	4	1.00	25	0.160
89	A	4	3	1.00	35	0.086
90	A	4	4	1.00	35	0.114
91	A	3	2	1.00	35	0.057
92	A	4	3	1.00	35	0.086
93	A	3	3	1.00	35	0.086
94	A	3	3	1.00	35	0.086
95	A	3	3	1.00	35	0.086

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	35	0.114
97	A	4	4	1.00	35	0.114
98	A	4	3	1.00	35	0.086
99	A	4	4	1.00	35	0.114
100	A	3	2	1.00	35	0.057
101	A	4	3	1.00	35	0.086
102	A	3	3	1.00	35	0.086
103	A	3	3	1.00	35	0.086
104	A	3	3	1.00	35	0.086
105	A	4	4	1.00	35	0.114
106	A	4	4	1.00	35	0.114
107	A	4	3	1.00	35	0.086
108	A	4	4	1.00	35	0.114
109	A	3	2	1.00	35	0.057
110	A	4	3	1.00	35	0.086
111	A	3	3	1.00	35	0.086
112	A	3	3	1.00	35	0.086
113	A	3	3	1.00	35	0.086
114	A	4	4	1.00	35	0.114
115	A	4	4	1.00	35	0.114
116	A	4	4	1.00	35	0.114
117	A	3	2	1.00	35	0.057
118	A	4	3	1.00	35	0.086
119	A	3	3	1.00	35	0.086
120	A	3	3	1.00	35	0.086
121	A	3	3	1.00	35	0.086
122	A	4	4	1.00	35	0.114
123	A	4	4	1.00	35	0.114
124	A	4	4	1.00	35	0.114
125	A	3	2	1.00	35	0.057
126	A	4	3	1.00	35	0.086
127	A	3	3	1.00	35	0.086
128	A	3	3	1.00	35	0.086
129	A	3	3	1.00	35	0.086
130	A	4	4	1.00	35	0.114

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	35	0.114
132	A	4	4	1.00	35	0.114
133	A	3	2	1.00	35	0.057
134	A	4	3	1.00	35	0.086
135	A	3	3	1.00	35	0.086
136	A	3	3	1.00	35	0.086
137	A	3	3	1.00	35	0.086
138	A	4	4	1.00	35	0.114
139	A	4	4	1.00	35	0.114
140	A	3	3	1.00	33	0.091
141	A	3	3	1.00	31	0.097
142	A	2	2	1.00	25	0.080
143	A	3	3	1.00	31	0.097
144	A	3	3	1.00	33	0.091
145	A	3	3	1.00	33	0.091
146	A	3	3	1.00	33	0.091
147	A	3	3	1.00	31	0.097
148	A	2	2	1.00	25	0.080
149	A	3	3	1.00	31	0.097
150	A	3	3	1.00	33	0.091
151	A	3	3	1.00	33	0.091
152	A	3	3	1.00	33	0.091
153	A	3	3	1.00	31	0.097
154	A	2	2	1.00	25	0.080
155	A	3	3	1.00	31	0.097
156	A	3	3	1.00	33	0.091
157	A	3	3	1.00	33	0.091
158	A	3	3	1.00	33	0.091
159	A	3	3	1.00	31	0.097
160	A	2	2	1.00	25	0.080
161	A	3	3	1.00	31	0.097
162	A	3	3	1.00	33	0.091
163	A	3	3	1.00	33	0.091
164	A	3	3	1.00	33	0.091
165	A	3	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	25	0.080
167	A	3	3	1.00	31	0.097
168	A	3	3	1.00	33	0.091
169	A	3	3	1.00	33	0.091
170	A	3	3	1.00	33	0.091
171	A	3	3	1.00	31	0.097
172	A	2	2	1.00	25	0.080
173	A	3	3	1.00	31	0.097
174	A	3	3	1.00	33	0.091
175	A	3	3	1.00	33	0.091
176	A	3	3	0.93	33	0.091
177	A	3	3	0.93	33	0.091
178	A	3	3	0.93	33	0.091
179	A	3	3	0.93	33	0.091
180	A	3	3	1.00	33	0.091
181	A	3	3	0.93	33	0.091
182	A	3	3	1.00	33	0.091
183	A	3	3	1.00	31	0.097
184	A	3	3	1.00	29	0.103
185	A	2	2	1.00	23	0.087
186	A	3	3	1.00	29	0.103
187	A	3	3	1.00	31	0.097
188	A	3	3	1.00	31	0.097
189	A	3	3	1.00	31	0.097
190	A	3	3	0.93	33	0.091
191	A	3	3	0.93	33	0.091
192	A	3	3	0.93	33	0.091
193	A	3	3	1.00	33	0.091
194	A	3	3	1.00	33	0.091
195	A	3	3	0.94	33	0.091
196	A	3	3	0.93	33	0.091
197	A	3	3	0.93	33	0.091
198	A	4	4	1.00	25	0.160
199	A	4	4	1.00	27	0.148
200	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.00	27	0.148
202	A	4	4	1.00	27	0.148
203	A	8	5	1.00	27	0.185
204	A	8	5	1.00	27	0.185
205	A	8	5	1.00	27	0.185
206	A	8	5	1.00	27	0.185
207	A	7	5	1.00	26	0.192
208	A	8	5	1.00	25	0.200
209	A	4	3	1.00	30	0.100
210	A	5	4	1.00	40	0.100
211	A	5	4	1.00	40	0.100
212	A	5	4	1.00	40	0.100
213	A	5	4	1.00	40	0.100
214	A	5	4	1.00	40	0.100
215	A	5	4	1.00	40	0.100
216	A	5	4	1.00	40	0.100
217	A	5	4	1.00	38	0.105
218	A	5	4	1.00	36	0.111
219	A	4	3	1.00	30	0.100
220	A	5	4	1.00	36	0.111
221	A	5	4	1.00	38	0.105
222	A	5	4	1.00	38	0.105
223	A	5	4	1.00	38	0.105
224	A	5	4	1.00	40	0.100
225	A	5	4	1.00	40	0.100
226	A	5	4	1.00	40	0.100
227	A	5	4	1.00	40	0.100
228	A	5	4	1.00	40	0.100
229	A	5	4	1.00	40	0.100
230	A	5	4	1.00	40	0.100
231	A	5	4	1.00	40	0.100
232	A	4	4	1.00	32	0.125
233	A	8	5	1.00	32	0.156
234	A	8	5	1.00	34	0.147
235	A	8	5	1.00	34	0.147

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	8	5	1.00	34	0.147
237	A	8	5	1.00	34	0.147
238	A	4	3	1.00	31	0.097
239	A	10	7	1.00	41	0.171
240	A	9	7	1.00	39	0.180
241	A	7	6	1.00	33	0.182
242	A	7	6	1.00	39	0.154
243	A	7	6	1.00	41	0.146
244	A	8	7	1.00	41	0.171
245	A	9	7	1.00	41	0.171
246	A	10	7	1.00	41	0.171
247	A	10	7	1.00	39	0.180
248	A	8	6	1.00	33	0.182
249	A	8	7	1.00	39	0.180
250	A	7	6	1.00	41	0.146
251	A	7	6	1.00	41	0.146
252	A	8	7	1.00	41	0.171
253	A	9	7	1.00	41	0.171
254	A	10	7	1.00	41	0.171
255	A	9	6	1.00	33	0.182
256	A	9	7	1.00	39	0.180
257	A	8	7	1.00	41	0.171
258	A	7	6	1.00	41	0.146
259	A	7	6	1.00	41	0.146
260	A	8	7	1.00	41	0.171
261	A	9	7	1.00	41	0.171
262	A	10	7	1.00	41	0.171
263	A	10	7	1.00	41	0.171
264	A	9	7	1.00	41	0.171
265	A	8	7	1.00	39	0.180
266	A	6	5	1.00	33	0.152
267	A	7	6	1.00	39	0.154
268	A	8	7	1.00	41	0.171
269	A	9	7	1.00	41	0.171
270	A	10	7	1.00	41	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	10	7	1.00	41	0.171
272	A	9	7	1.00	41	0.171
273	A	8	7	1.00	41	0.171
274	A	7	6	1.00	39	0.154
275	A	6	5	1.00	33	0.152
276	A	8	7	1.00	39	0.180
277	A	9	7	1.00	41	0.171
278	A	10	7	1.00	41	0.171
279	A	10	7	1.00	41	0.171
280	A	9	7	1.00	41	0.171
281	A	8	7	1.00	41	0.171
282	A	7	6	1.00	41	0.146
283	A	7	6	1.00	39	0.154
284	A	7	6	1.00	33	0.182
285	A	9	7	1.00	39	0.180
286	A	10	7	1.00	41	0.171
287	A	8	6	1.00	33	0.182
288	A	8	6	1.00	43	0.140
289	A	7	6	1.00	43	0.140
290	A	3	3	1.00	43	0.070
291	A	5	4	1.00	43	0.093
292	A	4	4	1.00	43	0.093
293	A	4	4	1.00	43	0.093
294	A	6	6	1.00	43	0.140
295	A	7	7	1.00	43	0.163
296	A	7	6	1.00	43	0.140
297	A	8	6	1.00	43	0.140
298	A	7	6	1.00	43	0.140
299	A	3	3	1.00	43	0.070
300	A	5	4	1.00	43	0.093
301	A	4	4	1.00	43	0.093
302	A	4	4	1.00	43	0.093
303	A	6	6	1.00	43	0.140
304	A	7	7	1.00	43	0.163
305	A	7	6	1.00	43	0.140

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	8	6	1.00	43	0.140
307	A	7	6	1.00	43	0.140
308	A	3	3	1.00	43	0.070
309	A	5	4	1.00	43	0.093
310	A	4	4	1.00	43	0.093
311	A	4	4	1.00	43	0.093
312	A	6	6	1.00	43	0.140
313	A	7	7	1.00	43	0.163
314	A	7	6	1.00	43	0.140
315	A	7	6	1.00	43	0.140
316	A	3	3	1.00	43	0.070
317	A	5	4	1.00	43	0.093
318	A	4	4	1.00	43	0.093
319	A	4	4	1.00	43	0.093
320	A	6	6	1.00	43	0.140
321	A	7	7	1.00	43	0.163
322	A	7	6	1.00	43	0.140
323	A	7	6	1.00	43	0.140
324	A	3	3	1.00	43	0.070
325	A	5	4	1.00	43	0.093
326	A	4	4	1.00	43	0.093
327	A	4	4	1.00	43	0.093
328	A	6	6	1.00	43	0.140
329	A	7	7	1.00	43	0.163
330	A	7	6	1.00	43	0.140
331	A	7	6	1.00	43	0.140
332	A	3	3	1.00	43	0.070
333	A	5	4	1.00	43	0.093
334	A	4	4	1.00	43	0.093
335	A	4	4	1.00	43	0.093
336	A	6	6	1.00	43	0.140
337	A	7	7	1.00	43	0.163
338	A	7	6	1.00	43	0.140
339	A	5	4	1.00	39	0.103
340	A	4	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	5	4	1.00	39	0.103
342	A	5	4	1.00	41	0.098
343	A	5	4	1.00	41	0.098
344	A	5	4	1.00	41	0.098
345	A	5	4	1.00	39	0.103
346	A	4	3	1.00	33	0.091
347	A	5	4	1.00	39	0.103
348	A	5	4	1.00	41	0.098
349	A	5	4	1.00	41	0.098
350	A	5	4	1.00	41	0.098
351	A	5	4	1.00	41	0.098
352	A	5	4	1.00	39	0.103
353	A	4	3	1.00	33	0.091
354	A	5	4	1.00	39	0.103
355	A	5	4	1.00	41	0.098
356	A	5	4	1.00	41	0.098
357	A	5	4	1.00	41	0.098
358	A	5	4	1.00	41	0.098
359	A	5	4	1.00	39	0.103
360	A	4	3	1.00	33	0.091
361	A	5	4	1.00	39	0.103
362	A	5	4	1.00	41	0.098
363	A	5	4	0.96	41	0.098
364	A	5	4	0.96	41	0.098
365	A	5	4	0.96	41	0.098
366	A	5	4	0.96	41	0.098
367	A	5	4	1.00	41	0.098
368	A	5	4	0.96	41	0.098
369	A	5	4	1.00	41	0.098
370	A	5	4	1.00	39	0.103
371	A	5	4	1.00	37	0.108
372	A	4	3	1.00	31	0.097
373	A	5	4	1.00	37	0.108
374	A	5	4	1.00	39	0.103
375	A	5	4	1.00	39	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	4	1.00	39	0.103
377	A	5	4	0.96	41	0.098
378	A	5	4	0.96	41	0.098
379	A	5	4	1.00	41	0.098
380	A	5	4	1.00	41	0.098
381	A	5	4	1.00	41	0.098
382	A	5	4	0.96	41	0.098
383	A	4	4	1.00	33	0.121
384	A	4	4	1.00	35	0.114
385	A	4	4	1.00	35	0.114
386	A	4	4	1.00	35	0.114
387	A	4	4	1.00	35	0.114
388	A	8	5	1.00	35	0.143
389	A	8	5	1.00	35	0.143
390	A	8	5	1.00	35	0.143
391	A	8	5	1.00	35	0.143
392	A	7	5	1.00	35	0.143
393	A	8	5	1.00	33	0.152

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{(A+C)\sin(c+dx)}{d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{C\sin^9(c+$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(3*A+4*C)*sin(d*x+c)^3/d+3/5*(A+2*C)*sin(d*x+c)^5/d-
1/7*(A+4*C)*sin(d*x+c)^7/d+1/9*C*sin(d*x+c)^9/d
```

Rubi [A]

time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3092, 380}

$$-\frac{(A+4C)\sin^7(c+dx)}{7d}+\frac{3(A+2C)\sin^5(c+dx)}{5d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{C\sin^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*d) + (3*(A + 2*C
)*Sin[c + d*x]^5)/(5*d) - ((A + 4*C)*Sin[c + d*x]^7)/(7*d) + (C*Sin[c + d*x
]^9)/(9*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)^3\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(3A+4C)x^2+3(A+2C)x^4-(A+4C\right.\right.}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(3A+4C)\sin^3(c+dx)}{3d}+\frac{3(A+2C)\sin^5}{5d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 141, normalized size = 1.53

$$\frac{35A\sin(c+dx)}{64d}+\frac{63C\sin(c+dx)}{128d}+\frac{7A\sin(3(c+dx))}{64d}+\frac{7C\sin(3(c+dx))}{64d}+\frac{7A\sin(5(c+dx))}{320d}+\frac{9C\sin(5(c+dx))}{320d}+\frac{A\sin(7(c+dx))}{448d}+\frac{9C\sin(7(c+dx))}{1792d}+\frac{C\sin(9(c+dx))}{2304d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (35*A*Sin[c + d*x])/(64*d) + (63*C*Sin[c + d*x])/(128*d) + (7*A*Sin[3*(c +
d*x)])/(64*d) + (7*C*Sin[3*(c + d*x)])/(64*d) + (7*A*Sin[5*(c + d*x)])/(320
*d) + (9*C*Sin[5*(c + d*x)])/(320*d) + (A*Sin[7*(c + d*x)])/(448*d) + (9*C*
Sin[7*(c + d*x)])/(1792*d) + (C*Sin[9*(c + d*x)])/(2304*d)
```

Maple [A]
time = 0.32, size = 94, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{C\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(dx+c)\right)}{35}+\frac{64\left(\cos^2(dx+c)\right)}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(\ldots}{\ldots}\right.}{7}}{d}$
default	$\frac{\frac{C\left(\frac{128}{35}+\cos^8(dx+c)+\frac{8\left(\cos^6(dx+c)\right)}{7}+\frac{48\left(\cos^4(dx+c)\right)}{35}+\frac{64\left(\cos^2(dx+c)\right)}{35}\right)\sin(dx+c)}{9}+\frac{A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8(\ldots}{\ldots}\right.}{7}}{d}$
risch	$\frac{35\sin(dx+c)A}{64d}+\frac{63C\sin(dx+c)}{128d}+\frac{C\sin(9dx+9c)}{2304d}+\frac{\sin(7dx+7c)A}{448d}+\frac{9\sin(7dx+7c)C}{1792d}+\frac{7\sin(5dx+5c)A}{320d}+\frac{9\ldots}{}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(3A+2C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(3A+2C)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(17A+19C)\left(\tan^5(\ldots\right.}{5d}}{\left(1\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/9*C*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*c
os(d*x+c)^2)*sin(d*x+c)+1/7*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d
*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.82

$$\frac{35\,C\sin\left(dx+c\right)^{9}-45\left(A+4\,C\right)\sin\left(dx+c\right)^{7}+189\left(A+2\,C\right)\sin\left(dx+c\right)^{5}-105\left(3\,A+4\,C\right)\sin\left(dx+c\right)^{3}+315\left(A+C\right)\sin\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*C*sin(d*x + c)^9 - 45*(A + 4*C)*sin(d*x + c)^7 + 189*(A + 2*C)*si
n(d*x + c)^5 - 105*(3*A + 4*C)*sin(d*x + c)^3 + 315*(A + C)*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 80, normalized size = 0.87

$$\frac{\left(35\,C\cos\left(dx+c\right)^{8}+5\,\left(9\,A+8\,C\right)\cos\left(dx+c\right)^{6}+6\,\left(9\,A+8\,C\right)\cos\left(dx+c\right)^{4}+8\,\left(9\,A+8\,C\right)\cos\left(dx+c\right)^{2}+144\,A+128\,C\right)\sin\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*C*cos(d*x + c)^8 + 5*(9*A + 8*C)*cos(d*x + c)^6 + 6*(9*A + 8*C)*c
os(d*x + c)^4 + 8*(9*A + 8*C)*cos(d*x + c)^2 + 144*A + 128*C)*sin(d*x + c)/
d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(80) = 160$.
time = 1.27, size = 199, normalized size = 2.16

$$\begin{cases} \frac{16A\sin^{7}\left(c+dx\right)}{35d}+\frac{8A\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{2A\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}+\frac{A\sin\left(c+dx\right)\cos^{6}\left(c+dx\right)}{d}+\frac{128C\sin^{9}\left(c+dx\right)}{315d}+\frac{64C\sin^{7}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{35d}+\frac{16C\sin^{5}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{5d}+\frac{8C\sin^{3}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{3d}+\frac{C\sin\left(c+dx\right)\cos^{8}\left(c+dx\right)}{d} & \text{for}\: d\neq 0 \\ x\left(A+C\cos^{2}\left(c\right)\right)\cos^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((16*A*sin(c + d*x)**7/(35*d) + 8*A*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*A*sin(c + d*x)**3*cos(c + d*x)**4/d + A*sin(c + d*x)*cos(c + d*
x)**6/d + 128*C*sin(c + d*x)**9/(315*d) + 64*C*sin(c + d*x)**7*cos(c + d*x)
**2/(35*d) + 16*C*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*C*sin(c + d*x)*
*3*cos(c + d*x)**6/(3*d) + C*sin(c + d*x)*cos(c + d*x)**8/d, Ne(d, 0)), (x*
(A + C*cos(c)**2)*cos(c)**7, True))
```

Giac [A]
time = 0.40, size = 93, normalized size = 1.01

$$\frac{C\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{\left(4\,A+9\,C\right)\sin\left(7\,dx+7\,c\right)}{1792\,d}+\frac{\left(7\,A+9\,C\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{7\,\left(A+C\right)\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{7\,\left(10\,A+9\,C\right)\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2304*C*sin(9*d*x + 9*c)/d + 1/1792*(4*A + 9*C)*sin(7*d*x + 7*c)/d + 1/320
*(7*A + 9*C)*sin(5*d*x + 5*c)/d + 7/64*(A + C)*sin(3*d*x + 3*c)/d + 7/128*(
10*A + 9*C)*sin(d*x + c)/d
```

Mupad [B]
time = 0.68, size = 74, normalized size = 0.80

$$\frac{\frac{C\,{\sin\left(c+d\,x\right)}^9}{9}+\left(-\frac{A}{7}-\frac{4\,C}{7}\right)\,{\sin\left(c+d\,x\right)}^7+\left(\frac{3\,A}{5}+\frac{6\,C}{5}\right)\,{\sin\left(c+d\,x\right)}^5+\left(-A-\frac{4\,C}{3}\right)\,{\sin\left(c+d\,x\right)}^3+\left(A+C\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + C*cos(c + d*x)^2),x)
```

```
[Out] ((C*sin(c + d*x)^9)/9 - sin(c + d*x)^3*(A + (4*C)/3) + sin(c + d*x)*(A + C)
 + sin(c + d*x)^5*((3*A)/5 + (6*C)/5) - sin(c + d*x)^7*(A/7 + (4*C)/7))/d
```

3.2 $\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=72

$$\frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{C\sin^7(c+dx)}{7d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(2*A+3*C)*sin(d*x+c)^3/d+1/5*(A+3*C)*sin(d*x+c)^5/d-
1/7*C*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3092, 380}

$$\frac{(A+3C)\sin^5(c+dx)}{5d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((2*A + 3*C)*Sin[c + d*x]^3)/(3*d) + ((A + 3*C)*
Sin[c + d*x]^5)/(5*d) - (C*Sin[c + d*x]^7)/(7*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right)-(2A+3C)x^2+(A+3C)x^4-Cx^6\right)\,dx\right.}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(2A+3C)\sin^3(c+dx)}{3d}+\frac{(A+3C)\sin^5}{5d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 109, normalized size = 1.51

$$\frac{5A\sin(c+dx)}{8d}+\frac{35C\sin(c+dx)}{64d}+\frac{5A\sin(3(c+dx))}{48d}+\frac{7C\sin(3(c+dx))}{64d}+\frac{A\sin(5(c+dx))}{80d}+\frac{7C\sin(5(c+dx))}{320d}+\frac{C\sin(7(c+dx))}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (5*A*Sin[c + d*x])/(8*d) + (35*C*Sin[c + d*x])/(64*d) + (5*A*Sin[3*(c + d*x
)])/(48*d) + (7*C*Sin[3*(c + d*x)])/(64*d) + (A*Sin[5*(c + d*x)])/(80*d) +
(7*C*Sin[5*(c + d*x)])/(320*d) + (C*Sin[7*(c + d*x)])/(448*d)
```

Maple [A]
time = 0.18, size = 74, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{C\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{\frac{C\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+\frac{A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$\frac{5\sin(dx+c)A}{8d}+\frac{35C\sin(dx+c)}{64d}+\frac{\sin(7dx+7c)C}{448d}+\frac{\sin(5dx+5c)A}{80d}+\frac{7\sin(5dx+5c)C}{320d}+\frac{5\sin(3dx+3c)A}{48d}+ 7\sin$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(5A+3C)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4(5A+3C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(91A+53C)\left(\tan^{7}\left(\frac{d}{2}\right.\right.}{35d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*C*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+1/5*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 60, normalized size = 0.83

$$-\frac{15\,C\sin\left(dx+c\right)^{7}-21\,(A+3\,C)\sin\left(dx+c\right)^{5}+35\,(2\,A+3\,C)\sin\left(dx+c\right)^{3}-105\,(A+C)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/105*(15*C*sin(d*x + c)^7 - 21*(A + 3*C)*sin(d*x + c)^5 + 35*(2*A + 3*C)*
sin(d*x + c)^3 - 105*(A + C)*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 63, normalized size = 0.88

$$\frac{\left(15\,C\cos\left(dx+c\right)^{6}+3\left(7\,A+6\,C\right)\cos\left(dx+c\right)^{4}+4\left(7\,A+6\,C\right)\cos\left(dx+c\right)^{2}+56\,A+48\,C\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*C*cos(d*x + c)^6 + 3*(7*A + 6*C)*cos(d*x + c)^4 + 4*(7*A + 6*C)*c
os(d*x + c)^2 + 56*A + 48*C)*sin(d*x + c)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(60) = 120$.
time = 0.58, size = 151, normalized size = 2.10

$$\begin{cases} \frac{8A\sin^{5}(c+dx)}{15d}+\frac{4A\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}+\frac{A\sin(c+dx)\cos^{4}(c+dx)}{d}+\frac{16C\sin^{7}(c+dx)}{35d}+\frac{8C\sin^{5}(c+dx)\cos^{2}(c+dx)}{5d}+\frac{2C\sin^{3}(c+dx)\cos^{4}(c+dx)}{d}+\frac{C\sin(c+dx)\cos^{6}(c+dx)}{d} & \text{for } d\neq 0 \\ x(A+C\cos^{2}(c))\cos^{5}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((8*A*sin(c + d*x)**5/(15*d) + 4*A*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + A*sin(c + d*x)*cos(c + d*x)**4/d + 16*C*sin(c + d*x)**7/(35*d) + 8
*C*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*C*sin(c + d*x)**3*cos(c + d*x)
**4/d + C*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*c
os(c)**5, True))
```

Giac [A]
time = 0.42, size = 76, normalized size = 1.06

$$\frac{C\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(4\,A+7\,C\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(20\,A+21\,C\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\left(8\,A+7\,C\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/448*C*sin(7*d*x + 7*c)/d + 1/320*(4*A + 7*C)*sin(5*d*x + 5*c)/d + 1/192*(
20*A + 21*C)*sin(3*d*x + 3*c)/d + 5/64*(8*A + 7*C)*sin(d*x + c)/d
```

Mupad [B]
time = 0.66, size = 59, normalized size = 0.82

$$-\frac{\frac{C\sin(c+d\,x)^7}{7}+\left(-\frac{A}{5}-\frac{3\,C}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(\frac{2\,A}{3}+C\right)\,\sin\left(c+d\,x\right)^3+\left(-A-C\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + C*cos(c + d*x)^2),x)
```

```
[Out] -(sin(c + d*x)^3*((2*A)/3 + C) + (C*sin(c + d*x)^7)/7 - sin(c + d*x)*(A + C
) - sin(c + d*x)^5*(A/5 + (3*C)/5))/d
```

3.3 $\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{(A+C)\sin(c+dx)}{d}-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*(A+2*C)*sin(d*x+c)^3/d+1/5*C*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3092, 380}

$$-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{(A+C)\sin(c+dx)}{d}+\frac{C\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((A + 2*C)*Sin[c + d*x]^3)/(3*d) + (C*Sin[c + d*
x]^5)/(5*d)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx &= -\frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int\left(A\left(1+\frac{C}{A}\right)-(A+2C)x^2+Cx^4\right)\,dx,x,-\sin(c+dx\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{(A+2C)\sin^3(c+dx)}{3d}+\frac{C\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 1.48

$$\frac{A\sin(c+dx)}{d}+\frac{5C\sin(c+dx)}{8d}-\frac{A\sin^3(c+dx)}{3d}+\frac{5C\sin(3(c+dx))}{48d}+\frac{C\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (A*Sin[c + d*x])/d + (5*C*Sin[c + d*x])/(8*d) - (A*Sin[c + d*x]^3)/(3*d) +
(5*C*Sin[3*(c + d*x)])/(48*d) + (C*Sin[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.14, size = 54, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{C\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{A\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$
default	$\frac{\frac{C\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{A\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}}{d}$
risch	$\frac{3\sin(dx+c)A}{4d}+\frac{5C\sin(dx+c)}{8d}+\frac{\sin(5dx+5c)C}{80d}+\frac{\sin(3dx+3c)A}{12d}+\frac{5\sin(3dx+3c)C}{48d}$
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(2A+C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(2A+C)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4(25A+29C)\left(\tan^5\left(\frac{dx}{2}\right.\right.}{15d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*C*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*A*(cos(d*x+c)
^2+2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.86

$$\frac{3\,C\sin\left(dx+c\right)^5-5\left(A+2\,C\right)\sin\left(dx+c\right)^3+15\left(A+C\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*C*sin(d*x + c)^5 - 5*(A + 2*C)*sin(d*x + c)^3 + 15*(A + C)*sin(d*x
+ c))/d
```

Fricas [A]
time = 0.37, size = 45, normalized size = 0.90

$$\frac{\left(3\,C\cos\left(dx+c\right)^{4}+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^{2}+10\,A+8\,C\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sin(d*x
 + c)/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.
time = 0.28, size = 105, normalized size = 2.10

$$\begin{cases}\frac{2A\sin^{3}(c+dx)}{3d}+\frac{A\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{8C\sin^{5}(c+dx)}{15d}+\frac{4C\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}+\frac{C\sin(c+dx)\cos^{4}(c+dx)}{d} & \text{for } d\neq 0\\ x(A+C\cos^{2}(c))\cos^{3}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((2*A*sin(c + d*x)**3/(3*d) + A*sin(c + d*x)*cos(c + d*x)**2/d + 8
*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*s
in(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3, T
rue))
```

Giac [A]
time = 0.42, size = 57, normalized size = 1.14

$$\frac{3\,C\sin\left(dx+c\right)^{5}-5\,A\sin\left(dx+c\right)^{3}-10\,C\sin\left(dx+c\right)^{3}+15\,A\sin\left(dx+c\right)+15\,C\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*C*sin(d*x + c)^5 - 5*A*sin(d*x + c)^3 - 10*C*sin(d*x + c)^3 + 15*A*
sin(d*x + c) + 15*C*sin(d*x + c))/d
```

Mupad [B]
time = 0.68, size = 43, normalized size = 0.86

$$\frac{\frac{C\sin(c+d\,x)^{5}}{5}+\left(-\frac{A}{3}-\frac{2\,C}{3}\right)\,\sin\left(c+d\,x\right)^{3}+\left(A+C\right)\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + C*cos(c + d*x)^2),x)
```

```
[Out] ((C*sin(c + d*x)^5)/5 + sin(c + d*x)*(A + C) - sin(c + d*x)^3*(A/3 + (2*C)/
3))/d
```

3.4 $\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

```
[Out] (A+C)*sin(d*x+c)/d-1/3*C*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3092}

$$\frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d)
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= \frac{(A+C)\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.67

$$\frac{A\cos(dx)\sin(c)}{d}+\frac{A\cos(c)\sin(dx)}{d}+\frac{C\sin(c+dx)}{d}-\frac{C\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]
```

[Out] (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d + (C*Sin[c + d*x])/d - (C*Sin[c + d*x]^3)/(3*d)

Maple [A]
time = 0.10, size = 33, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\frac{C\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$	33
default	$\frac{\frac{C\left(\cos^2(dx+c)+2\right)\sin(dx+c)}{3}+A\sin(dx+c)}{d}$	33
risch	$\frac{\sin(dx+c)A}{d}+\frac{3C\sin(dx+c)}{4d}+\frac{\sin(3dx+3c)C}{12d}$	40
norman	$\frac{\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(A+C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(3A+C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/3*C*(cos(d*x+c)^2+2)*sin(d*x+c)+A*sin(d*x+c))

Maxima [A]
time = 0.28, size = 34, normalized size = 1.13

$$-\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)C-3\,A\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d

Fricas [A]
time = 0.37, size = 28, normalized size = 0.93

$$\frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sin(d*x + c)/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

time = 0.12, size = 56, normalized size = 1.87

$$\begin{cases} \frac{A\sin{(c+dx)}}{d} + \frac{2C\sin^3{(c+dx)}}{3d} + \frac{C\sin{(c+dx)}\cos^2{(c+dx)}}{d} & \text{for } d\neq 0 \\ x(A+C\cos^2{(c)})\cos{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*sin(c + d*x)/d + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c + d*x)*co
s(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c), True))
```

Giac [A]
time = 0.39, size = 34, normalized size = 1.13

$$-\frac{\left(\sin{(dx+c)}^3 - 3\,\sin{(dx+c)}\right)C - 3\,A\sin{(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.93

$$-\frac{\frac{C\sin(c+d\,x)^3}{3} - \sin{(c+d\,x)}\,\left(A+C\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2),x)
```

```
[Out] -((C*sin(c + d*x)^3)/3 - sin(c + d*x)*(A + C))/d
```

3.5 $\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{A\tanh^{-1}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}$$

```
[Out] A*arctanh(sin(d*x+c))/d+C*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3093, 3855}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx&=\frac{C\sin(c+dx)}{d}+A\int\sec(c+dx)\,dx\\&=\frac{A\tanh^{-1}(\sin(c+dx))}{d}+\frac{C\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.46

$$\frac{A\tanh^{-1}(\sin(c+dx))}{d}+\frac{C\cos(dx)\sin(c)}{d}+\frac{C\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.14, size = 30, normalized size = 1.25

method	result	size
derivativedivides	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\sin(dx+c)}{d}$	30
default	$\frac{A\ln(\sec(dx+c)+\tan(dx+c))+C\sin(dx+c)}{d}$	30
risch	$-\frac{iC\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{iC\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{A\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{A\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	71
norman	$\frac{\frac{2C\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2C\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*ln(sec(d*x+c)+tan(d*x+c))+C*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.58

$$\frac{A\log\left(\sin\left(dx+c\right)+1\right)-A\log\left(\sin\left(dx+c\right)-1\right)+2\,C\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(sin(d*x + c) - 1) + 2*C*sin(d*x + c))/
d
```

Fricas [A]
time = 0.40, size = 40, normalized size = 1.67

$$\frac{A\log\left(\sin\left(dx+c\right)+1\right)-A\log\left(-\sin\left(dx+c\right)+1\right)+2\,C\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c))
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C\cos^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Giac [A]
time = 0.44, size = 40, normalized size = 1.67

$$\frac{A\log\left(\left|\sin\left(dx + c\right) + 1\right|\right) - A\log\left(\left|\sin\left(dx + c\right) - 1\right|\right) + 2\,C\sin\left(dx + c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(A*log(abs(sin(d*x + c) + 1)) - A*log(abs(sin(d*x + c) - 1)) + 2*C*sin(
d*x + c))/d
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.92

$$\frac{C\,\sin\left(c + d\,x\right) + A\,\mathrm{atanh}\left(\sin\left(c + d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x),x)
```

```
[Out] (C*sin(c + d*x) + A*atanh(sin(c + d*x)))/d
```

3.6 $\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=40

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(A+2*C)*arctanh(sin(d*x+c))/d+1/2*A*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3091, 3855}

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]])/(2*d) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*
d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx&=\frac{A\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}(A+2C)\int \sec(c+dx)\,dx\\&=\frac{(A+2C)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 1.20

$$\frac{A\tanh^{-1}(\sin(c+dx))}{2d}+\frac{C\tanh^{-1}(\sin(c+dx))}{d}+\frac{A\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(2*d) + (C*ArcTanh[Sin[c + d*x]])/d + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.17, size = 55, normalized size = 1.38

method	result
derivativedivides	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+C\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$-\frac{iA\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{A\ln\left(e^{i(dx+c)}-i\right)}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)C}{d}+\frac{A\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)C}{d}$
norman	$\frac{\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{(A+2C)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+(A+2C)\,\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+C*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.29, size = 58, normalized size = 1.45

$$\frac{(A+2C)\log\left(\sin\left(dx+c\right)+1\right)-(A+2C)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2A\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((A + 2*C)*log(sin(d*x + c) + 1) - (A + 2*C)*log(sin(d*x + c) - 1) - 2*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.38, size = 72, normalized size = 1.80

$$\frac{(A+2C)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(A+2C)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2A\sin\left(dx+c\right)}{4d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((A + 2*C)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*C)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) + 2*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+C\cos^{2}\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.45, size = 60, normalized size = 1.50

$$\frac{(A+2C)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(A+2C)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2A\sin(dx+c)}{\sin(dx+c)^{2}-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*((A + 2*C)*log(abs(sin(d*x + c) + 1)) - (A + 2*C)*log(abs(sin(d*x + c)
- 1)) - 2*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 0.10, size = 41, normalized size = 1.02

$$\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{A}{2}+C\right)}{d}-\frac{A\,\sin\left(c+d\,x\right)}{2\,d\,\left({\sin\left(c+d\,x\right)}^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^3,x)
```

```
[Out] (atanh(sin(c + d*x))*(A/2 + C))/d - (A*sin(c + d*x))/(2*d*(sin(c + d*x)^2 -
 1))
```

3.7 $\int (A + C\cos^2(c + dx))\sec^5(c + dx)\,dx$

Optimal. Leaf size=70

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}+\frac{A\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 1/8*(3*A+4*C)*arctanh(sin(d*x+c))/d+1/8*(3*A+4*C)*sec(d*x+c)*tan(d*x+c)/d+1
/4*A*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3091, 3853, 3855}

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3A+4C)\tan(c+dx)\sec(c+dx)}{8d}+\frac{A\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]])/(8*d) + ((3*A + 4*C)*Sec[c + d*x]*Tan[c
 + d*x])/(8*d) + (A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx = \frac{A\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3A+4C)\int \sec^3(c+dx)\,dx$$
$$= \frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d} + \frac{A\sec^3(c+dx)\tan(c+dx)}{4d}$$
$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(3A+4C)\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.77

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\left(3A+4C+2A\sec^2(c+dx)\right)\tan(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3*A + 4*C + 2*A*Sec[c +
d*x]^2)*Tan[c + d*x])/(8*d)
```

Maple [A]
time = 0.25, size = 85, normalized size = 1.21

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+C\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan}{2}\right.}{d}$
risch	$-\frac{i(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C\,e^{i(dx+c)}}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{(5A+4C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(5A+4C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(7A-4C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{(7A-4C)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{(13A+4C)\left(\tan^3\left(\frac{dx}{2}\right.\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
[Out] 1/d*(A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+C*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.39

$$\frac{(3A+4C)\log(\sin(dx+c)+1) - (3A+4C)\log(\sin(dx+c)-1) - \frac{2\left((3A+4C)\sin(dx+c)^3-(5A+4C)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*((3*A + 4*C)*log(sin(d*x + c) + 1) - (3*A + 4*C)*log(sin(d*x + c) - 1)
 - 2*((3*A + 4*C)*sin(d*x + c)^3 - (5*A + 4*C)*sin(d*x + c))/(sin(d*x + c)^
4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 95, normalized size = 1.36

$$\frac{(3A+4C)\cos(dx+c)^4\log(\sin(dx+c)+1)-(3A+4C)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left((3A+4C)\cos(dx+c)^2+2A\right)\sin(dx+c)}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*((3*A + 4*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A + 4*C)*cos(d*
x + c)^4*log(-sin(d*x + c) + 1) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sin(
d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**5, x)
```

Giac [A]
time = 0.43, size = 98, normalized size = 1.40

$$\frac{(3A+4C)\log(|\sin(dx+c)+1|)-(3A+4C)\log(|\sin(dx+c)-1|)-\frac{2\left(3A\sin(dx+c)^3+4C\sin(dx+c)^3-5A\sin(dx+c)-4C\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/16*((3*A + 4*C)*log(abs(sin(d*x + c) + 1)) - (3*A + 4*C)*log(abs(sin(d*x
+ c) - 1)) - 2*(3*A*sin(d*x + c)^3 + 4*C*sin(d*x + c)^3 - 5*A*sin(d*x + c)
- 4*C*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.74, size = 77, normalized size = 1.10

$$\frac{\sin(c+dx)\left(\frac{5A}{8}+\frac{C}{2}\right)-\sin(c+dx)^3\left(\frac{3A}{8}+\frac{C}{2}\right)}{d\left(\sin(c+dx)^4-2\sin(c+dx)^2+1\right)}+\frac{\mathrm{atanh}(\sin(c+dx))\left(\frac{3A}{8}+\frac{C}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^5,x)
```

```
[Out] (sin(c + d*x)*((5*A)/8 + C/2) - sin(c + d*x)^3*((3*A)/8 + C/2))/(d*(sin(c +
 d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(sin(c + d*x))*((3*A)/8 + C/2))/d
```

3.8 $\int (A+C\cos^2(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{(5A+6C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d}+\frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d}+$$

```
[Out] 1/16*(5*A+6*C)*arctanh(sin(d*x+c))/d+1/16*(5*A+6*C)*sec(d*x+c)*tan(d*x+c)/d
+1/24*(5*A+6*C)*sec(d*x+c)^3*tan(d*x+c)/d+1/6*A*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3091, 3853, 3855}

$$\frac{(5A+6C)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(5A+6C)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(5A+6C)\tan(c+dx)\sec(c+dx)}{16d}+\frac{A\tan(c+dx)\sec^5(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] ((5*A + 6*C)*ArcTanh[Sin[c + d*x]])/(16*d) + ((5*A + 6*C)*Sec[c + d*x]*Tan[
c + d*x])/(16*d) + ((5*A + 6*C)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (A*Se
c[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (A + C\cos^2(c+dx))\sec^7(c+dx)\,dx = \frac{A\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}(5A+6C)\int \sec^5(c+dx)\,dx$$
$$= \frac{(5A+6C)\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{A\sec^5(c+dx)\tan(c+dx)}{6d} \ldots$$
$$= \frac{(5A+6C)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(5A+6C)\sec^3(c+dx)\tan\ldots}{24d}$$
$$= \frac{(5A+6C)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{(5A+6C)\sec(c+dx)\tan(c\ldots}{16d}$$

Mathematica [A]
time = 0.36, size = 75, normalized size = 0.77

$$\frac{3(5A+6C)\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\left(3(5A+6C) + 2(5A+6C)\sec^2(c+dx) + 8A\sec^4(c+dx)\right)\tan(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
[Out] (3*(5*A + 6*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3*(5*A + 6*C) + 2*(5*A
 + 6*C)*Sec[c + d*x]^2 + 8*A*Sec[c + d*x]^4)*Tan[c + d*x])/(48*d)
```

Maple [A]
time = 0.30, size = 108, normalized size = 1.10

method	result
derivativedivides	$\frac{A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6} - \frac{5\left(\sec^3(dx+c)\right)}{24} - \frac{5\sec(dx+c)}{16}\right)\tan(dx+c) + \frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right) + C\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4} - \ldots\right.\right.}{d}$
default	$\frac{A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6} - \frac{5\left(\sec^3(dx+c)\right)}{24} - \frac{5\sec(dx+c)}{16}\right)\tan(dx+c) + \frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right) + C\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4} - \ldots\right.\right.}{d}$
risch	$-\frac{i\left(15A\,e^{11i(dx+c)} + 18C\,e^{11i(dx+c)} + 85A\,e^{9i(dx+c)} + 102C\,e^{9i(dx+c)} + 198A\,e^{7i(dx+c)} + 84C\,e^{7i(dx+c)} - 198A\,e^{5i(dx+c)} - \ldots\right.}{24d\left(e^{2i(dx+c)}+1\right)^6}$
norman	$\frac{\frac{(11A+10C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d} + \frac{(11A+10C)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} + \frac{7(19A-6C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d} + \frac{7(19A-6C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d} + \frac{(71A+18C\ldots}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{d\ldots}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
[Out] 1/d*(A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5
/16*ln(sec(d*x+c)+tan(d*x+c)))+C*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d
*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 126, normalized size = 1.29

$$\frac{3\,(5\,A+6\,C)\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A+6\,C)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A+6\,C)\sin(dx+c)^5-8\,(5\,A+6\,C)\sin(dx+c)^3+3\,(11\,A+10\,C)\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(5*A + 6*C)*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*log(sin(d*x + c)
- 1) - 2*(3*(5*A + 6*C)*sin(d*x + c)^5 - 8*(5*A + 6*C)*sin(d*x + c)^3 + 3*(
11*A + 10*C)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x +
 c)^2 - 1))/d
```

Fricas [A]
time = 0.38, size = 114, normalized size = 1.16

$$\frac{3\,(5\,A+6\,C)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A+6\,C)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,(5\,A+6\,C)\cos\left(dx+c\right)^4+2\,(5\,A+6\,C)\cos\left(dx+c\right)^2+8\,A\right)\sin\left(dx+c\right)}{96\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(5*A + 6*C)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*co
s(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(5*A + 6*C)*cos(d*x + c)^4 + 2*(
5*A + 6*C)*cos(d*x + c)^2 + 8*A)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.44, size = 121, normalized size = 1.23

$$\frac{3\,(5\,A+6\,C)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\,(5\,A+6\,C)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(15\,A\sin(dx+c)^5+18\,C\sin(dx+c)^5-40\,A\sin(dx+c)^3-48\,C\sin(dx+c)^3+33\,A\sin(dx+c)+30\,C\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/96*(3*(5*A + 6*C)*log(abs(sin(d*x + c) + 1)) - 3*(5*A + 6*C)*log(abs(sin(
d*x + c) - 1)) - 2*(15*A*sin(d*x + c)^5 + 18*C*sin(d*x + c)^5 - 40*A*sin(d*
```

```
x + c)^3 - 48*C*sin(d*x + c)^3 + 33*A*sin(d*x + c) + 30*C*sin(d*x + c))/(si
n(d*x + c)^2 - 1)^3)/d
```

Mupad [B]
time = 0.77, size = 102, normalized size = 1.04

$$\frac{\operatorname{atanh}(\sin(c+d\,x))\,\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)}{d}-\frac{\left(\frac{5\,A}{16}+\frac{3\,C}{8}\right)\,\sin(c+d\,x)^5+\left(-\frac{5\,A}{6}-C\right)\,\sin(c+d\,x)^3+\left(\frac{11\,A}{16}+\frac{5\,C}{8}\right)\,\sin(c+d\,x)}{d\,\left(\sin(c+d\,x)^6-3\sin(c+d\,x)^4+3\sin(c+d\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^7,x)
```

```
[Out] (atanh(sin(c + d*x))*((5*A)/16 + (3*C)/8))/d - (sin(c + d*x)*((11*A)/16 + (
5*C)/8) - sin(c + d*x)^3*((5*A)/6 + C) + sin(c + d*x)^5*((5*A)/16 + (3*C)/8
))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))
```

3.9 $\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=117

$$\frac{5}{128}(8A+7C)x+\frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{(8A+7C)\cos^5}{}$$

```
[Out] 5/128*(8*A+7*C)*x+5/128*(8*A+7*C)*cos(d*x+c)*sin(d*x+c)/d+5/192*(8*A+7*C)*c
os(d*x+c)^3*sin(d*x+c)/d+1/48*(8*A+7*C)*cos(d*x+c)^5*sin(d*x+c)/d+1/8*C*cos
(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$\frac{(8A+7C)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5(8A+7C)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5(8A+7C)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}x(8A+7C)+\frac{C\sin(c+dx)\cos^7(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (5*(8*A + 7*C)*x)/128 + (5*(8*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) +
 (5*(8*A + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + ((8*A + 7*C)*Cos[c +
 d*x]^5*Sin[c + d*x])/(48*d) + (C*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^6(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(8A+7C)\int \cos^6(c+dx)\,dx$$
$$= \frac{(8A+7C)\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{C\cos^7(c+dx)\sin(c+dx}{8d}$$
$$= \frac{5(8A+7C)\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{(8A+7C)\cos^5(c+dx)}{48d}$$
$$= \frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d} + \frac{5(8A+7C)\cos^3(c+dx)}{192d}$$
$$= \frac{5}{128}(8A+7C)x + \frac{5(8A+7C)\cos(c+dx)\sin(c+dx)}{128d} + \frac{5(8A}{}$$

Mathematica [A]

time = 0.18, size = 93, normalized size = 0.79

$$\frac{960Ac+840cC+960Adx+840Cdx+48(15A+14C)\sin(2(c+dx))+24(6A+7C)\sin(4(c+dx))+16A\sin(6(c+dx))+32C\sin(6(c+dx))+3C\sin(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (960*A*c + 840*c*C + 960*A*d*x + 840*C*d*x + 48*(15*A + 14*C)*Sin[2*(c + d*
x)] + 24*(6*A + 7*C)*Sin[4*(c + d*x)] + 16*A*Sin[6*(c + d*x)] + 32*C*Sin[6*
(c + d*x)] + 3*C*Sin[8*(c + d*x)])/(3072*d)
```

Maple [A]

time = 0.25, size = 106, normalized size = 0.91

method	result
derivativedivides	$\frac{C\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c\right.}{4}\right.}{}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)+A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c\right.}{4}\right.}{}\right.}{d}$
risch	$\frac{5Ax}{16}+\frac{35Cx}{128}+\frac{C\sin(8dx+8c)}{1024d}+\frac{\sin(6dx+6c)A}{192d}+\frac{\sin(6dx+6c)C}{96d}+\frac{3\sin(4dx+4c)A}{64d}+\frac{7\sin(4dx+4c)C}{128d}+1$
norman	$\left(\frac{5A}{16}+\frac{35C}{128}\right)x+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5A}{2}+\frac{35C}{16}\right)x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5A}{16}+\frac{35C}{128}\right)x\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{35A}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(C*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x
+c))*sin(d*x+c)+35/128*d*x+35/128*c)+A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.49, size = 130, normalized size = 1.11

$$\frac{15\,(dx+c)(8\,A+7\,C)+\frac{15\,(8\,A+7\,C)\tan(dx+c)^{7}+55\,(8\,A+7\,C)\tan(dx+c)^{5}+73\,(8\,A+7\,C)\tan(dx+c)^{3}+3\,(88\,A+93\,C)\tan(dx+c)}{\tan(dx+c)^{8}+4\,\tan(dx+c)^{6}+6\,\tan(dx+c)^{4}+4\,\tan(dx+c)^{2}+1}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(d*x + c)*(8*A + 7*C) + (15*(8*A + 7*C)*tan(d*x + c)^7 + 55*(8*A
+ 7*C)*tan(d*x + c)^5 + 73*(8*A + 7*C)*tan(d*x + c)^3 + 3*(88*A + 93*C)*tan
(d*x + c))/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*
x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 85, normalized size = 0.73

$$\frac{15\,(8\,A+7\,C)dx+\left(48\,C\cos\left(dx+c\right)^{7}+8\,(8\,A+7\,C)\cos\left(dx+c\right)^{5}+10\,(8\,A+7\,C)\cos\left(dx+c\right)^{3}+15\,(8\,A+7\,C)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*(8*A + 7*C)*d*x + (48*C*cos(d*x + c)^7 + 8*(8*A + 7*C)*cos(d*x +
c)^5 + 10*(8*A + 7*C)*cos(d*x + c)^3 + 15*(8*A + 7*C)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(109) = 218.
time = 0.93, size = 354, normalized size = 3.03

$$\begin{cases} \frac{5Ax\sin^{6}(c+dx)}{16} + \frac{15Ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15Ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16} + \frac{5Ax\cos^{6}(c+dx)}{16} + \frac{5A\sin^{5}(c+dx)\cos(c+dx)}{16d} + \frac{5A\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d} + \frac{11A\sin(c+dx)\cos^{5}(c+dx)}{16d} + \frac{35Cx\sin^{8}(c+dx)}{128} + \frac{35Cx\sin^{6}(c+dx)\cos^{2}(c+dx)}{32} + \frac{105Cx\sin^{4}(c+dx)\cos^{4}(c+dx)}{64} + \frac{35Cx\sin^{2}(c+dx)\cos^{6}(c+dx)}{32} + \frac{35Cx\cos^{8}(c+dx)}{128} + \frac{35C\sin^{7}(c+dx)\cos(c+dx)}{128d} + \frac{385C\sin^{5}(c+dx)\cos^{3}(c+dx)}{384d} + \frac{511C\sin^{3}(c+dx)\cos^{5}(c+dx)}{384d} + \frac{93C\sin(c+dx)\cos^{7}(c+dx)}{128d} & \text{for}\ d \neq 0 \\ x(A+C\cos^{2}(c))\cos^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((5*A*x*sin(c + d*x)**6/16 + 15*A*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*A*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*x*cos(c + d*x)**6/16
 + 5*A*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*A*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 35*C*x*sin(c + d*x
)**8/128 + 35*C*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 105*C*x*sin(c + d*x)
**4*cos(c + d*x)**4/64 + 35*C*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*C*x
```

```
*cos(c + d*x)**8/128 + 35*C*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*C*si
n(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*C*sin(c + d*x)**3*cos(c + d*x)*
*5/(384*d) + 93*C*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(A +
C*cos(c)**2)*cos(c)**6, True))
```

Giac [A]
time = 0.41, size = 87, normalized size = 0.74

$$\frac{5}{128}(8A+7C)x+\frac{C\sin(8dx+8c)}{1024d}+\frac{(A+2C)\sin(6dx+6c)}{192d}+\frac{(6A+7C)\sin(4dx+4c)}{128d}+\frac{(15A+14C)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 5/128*(8*A + 7*C)*x + 1/1024*C*sin(8*d*x + 8*c)/d + 1/192*(A + 2*C)*sin(6*d
*x + 6*c)/d + 1/128*(6*A + 7*C)*sin(4*d*x + 4*c)/d + 1/64*(15*A + 14*C)*sin
(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.11, size = 119, normalized size = 1.02

$$x\left(\frac{5A}{16}+\frac{35C}{128}\right)+\frac{\left(\frac{5A}{16}+\frac{35C}{128}\right)\tan(c+dx)^7+\left(\frac{55A}{48}+\frac{385C}{384}\right)\tan(c+dx)^5+\left(\frac{73A}{48}+\frac{511C}{384}\right)\tan(c+dx)^3+\left(\frac{11A}{16}+\frac{93C}{128}\right)\tan(c+dx)}{d\left(\tan(c+dx)^8+4\tan(c+dx)^6+6\tan(c+dx)^4+4\tan(c+dx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + C*cos(c + d*x)^2),x)
```

```
[Out] x*((5*A)/16 + (35*C)/128) + (tan(c + d*x)*((11*A)/16 + (93*C)/128) + tan(c
+ d*x)^7*((5*A)/16 + (35*C)/128) + tan(c + d*x)^5*((55*A)/48 + (385*C)/384)
 + tan(c + d*x)^3*((73*A)/48 + (511*C)/384))/(d*(4*tan(c + d*x)^2 + 6*tan(c
 + d*x)^4 + 4*tan(c + d*x)^6 + tan(c + d*x)^8 + 1))
```

3.10 $\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{1}{16}(6A+5C)x+\frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{C\cos^5(c+dx)\sin(}{6d}$$

```
[Out] 1/16*(6*A+5*C)*x+1/16*(6*A+5*C)*cos(d*x+c)*sin(d*x+c)/d+1/24*(6*A+5*C)*cos(
d*x+c)^3*sin(d*x+c)/d+1/6*C*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$\frac{(6A+5C)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6A+5C)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6A+5C)+\frac{C\sin(c+dx)\cos^5(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((6*A + 5*C)*x)/16 + ((6*A + 5*C)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((6*A
 + 5*C)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (C*Cos[c + d*x]^5*Sin[c + d*x
])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(6A+5C)\int \cos^4(c+dx)\,dx$$
$$= \frac{(6A+5C)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{C\cos^5(c+dx)\sin(c+dx)}{6d}$$
$$= \frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(6A+5C)\cos^3(c+dx)\sin}{24d}$$
$$= \frac{1}{16}(6A+5C)x + \frac{(6A+5C)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(6A+5}{}$$

Mathematica [A]
time = 0.11, size = 68, normalized size = 0.76

$$\frac{72Ac+60cC+72Adx+60Cdx+(48A+45C)\sin(2(c+dx))+(6A+9C)\sin(4(c+dx))+C\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (72*A*c + 60*c*C + 72*A*d*x + 60*C*d*x + (48*A + 45*C)*Sin[2*(c + d*x)] + (
6*A + 9*C)*Sin[4*(c + d*x)] + C*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.17, size = 86, normalized size = 0.97

method	result
risch	$\frac{3Ax}{8}+\frac{5Cx}{16}+\frac{\sin(6dx+6c)C}{192d}+\frac{\sin(4dx+4c)A}{32d}+\frac{3\sin(4dx+4c)C}{64d}+\frac{\sin(2dx+2c)A}{4d}+\frac{15\sin(2dx+2c)C}{64d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{d}$
default	$\frac{C\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{d}$
norman	$\left(\frac{3A}{8}+\frac{5C}{16}\right)x+\left(\frac{3A}{8}+\frac{5C}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9A}{4}+\frac{15C}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15A}{2}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)
)
```

Maxima [A]
time = 0.50, size = 103, normalized size = 1.16

$$\frac{3\,(dx+c)(6\,A+5\,C)+\frac{3\,(6\,A+5\,C)\tan(dx+c)^{5}+8\,(6\,A+5\,C)\tan(dx+c)^{3}+3\,(10\,A+11\,C)\tan(dx+c)}{\tan(dx+c)^{6}+3\,\tan(dx+c)^{4}+3\,\tan(dx+c)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(6*A + 5*C) + (3*(6*A + 5*C)*tan(d*x + c)^5 + 8*(6*A + 5*
C)*tan(d*x + c)^3 + 3*(10*A + 11*C)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 68, normalized size = 0.76

$$\frac{3\,(6\,A+5\,C)dx+\left(8\,C\cos\left(dx+c\right)^{5}+2\,(6\,A+5\,C)\cos\left(dx+c\right)^{3}+3\,(6\,A+5\,C)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*A + 5*C)*d*x + (8*C*cos(d*x + c)^5 + 2*(6*A + 5*C)*cos(d*x + c)^
3 + 3*(6*A + 5*C)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(82) = 164$.
time = 0.41, size = 258, normalized size = 2.90

$$\begin{cases} \frac{3Ax\sin^{4}(c+dx)}{8}+\frac{3Ax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{3Ax\cos^{4}(c+dx)}{8}+\frac{3A\sin^{3}(c+dx)\cos(c+dx)}{8d}+\frac{5A\sin(c+dx)\cos^{3}(c+dx)}{8d}+\frac{5Cx\sin^{6}(c+dx)}{16}+\frac{15Cx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{15Cx\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{5Cx\cos^{6}(c+dx)}{16}+\frac{5C\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{5C\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d}+\frac{11C\sin(c+dx)\cos^{5}(c+dx)}{16d} & \text{for}\: d\neq 0 \\ x(A+C\cos^{2}(c))\cos^{4}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((3*A*x*sin(c + d*x)**4/8 + 3*A*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*A*x*cos(c + d*x)**4/8 + 3*A*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*C*x*sin(c + d*x)**6/16 + 15*C*x*sin(
c + d*x)**4*cos(c + d*x)**2/16 + 15*C*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 5*C*x*cos(c + d*x)**6/16 + 5*C*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*
sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*sin(c + d*x)*cos(c + d*x)**5/(
16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**4, True))
```

Giac [A]
time = 0.41, size = 68, normalized size = 0.76

$$\frac{1}{16}(6A+5C)x+\frac{C\sin(6dx+6c)}{192d}+\frac{(2A+3C)\sin(4dx+4c)}{64d}+\frac{(16A+15C)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*A + 5*C)*x + 1/192*C*sin(6*d*x + 6*c)/d + 1/64*(2*A + 3*C)*sin(4*d*
x + 4*c)/d + 1/64*(16*A + 15*C)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 1.22, size = 91, normalized size = 1.02

$$x\left(\frac{3A}{8}+\frac{5C}{16}\right)+\frac{\left(\frac{3A}{8}+\frac{5C}{16}\right)\tan(c+dx)^5+\left(A+\frac{5C}{6}\right)\tan(c+dx)^3+\left(\frac{5A}{8}+\frac{11C}{16}\right)\tan(c+dx)}{d\left(\tan(c+dx)^6+3\tan(c+dx)^4+3\tan(c+dx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + C*cos(c + d*x)^2),x)
```

```
[Out] x*((3*A)/8 + (5*C)/16) + (tan(c + d*x)*((5*A)/8 + (11*C)/16) + tan(c + d*x)
^3*(A + (5*C)/6) + tan(c + d*x)^5*((3*A)/8 + (5*C)/16))/(d*(3*tan(c + d*x)^
2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

3.11 $\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}(4A+3C)x+\frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d}+\frac{C\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(4*A+3*C)*x+1/8*(4*A+3*C)*cos(d*x+c)*sin(d*x+c)/d+1/4*C*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3093, 2715, 8}

$$\frac{(4A+3C)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4A+3C)+\frac{C\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x)/8 + ((4*A + 3*C)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (C*Cos[
c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx = \frac{C\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(4A+3C)\int \cos^2(c+dx)\,dx$$
$$= \frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{C\cos^3(c+dx)\sin(c+dx)}{4d}$$
$$= \frac{1}{8}(4A+3C)x + \frac{(4A+3C)\cos(c+dx)\sin(c+dx)}{8d} + \frac{C\cos^3(c+$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.74

$$\frac{4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2),x]
[Out] (4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A]
time = 0.12, size = 65, normalized size = 1.07

method	result
risch	$\frac{Ax}{2}+\frac{3Cx}{8}+\frac{\sin(4dx+4c)C}{32d}+\frac{\sin(2dx+2c)A}{4d}+\frac{\sin(2dx+2c)C}{4d}$
derivativedivides	$\frac{C\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{C\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(\frac{A}{2}+\frac{3C}{8}\right)x+\left(2A+\frac{3C}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2A+\frac{3C}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3A+\frac{9C}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{A}{2}+\frac{3C}{8}\right)x\left(t\right.}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d*(C*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*(1/2*
sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 73, normalized size = 1.20

$$\frac{(dx+c)(4\,A+3\,C)+\frac{(4\,A+3\,C)\tan(dx+c)^3+(4\,A+5\,C)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(4*A + 3*C) + ((4*A + 3*C)*tan(d*x + c)^3 + (4*A + 5*C)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 49, normalized size = 0.80

$$\frac{(4\,A+3\,C)dx+\left(2\,C\cos\left(dx+c\right)^{3}+(4\,A+3\,C)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*A + 3*C)*d*x + (2*C*cos(d*x + c)^3 + (4*A + 3*C)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(53) = 106$.
time = 0.18, size = 158, normalized size = 2.59

$$\begin{cases} \frac{Ax\sin^{2}\left(c+dx\right)}{2}+\frac{Ax\cos^{2}\left(c+dx\right)}{2}+\frac{A\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{3Cx\sin^{4}\left(c+dx\right)}{8}+\frac{3Cx\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3Cx\cos^{4}\left(c+dx\right)}{8}+\frac{3C\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5C\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d} & \text{for}\: d\neq 0 \\ x\left(A+C\cos^{2}\left(c\right)\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((A*x*sin(c + d*x)**2/2 + A*x*cos(c + d*x)**2/2 + A*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*C*x*cos(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 5*C*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2
)*cos(c)**2, True))
```

Giac [A]
time = 0.40, size = 43, normalized size = 0.70

$$\frac{1}{8}\,(4\,A+3\,C)x+\frac{C\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(A+C)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*A + 3*C)*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/4*(A + C)*sin(2*d*x + 2*c
)/d
```

Mupad [B]
time = 0.78, size = 67, normalized size = 1.10

$$x\left(\frac{A}{2}+\frac{3\,C}{8}\right)+\frac{\left(\frac{A}{2}+\frac{3\,C}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{A}{2}+\frac{5\,C}{8}\right)\,\tan(c+d\,x)}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2),x)
```

```
[Out] x*(A/2 + (3*C)/8) + (tan(c + d*x)*(A/2 + (5*C)/8) + tan(c + d*x)^3*(A/2 + (
3*C)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.12 $\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=15

$$Cx+\frac{A\tan(c+dx)}{d}$$

```
[Out] C*x+A*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3091, 8}

$$\frac{A\tan(c+dx)}{d}+Cx$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (A*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx &= \frac{A\tan(c+dx)}{d}+C\int 1\,dx\\ &= Cx+\frac{A\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$Cx+\frac{A\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] C*x + (A*Tan[c + d*x])/d
```

Maple [A]
time = 0.14, size = 21, normalized size = 1.40

method	result
derivativedivides	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
default	$\frac{A\tan(dx+c)+C(dx+c)}{d}$
risch	$Cx+\frac{2iA}{d(\mathrm{e}^{2i(dx+c)}+1)}$
norman	$\frac{Cx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+Cx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-Cx-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-Cx\left(\tan^2\left(\frac{dx}{2}\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*tan(d*x+c)+C*(d*x+c))
```

Maxima [A]
time = 0.49, size = 20, normalized size = 1.33

$$\frac{(dx+c)C+A\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*C + A*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.36, size = 31, normalized size = 2.07

$$\frac{Cdx\cos(dx+c)+A\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (C*d*x*cos(d*x + c) + A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C\cos^{2}\left(c + dx\right)\right)\sec^{2}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.41, size = 20, normalized size = 1.33

$$\frac{(dx + c)C + A\tan\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*C + A*tan(d*x + c))/d
```

Mupad [B]
time = 0.64, size = 17, normalized size = 1.13

$$\frac{A\tan(c + d\,x) + C\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^2,x)
```

```
[Out] (A*tan(c + d*x) + C*d*x)/d
```

3.13 $\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\sec^2(c+dx)\tan(c+dx)}{3d}$$

```
[Out] 1/3*(2*A+3*C)*tan(d*x+c)/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3091, 3852, 8}

$$\frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] ((2*A + 3*C)*Tan[c + d*x])/(3*d) + (A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx &= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}(2A+3C)\int \sec^2(c+dx)\,dx\\ &= \frac{A\sec^2(c+dx)\tan(c+dx)}{3d}-\frac{(2A+3C)\text{Subst}(\int 1\,dx,x,-\tan(}{3d}\\ &= \frac{(2A+3C)\tan(c+dx)}{3d}+\frac{A\sec^2(c+dx)\tan(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 36, normalized size = 0.84

$$\frac{C\tan(c+dx)}{d}+\frac{A\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
[Out] (C*Tan[c + d*x])/d + (A*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.18, size = 35, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
default	$\frac{-A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+C\tan(dx+c)}{d}$	35
risch	$\frac{2i\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	63
norman	$\frac{-\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8A\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4(A-3C)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+C*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.63

$$\frac{A\tan\left(dx+c\right)^3+3\left(A+C\right)\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")
[Out] 1/3*(A*tan(d*x + c)^3 + 3*(A + C)*tan(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.86

$$\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^2+A\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C\cos^2\left(c + dx\right)\right)\sec^4\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.40, size = 34, normalized size = 0.79

$$\frac{A\tan\left(dx + c\right)^3 + 3\,A\tan\left(dx + c\right) + 3\,C\tan\left(dx + c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(A*tan(d*x + c)^3 + 3*A*tan(d*x + c) + 3*C*tan(d*x + c))/d
```

Mupad [B]
time = 0.64, size = 28, normalized size = 0.65

$$\frac{A\tan(c + d\,x)^3}{3\,d} + \frac{\tan(c + d\,x)\ (A + C)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^4,x)
```

```
[Out] (A*tan(c + d*x)^3)/(3*d) + (tan(c + d*x)*(A + C))/d
```

3.14 $\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{(4A+5C)\tan(c+dx)}{5d}+\frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{(4A+5C)\tan^3(c+dx)}{15d}$$

[Out] 1/5*(4*A+5*C)*tan(d*x+c)/d+1/5*A*sec(d*x+c)^4*tan(d*x+c)/d+1/15*(4*A+5*C)*tan(d*x+c)^3/d

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3091, 3852}

$$\frac{(4A+5C)\tan^3(c+dx)}{15d}+\frac{(4A+5C)\tan(c+dx)}{5d}+\frac{A\tan(c+dx)\sec^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] ((4*A + 5*C)*Tan[c + d*x])/(5*d) + (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*A + 5*C)*Tan[c + d*x]^3)/(15*d)

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^6(c+dx)\,dx &= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}(4A+5C)\int \sec^4(c+dx)\,dx\\ &= \frac{A\sec^4(c+dx)\tan(c+dx)}{5d}-\frac{(4A+5C)\text{Subst}\left(\int (1+x^2)\,dx,x,-\right.}{5d}\\ &= \frac{(4A+5C)\tan(c+dx)}{5d}+\frac{A\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{(4A+5C}{}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 61, normalized size = 0.94

$$\frac{C\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}+\frac{A\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d + (A*(Tan[c + d*x] + (2*Tan[c + d*x
]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A]
time = 0.24, size = 58, normalized size = 0.89

method	result
derivativedivides	$\frac{-A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{-A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)-C\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$\frac{4i\left(15C\,e^{6i(dx+c)}+40A\,e^{4i(dx+c)}+35C\,e^{4i(dx+c)}+20A\,e^{2i(dx+c)}+25C\,e^{2i(dx+c)}+4A+5C\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{4(A-C)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4(A-C)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(A+C)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2(A+C)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(11A-5C)\left(\tan^5\left(\frac{dx}{2}\right.\right.}{5d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-C*(-2/3-1/3*s
ec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.66

$$\frac{3\,A\tan\left(dx+c\right)^5+5\left(2\,A+C\right)\tan\left(dx+c\right)^3+15\left(A+C\right)\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*A*tan(d*x + c)^5 + 5*(2*A + C)*tan(d*x + c)^3 + 15*(A + C)*tan(d*x
+ c))/d
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.86

$$\frac{\left(2\left(4A+5C\right)\cos\left(dx+c\right)^{4}+\left(4A+5C\right)\cos\left(dx+c\right)^{2}+3A\right)\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(2*(4*A + 5*C)*cos(d*x + c)^4 + (4*A + 5*C)*cos(d*x + c)^2 + 3*A)*sin(
d*x + c)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 57, normalized size = 0.88

$$\frac{3A\tan\left(dx+c\right)^{5}+10A\tan\left(dx+c\right)^{3}+5C\tan\left(dx+c\right)^{3}+15A\tan\left(dx+c\right)+15C\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/15*(3*A*tan(d*x + c)^5 + 10*A*tan(d*x + c)^3 + 5*C*tan(d*x + c)^3 + 15*A*
tan(d*x + c) + 15*C*tan(d*x + c))/d
```

Mupad [B]
time = 0.66, size = 42, normalized size = 0.65

$$\frac{\frac{A\tan(c+d\,x)^5}{5}+\left(\frac{2\,A}{3}+\frac{C}{3}\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^6,x)
```

```
[Out] ((A*tan(c + d*x)^5)/5 + tan(c + d*x)*(A + C) + tan(c + d*x)^3*((2*A)/3 + C/
3))/d
```

3.15 $\int (A+C\cos^2(c+dx))\sec^8(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{(6A+7C)\tan(c+dx)}{7d}+\frac{A\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{2(6A+7C)\tan^3(c+dx)}{21d}+\frac{(6A+7C)\tan^5(c+dx)}{35d}$$

```
[Out] 1/7*(6*A+7*C)*tan(d*x+c)/d+1/7*A*sec(d*x+c)^6*tan(d*x+c)/d+2/21*(6*A+7*C)*t
an(d*x+c)^3/d+1/35*(6*A+7*C)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.04, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3091, 3852}

$$\frac{(6A+7C)\tan^5(c+dx)}{35d}+\frac{2(6A+7C)\tan^3(c+dx)}{21d}+\frac{(6A+7C)\tan(c+dx)}{7d}+\frac{A\tan(c+dx)\sec^6(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] ((6*A + 7*C)*Tan[c + d*x])/(7*d) + (A*Sec[c + d*x]^6*Tan[c + d*x])/(7*d) +
(2*(6*A + 7*C)*Tan[c + d*x]^3)/(21*d) + ((6*A + 7*C)*Tan[c + d*x]^5)/(35*d)
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (A+C\cos^2(c+dx))\sec^8(c+dx)\,dx &= \frac{A\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{1}{7}(6A+7C)\int \sec^6(c+dx)\,dx\\ &= \frac{A\sec^6(c+dx)\tan(c+dx)}{7d}-\frac{(6A+7C)\text{Subst}\left(\int (1+2x^2+x^4)\right.}{7d}\\ &= \frac{(6A+7C)\tan(c+dx)}{7d}+\frac{A\sec^6(c+dx)\tan(c+dx)}{7d}+\frac{2(6A+\end{aligned}$$

Mathematica [A]
time = 0.33, size = 81, normalized size = 0.93

$$\frac{C\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}+\frac{A\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]
```

```
[Out] (C*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d + (A*(Tan[c
+ d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A]
time = 0.20, size = 78, normalized size = 0.90

method	result
derivativedivides	$\frac{-A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
default	$\frac{-A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)-C\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
risch	$\frac{16i\left(70C\,e^{8i(dx+c)}+210A\,e^{6i(dx+c)}+175C\,e^{6i(dx+c)}+126A\,e^{4i(dx+c)}+147C\,e^{4i(dx+c)}+42A\,e^{2i(dx+c)}+49C\,e^{2i(dx+c)}+6A\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d
*x+c)-C*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 60, normalized size = 0.69

$$\frac{15\,A\tan\left(dx+c\right)^7+21\,(3\,A+C)\tan\left(dx+c\right)^5+35\,(3\,A+2\,C)\tan\left(dx+c\right)^3+105\,(A+C)\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*A*tan(d*x + c)^7 + 21*(3*A + C)*tan(d*x + c)^5 + 35*(3*A + 2*C)*t
an(d*x + c)^3 + 105*(A + C)*tan(d*x + c))/d
```

Fricas [A]
time = 0.34, size = 74, normalized size = 0.85

$$\frac{\left(8\,(6\,A+7\,C)\cos\left(dx+c\right)^6+4\,(6\,A+7\,C)\cos\left(dx+c\right)^4+3\,(6\,A+7\,C)\cos\left(dx+c\right)^2+15\,A\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*A + 7*C)*cos(d*x + c)^6 + 4*(6*A + 7*C)*cos(d*x + c)^4 + 3*(6*A
 + 7*C)*cos(d*x + c)^2 + 15*A)*sin(d*x + c)/(d*cos(d*x + c)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.41, size = 79, normalized size = 0.91

$$\frac{15\,A\tan\left(dx+c\right)^{7}+63\,A\tan\left(dx+c\right)^{5}+21\,C\tan\left(dx+c\right)^{5}+105\,A\tan\left(dx+c\right)^{3}+70\,C\tan\left(dx+c\right)^{3}+105\,A\tan\left(dx+c\right)+105\,C\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/105*(15*A*tan(d*x + c)^7 + 63*A*tan(d*x + c)^5 + 21*C*tan(d*x + c)^5 + 10
5*A*tan(d*x + c)^3 + 70*C*tan(d*x + c)^3 + 105*A*tan(d*x + c) + 105*C*tan(d
*x + c))/d
```

Mupad [B]
time = 0.66, size = 56, normalized size = 0.64

$$\frac{\frac{A\tan(c+d\,x)^7}{7}+\left(\frac{3\,A}{5}+\frac{C}{5}\right)\,\tan(c+d\,x)^5+\left(A+\frac{2\,C}{3}\right)\,\tan(c+d\,x)^3+(A+C)\,\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/cos(c + d*x)^8,x)
```

```
[Out] ((A*tan(c + d*x)^7)/7 + tan(c + d*x)^3*(A + (2*C)/3) + tan(c + d*x)*(A + C)
 + tan(c + d*x)^5*((3*A)/5 + C/5))/d
```

3.16 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=113

$$\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx}{9}$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3093, 2715, 2721, 2719}

$$\frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{1}{9}(9A+7C)\int (b\cos(c$$

$$=\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+$$

$$=\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+$$

$$=\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+$$

Mathematica [A]
time = 0.44, size = 88, normalized size = 0.78

$$\frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))\right)}{180d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.
time = 0.40, size = 324, normalized size = 2.87

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 128, normalized size = 1.13

$$\frac{3i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(5Cb^2\cos(dx+c)^3+(9A+7C)b^2\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(5
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x + c))*sqrt
(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

3.17 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=113

$$\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))}{7bd}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = $\{3093, 2715, 2721, 2720\}$

$$\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{1}{7}(7A+5C)\int (b\cos(c$$

$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+d}{}$$

$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+d}{}$$

$$=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5}{}$$

Mathematica [A]
time = 0.39, size = 86, normalized size = 0.76

$$\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)\right)}{42d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[
c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(125) = 250$.
time = 0.35, size = 296, normalized size = 2.62

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 112, normalized size = 0.99

$$\frac{-i\sqrt{2}\,(7\,A+5\,C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(7\,A+5\,C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\left(3\,Cb\cos\left(dx+c\right)^{2}+(7\,A+5\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b*cos(d*x + c)^2 + (7*A + 5*C)*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3093, 2721, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{\left((5A+3C)\sqrt{b\cos(c+}\right.}{5\sqrt{\cos(}}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+}{}$$

Mathematica [A]
time = 0.15, size = 70, normalized size = 0.91

$$\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(93) = 186.
time = 0.35, size = 261, normalized size = 3.39

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{5\sqrt{-b\left(2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 101, normalized size = 1.31

$$\frac{2\sqrt{b\cos(dx+c)}\,C\cos(dx+c)\sin(dx+c)+\sqrt{2}\,(5i\,A+3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\sqrt{2}\,(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.19 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3093, 2721, 2720}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}$$

Mathematica [**A**]
time = 0.16, size = 58, normalized size = 0.77

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.
time = 0.34, size = 236, normalized size = 3.15

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2(\mathrm{si}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d
*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 92, normalized size = 1.23

$$\frac{\sqrt{2}\,(-3i\,A - i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)} + i\,\sin{(dx+c)}) + \sqrt{2}\,(3i\,A + i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)} - i\,\sin{(dx+c)}) + 2\,\sqrt{b\cos{(dx+c)}}\,C\sin{(dx+c)}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.32, size = 94, normalized size = 1.25

$$\frac{2\,C\,\sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{3\,b\,d}+\frac{2\,A\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}+\frac{2\,C\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d*x
)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.20 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2719}

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b^2} \\
&= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.16, size = 57, normalized size = 0.77

$$
\frac{-2(A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(94) = 188$.
time = 0.39, size = 216, normalized size = 2.92

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\right.}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 117, normalized size = 1.58

$$\frac{\sqrt{2}\,(-i\,A+i\,C)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,(i\,A-i\,C)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{b^{2}d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*co
s(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.21 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3091, 2721, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$

$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.24, size = 58, normalized size = 0.74

$$\frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]
))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(94) = 188.
time = 0.35, size = 294, normalized size = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+\right.}{3b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 116, normalized size = 1.49

$$\frac{\sqrt{2}\,(-i\,A-3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,A+3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{3\,b^{3}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.22 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(
d*x+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3091, 2716, 2721, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx)}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right.}{5b^4\sqrt{\cos(c+}$$
$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A}{5b^3d}$$

Mathematica [A]
time = 0.29, size = 81, normalized size = 0.70

$$\frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\sec(c+dx)\tan(c+dx)\right)}{5b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(127) = 254$.
time = 0.73, size = 601, normalized size = 5.23

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2
-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 139, normalized size = 1.21

$$\frac{\sqrt{2}\,(-3i\,A-5i\,C)\sqrt{b}\,\cos(dx+c)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(3i\,A+5i\,C)\sqrt{b}\,\cos(dx+c)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left((3\,A+5\,C)\cos(dx+c)^{2}+A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{5\,b^{4}d\cos(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.23 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b^3/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3091, 2716, 2721, 2720}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{7b^2}$$

$$= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx)}}}{21b^4}$$

$$= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right)}{21b^4\sqrt{b\cos(c}}$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7}{21b^3d(b\,c}$$

Mathematica [A]
time = 0.42, size = 77, normalized size = 0.67

$$\frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + (5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21b^4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(127) = 254$.
time = 0.65, size = 413, normalized size = 3.59

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 135, normalized size = 1.17

$$\frac{\sqrt{2}\left(-5i\,A-7i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^4\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(5i\,A+7i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^4\operatorname{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\left(\left(5\,A+7\,C\right)\cos\left(dx+c\right)^2+3\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,b^5d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^5*d*
cos(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\left(b\,\cos\left(c+d\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

3.24 $\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=21

$$-\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}$$

```
[Out] -2*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3090}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/d
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\left(3-5\cos^2(c+dx)\right)\,dx=-\frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.06, size = 23, normalized size = 1.10

$$-\frac{\sqrt{\cos(c+dx)}\,\sin(2(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(3 - 5*Cos[c + d*x]^2),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)])/d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(19) = 38.
time = 0.22, size = 99, normalized size = 4.71

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Fricas [A]
time = 0.36, size = 19, normalized size = 0.90

$$-\frac{2\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*cos(d*x + c)^(3/2)*sin(d*x + c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(-(5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$-\int \sqrt{\cos\left(c+d\,x\right)}\,\left(5\cos\left(c+d\,x\right)^2-3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3),x)
[Out] -int(cos(c + d*x)^(1/2)*(5*cos(c + d*x)^2 - 3), x)
```

3.25 $\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=21

$$-\frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] -2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3090}

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int \frac{1-3\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 1.00

$$-\frac{2\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*Cos[c + d*x]^2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(19) = 38$.
time = 0.24, size = 99, normalized size = 4.71

method	result	s
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.90

$$-\frac{2\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-3*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.77, size = 19, normalized size = 0.90

$$-\frac{2\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*cos(c + d*x)^2 - 1)/cos(c + d*x)^(1/2),x)
```

```
[Out] -(2*cos(c + d*x)^(1/2)*sin(c + d*x))/d
```

3.26 $\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{9/2}\,dx$

Optimal. Leaf size=115

$$\frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}$$

```
[Out] 2/21*b^3*(5*A+7*C)*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/21*b^4*(5*A+7*C)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/7*A*b^2*(b*sec(d*x+c))^(5
/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3317, 4131, 3853, 3856, 2720}

$$\frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3(5A+7C)\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}+\frac{2Ab^2\tan(c+dx)(b\sec(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*b^4*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(21*d) + (2*b^3*(5*A + 7*C)*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])
/(21*d) + (2*A*b^2*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{9/2}\,dx = b^2\int (b\sec(c+dx))^{5/2}\left(C+A\sec^2(c+dx)\right)\,dx$$
$$= \frac{2Ab^2(b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{1}{7}\left(b^2(5A+7C)\right)\int (b\,\mathrm{se}$$
$$= \frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2Ab^2(b\sec(c+}{}$$
$$= \frac{2b^3(5A+7C)(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2Ab^2(b\sec(c+}{}$$
$$= \frac{2b^4(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Mathematica [A]
time = 0.89, size = 78, normalized size = 0.68

$$\frac{b^2(b\sec(c+dx))^{5/2}\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C)\sin(2(c+dx))+6A\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2),x]
```

```
[Out] (b^2*(b*Sec[c + d*x])^(5/2)*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)
```

Maple [C] Result contains complex when optimal does not.
time = 4.18, size = 249, normalized size = 2.17

method	result

default	$-\frac{2(-1+\cos(dx+c))\left(5iA\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)(\cos^3(dx+c))\sin(dx+c)+7iC\sqrt{\frac{}{1+c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(-1+cos(d*x+c))*(5*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d
*x+c)+7*I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Elli
pticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)-5*A*cos(d*x+c
)^3-7*C*cos(d*x+c)^3+5*A*cos(d*x+c)^2+7*C*cos(d*x+c)^2-3*A*cos(d*x+c)+3*A)*
cos(d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(9/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 142, normalized size = 1.23

$$\frac{-i\sqrt{2}\,(5A+7C)b^{\frac{9}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(5A+7C)b^{\frac{9}{2}}\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left((5A+7C)b^4\cos(dx+c)^2+3Ab^4\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{21\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*x + c)^3*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(9/2)*cos(d*x
+ c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b^4*cos(d*x + c)^2 + 3*A*b^4)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{9/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(9/2), x)
```

3.27 $\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx$

Optimal. Leaf size=115

$$-\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] -2/5*b^4*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b
^3*(3*A+5*C)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/5*A*b^2*(b*sec(d*x+c))^(3/
2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3317, 4131, 3853, 3856, 2719}

$$-\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3(3A+5C)\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2Ab^2\tan(c+dx)(b\sec(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*b^4*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*b^3*(3*A + 5*C)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(
5*d) + (2*A*b^2*(b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{7/2}\,dx = b^2\int (b\sec(c+dx))^{3/2}\left(C+A\sec^2(c+dx)\right)\,dx$$
$$= \frac{2Ab^2(b\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{1}{5}\left(b^2(3A+5C)\right)\int (b\,\mathrm{se}$$
$$= \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2Ab^2(b\sec(c+}{}$$
$$= \frac{2b^3(3A+5C)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2Ab^2(b\sec(c+}{}$$
$$= -\frac{2b^4(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2b^3(3A+5C)\sqrt{b\sec(}}{5d}$$

Mathematica [A]
time = 0.52, size = 79, normalized size = 0.69

$$-\frac{b^2(b\sec(c+dx))^{3/2}\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2A\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] -1/5*(b^2*(b*Sec[c + d*x])^(3/2)*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 0.48, size = 666, normalized size = 5.79

method	result
default	$\frac{2(-1+\cos(dx+c))^2\left(3iA\left(\cos^3(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-3iA\sqrt{\frac{}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(-1+cos(d*x+c))^2*(3*I*A*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+
c),I)-3*I*A*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*C*cos(d*x
+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*C*cos(d*x+c)^3*sin(d*x+c)*(1
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)+3*I*A*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)-3*I*A*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*C*cos(d*x+
c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*C*cos(d*x+c)^2*sin(d*x+c)*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)-3*A*cos(d*x+c)^3-5*C*cos(d*x+c)^3+2*A*cos(d*x+c)^2+5
*C*cos(d*x+c)^2+A)*cos(d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(7/2)/sin(d*x
+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 147, normalized size = 1.28

$$\frac{-i\sqrt{2}\,(3A+5C)b^{\frac{7}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(3A+5C)b^{\frac{7}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left((3A+5C)b^3\cos(dx+c)^2+Ab^3\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b^3*cos(d*x + c)^2 +
A*b^3)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(7/2), x)
```

3.28 $\int (A + C\cos^2(c + dx)) (b\sec(c + dx))^{5/2} dx$

Optimal. Leaf size=78

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}\, F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b\sec(c + dx)}}{3d} + \frac{2Ab^2\sqrt{b\sec(c + dx)}\,\tan(c + dx)}{3d}$$

```
[Out] 2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/3*A*b^
2*(b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4131, 3856, 2720}

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}\, F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b\sec(c + dx)}}{3d} + \frac{2Ab^2\tan(c + dx)\sqrt{b\sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c
+ d*x]])/(3*d) + (2*A*b^2*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
```

```
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{5/2}\,dx &= b^2\int \sqrt{b\sec(c+dx)}\left(C+A\sec^2(c+dx)\right)\,dx\\
&= \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\left(b^2(A+3C)\right)\int \sqrt{b\mathrm{se}\\
&= \frac{2Ab^2\sqrt{b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\Big(b^2(A+3C)\sqrt{\cos(c+\\
&= \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 58, normalized size = 0.74

$$\frac{2b^2\sqrt{b\sec(c+dx)}\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + A*Tan[c + d*x]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 199, normalized size = 2.55

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(iA\cos(dx+c)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)+3iC\cos(dx+c)\sin}{3d\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)
+3*I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*
```

```
x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-A*cos(d*x+c)+A)*cos(
d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 1.44

$$\frac{-i\sqrt{2}\,(A+3\,C)b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\,\sqrt{2}\,(A+3\,C)b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,Ab^2\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*A*b^2*sqrt(b/co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(5/2), x)
```

3.29 $\int (A+C\cos^2(c+dx))\,(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=74

$$-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] -2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*A*b^2*tan
(d*x+c)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4131, 3856, 2719}

$$\frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*b^2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec
[c + d*x]]) + (2*A*b^2*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
```

```
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(A+C\cos^2(c+dx)\right)(b\sec(c+dx))^{3/2}\,dx &= b^2\int \frac{C+A\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\left(b^2(A-C)\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{\left(b^2(A-C)\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= -\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab^2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 55, normalized size = 0.74

$$
\frac{2b\sqrt{b\sec(c+dx)}\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + A*Sin[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 591, normalized size = 7.99

method	result
default	$-\frac{2\left(iA\cos(dx+c)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-iA\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+}{1+\cos(dx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*A*(1/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/s
```

```
in(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)
*sin(d*x+c)+I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*A*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*sin(d*x+c)-I*A*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-I*C*
sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+I*C*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
I)+C*cos(d*x+c)^2+A*cos(d*x+c)-C*cos(d*x+c)-A)*cos(d*x+c)*(b/cos(d*x+c))^(3
/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 95, normalized size = 1.28

$$\frac{-i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,Ab\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*(A - C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(3/2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A
*b*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(3/2), x)
```

3.30 $\int (A+C\cos^2(c+dx))\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d+2/3*b^2*C*ta
n(d*x+c)/d/(b*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4130, 3856, 2720}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*
x]])/(3*d) + (2*b^2*C*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \left(A+C\cos^2(c+dx)\right)\sqrt{b\sec(c+dx)}\,dx = b^2\int \frac{C+A\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}+\frac{1}{3}(3A+C)\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^2C\tan(c+dx)}{3d(b\sec(c+dx))^{3/2}}+\frac{1}{3}\Big((3A+C)\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+$$
$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+$$

Mathematica [A]
time = 0.15, size = 58, normalized size = 0.77

$$\frac{\sqrt{b\sec(c+dx)}\left(2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2] + C*Sin[2*(c + d*x)]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 190, normalized size = 2.53

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(3iA\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)+iC\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx-}}\right.}{3d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(3*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*C*sin(
d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF
```

```
(I*(-1+cos(d*x+c))/sin(d*x+c),I)-C*cos(d*x+c)^2+C*cos(d*x+c))*(1+cos(d*x+c)
)^2*(b/cos(d*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 97, normalized size = 1.29

$$\frac{2C\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+\sqrt{2}\,(-3iA-iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}\,(3iA+iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-3*I*A -
 I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + s
qrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sec(c + d*x))*(A + C*cos(c + d*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b/cos(c + d*x))^(1/2), x)
```

3.31 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

```
[Out] 2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/5*b^2*C*
tan(d*x+c)/d/(b*sec(d*x+c))^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3317, 4130, 3856, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Se
c[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^2\int \frac{C+A\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}+\frac{1}{5}(5A+3C)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}+\frac{(5A+3C)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{5d(b\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 61, normalized size = 0.79

$$\frac{\frac{4(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}+2C\sin(2(c+dx))}{10d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]
[Out] ((4*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*C*Sin[2*(
c + d*x)])/(10*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 609, normalized size = 7.91

method	result
default	$\frac{2\left(5iA\cos(dx+c)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)-5iA\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+}{1+\cos(dx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(5*I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-5*I*A*(1/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*El
lipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x
+c))/sin(d*x+c),I)-3*I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+5*
I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*
(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-5*I*A*(1/(1+cos(d*x+c)))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d
*x+c),I)+3*I*C*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*C*sin(d*x+c)*(1/(1+
cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*
x+c))/sin(d*x+c),I)-C*cos(d*x+c)^4-5*A*cos(d*x+c)^2-2*C*cos(d*x+c)^2+5*A*co
s(d*x+c)+3*C*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 108, normalized size = 1.40

$$\frac{2\,C\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\sqrt{2}\left(5i\,A+3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\left(-5i\,A-3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*C*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + sqrt(2)*(5*I*A
+ 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^{2}\left(c + dx\right)}{\sqrt{b\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(1/2), x)
```

3.32 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

```
[Out] 2/21*(7*A+5*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)+2/21*(7*A+5*C)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d+2/7*b^2*C*tan(d*x+c)/d/(b*sec
(d*x+c))^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3317, 4130, 3854, 3856, 2720}

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(21*b^2*d) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*b*d*Sqrt[b*Sec[c + d*x
]]) + (2*b^2*C*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = b^2\int \frac{C+A\sec^2(c+dx)}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{1}{7}(7A+5C)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{(7A+5C)\int\sqrt{b\sec(c+dx)}$$
$$= \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}} + \frac{2b^2C\tan(c+dx)}{7d(b\sec(c+dx))^{7/2}} + \frac{\left((7A+5C)\sqrt{\cos(c+dx)}\,\sqrt{\right.$$
$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sin(}{21bd\sqrt{b\sec(c+}$$

Mathematica [A]
time = 0.58, size = 79, normalized size = 0.69

$$\frac{\frac{4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)}{42bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(14*A + 1
3*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.67, size = 241, normalized size = 2.10

method	result
default	$-\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(7iA\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)+5iC\sin(d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(7*I*A*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*s
in(d*x+c)+5*I*C*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*C*cos(d*x+c)^4+3*C*c
os(d*x+c)^3-7*A*cos(d*x+c)^2-5*C*cos(d*x+c)^2+7*A*cos(d*x+c)+5*C*cos(d*x+c)
)/sin(d*x+c)^3/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 119, normalized size = 1.03

$$\frac{\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(3\,C\cos(dx+c)^3+(7\,A+5\,C)\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{21\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^3 + (7*A + 5*C
)*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(3/2), x)
```

3.33 $\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

```
[Out] 2/45*(9*A+7*C)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)+2/15*(9*A+7*C)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/9*b^2*C*tan(d*x+c)/d/(b*sec
(d*x+c))^(9/2)
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3317, 4130, 3854, 3856, 2719}

$$\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt
[b*Sec[c + d*x]]) + (2*(9*A + 7*C)*Sin[c + d*x])/(45*b*d*(b*Sec[c + d*x])^(
3/2)) + (2*b^2*C*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= b^2\int \frac{C+A\sec^2(c+dx)}{(b\sec(c+dx))^{9/2}}\,dx\\ &= \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{1}{9}(9A+7C)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{(9A+7C)\int \frac{1}{\sqrt{b\sec(c+dx)}}}{15b^2}\\ &= \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}+\frac{(9A+7C)\int \sqrt{\cos(c+dx)}}{15b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+}}\\ &= \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b^2C\tan(c}{9d(b\sec(c+}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 81, normalized size = 0.70

$$\frac{\frac{48(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}+4(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))}{360b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((48*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*(18*A +
19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(360*b^2*d*Sqrt[b*Sec[c + d*
x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 636, normalized size = 5.53

method	result
default	$-\frac{2\left(27iA\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)-27iA\cos(dx+c)\sin(dx+c)\sqrt{\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(27*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-27*I*A*cos(
d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*C*cos(d*x+c)^6+21*I*C*(1/(1+c
os(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-21*I*C*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(-1+cos(d
*x+c))/sin(d*x+c),I)+27*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)-27*I*A*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos
(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+21*I*C*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),
I)-21*I*C*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*A*cos(d*x+c)^4+2*C*cos(d*x
+c)^4+18*A*cos(d*x+c)^2+14*C*cos(d*x+c)^2-27*A*cos(d*x+c)-21*C*cos(d*x+c))/
cos(d*x+c)^3/(b/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 129, normalized size = 1.12

$$-\frac{3\sqrt{2}\left(-9iA-7iC\right)\sqrt{b}\ \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(9iA+7iC\right)\sqrt{b}\ \text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(5C\cos\left(dx+c\right)^{4}+\left(9A+7C\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{45b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^4 + (9*A + 7*C)*cos(d*x + c)^2)*sqrt
(b/cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b/cos(c + d*x))^(5/2), x)
```

3.34 $\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=117

$$\frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}-\frac{(C(1+m)+A(2+m))(b\cos(c+dx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+m)(2+m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)-(C*(1+m)+A*(2+m))*(b*cos(d*x+c)
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3093, 2722}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}-\frac{(A(m+2)+C(m+1))\sin(c+dx)(b\cos(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{bd(m+1)(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m)) - ((C*(1 + m) + A*(
2 + m))*(b*Cos[c + d*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^m \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} + \left(A+\frac{C(1+m)}{2+m}\right)\int (b\cos$$

$$= \frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)} - \frac{\left(A+\frac{C(1+m)}{2+m}\right)(b\cos(c+d}{b}$$

Mathematica [A]
time = 0.20, size = 114, normalized size = 0.97

$$-\frac{(b\cos(c+dx))^m\cot(c+dx)\left(A(3+m)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)+C(1+m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((b*Cos[c + d*x])^m*Cot[c + d*x]*(A*(3 + m)*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, Cos[c + d*x]^2] + C*(1 + m)*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 m)*(3 + m)))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^m \left(A+C\left(\cos^2(dx+c)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos{(c+dx)})^m \left(A+C\cos^2{(c+dx)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**m*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((b*cos(c + d*x))**m*(A + C*cos(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m,x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^m, x)
```

3.35 $\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}$$

```
[Out] C*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3090}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m))
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int (b\cos(c+dx))^m \left(-\frac{C(1+m)}{2+m}+C\cos^2(c+dx)\right)\,dx=\frac{C(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(2+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 113, normalized size = 3.65

$$\frac{C(b\cos(c+dx))^m\cot(c+dx)\left((3+m)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)-(2+m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[1/
2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m)*
(3 + m))
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^m \left(-\frac{C(1+m)}{2+m}+C\left(\cos^2(dx+c)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(31) = 62.
time = 0.63, size = 175, normalized size = 5.65

$$-\frac{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{2}\,m} Cb^m \sin\left(-(dx+c)(m+2)+m\arctan\left(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1\right)\right)-\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{2}\,m} Cb^m \sin\left(-(dx+c)(m-2)+m\arctan\left(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1\right)\right)}{4\cdot 2^m d(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] -1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/2*m)*C*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/2*m)*C*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))/(2^m*d*(m + 2))
```

Fricas [A]
time = 0.38, size = 33, normalized size = 1.06

$$\frac{(b\cos(dx+c))^m\, C\cos(dx+c)\sin(dx+c)}{dm+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] (b*cos(d*x + c))^m*C*cos(d*x + c)*sin(d*x + c)/(d*m + 2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(26) = 52.

time = 20.84, size = 279, normalized size = 9.00

$$\begin{cases} -\frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2d}+\frac{2C\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2d} & \text{for } d \neq 0 \\ x(b\cos(c))^m\left(-\frac{C(m+1)}{m+2}+C\cos^2(c)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**m*(-C*(1+m)/(2+m)+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(
c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2*d
*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2 + d*x/
2)**2 + 2*d) + 2*C*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(t
an(c/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4 + 2*d
*m*tan(c/2 + d*x/2)**2 + d*m + 2*d*tan(c/2 + d*x/2)**4 + 4*d*tan(c/2 + d*x/
2)**2 + 2*d), Ne(d, 0)), (x*(b*cos(c))**m*(-C*(m + 1)/(m + 2) + C*cos(c)**2
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2494 vs. 2(31) = 62.
time = 11.38, size = 2494, normalized size = 80.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] 2*(C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))^
m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2
 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*
floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x +
 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c
)^3 - C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1
))^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c
)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
 + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)
) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi
*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4
```

```
*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c) - C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
1))^m*tan(1/2*d*x + 1/2*c)^3 + C*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(t
an(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn(
2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*ta
n(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*
c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2
 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/4*p
i*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sg
n(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1
/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*m*
tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1
/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*fl
oor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^
2 + d*m*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/
2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4
 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn
(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*
x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d
*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + 2*d*tan(1/2*d*x + 1/2*c)^4 + d*m*t
an(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4
```

```
*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*flo
or(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d...
```

Mupad [B]
time = 1.01, size = 30, normalized size = 0.97

$$\frac{C\,\sin\left(2\,c+2\,d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^m}{2\,d\,\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^m*(C*cos(c + d*x)^2 - (C*(m + 1))/(m + 2)),x)
[Out] (C*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 2))
```

3.36 $\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx$

Optimal. Leaf size=32

$$-\frac{A(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(1+m)}$$

```
[Out] -A*(b*cos(d*x+c))^(1+m)*sin(d*x+c)/b/d/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {3090}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{m+1}}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(1 + m)))
```

Rule 3090

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0
]
```

Rubi steps

$$\int (b\cos(c+dx))^m \left(A-\frac{A(2+m)\cos^2(c+dx)}{1+m}\right) dx = -\frac{A(b\cos(c+dx))^{1+m}\sin(c+dx)}{bd(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.23, size = 119, normalized size = 3.72

$$\frac{A\cos(c+dx)(b\cos(c+dx))^m\left(-\left((3+m)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\right)+(2+m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(c+dx)\right)\right)\sin(c+dx)}{d(1+m)(3+m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]
```

```
[Out] (A*Cos[c + d*x]*(b*Cos[c + d*x])^m*(-((3 + m)*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, Cos[c + d*x]^2]) + (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1
[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sin[c + d*x])/(d*(1 + m)*(3 +
m)*Sqrt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(d x +c \right)\right)^{m} \left(A -\frac{A \left(2+m \right) \left(\cos^{2}\left(d x +c \right)\right)}{1+m}\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x)
```

```
[Out] int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(32) = 64.
time = 0.64, size = 175, normalized size = 5.47

$$\frac{{\left(\cos \left(2 \, d x + 2 \, c\right)^{2} + \sin \left(2 \, d x + 2 \, c\right)^{2} + 2 \, \cos \left(2 \, d x + 2 \, c\right) + 1\right)}^{\frac{1}{2} \, m} A b^{m} \sin \left(-{\left(d x + c\right)} {\left(m + 2\right)} + m \arctan\left(\sin \left(2 \, d x + 2 \, c\right), \cos \left(2 \, d x + 2 \, c\right) + 1\right)\right) - {\left(\cos \left(2 \, d x + 2 \, c\right)^{2} + \sin \left(2 \, d x + 2 \, c\right)^{2} + 2 \, \cos \left(2 \, d x + 2 \, c\right) + 1\right)}^{\frac{1}{2} \, m} A b^{m} \sin \left(-{\left(d x + c\right)} {\left(m - 2\right)} + m \arctan\left(\sin \left(2 \, d x + 2 \, c\right), \cos \left(2 \, d x + 2 \, c\right) + 1\right)\right)}{4 \cdot 2^{m} d {\left(m + 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="max
ima")
```

```
[Out] 1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
2*m)*A*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/2*m)*A*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))/(2^m*d*(m + 1))
```

Fricas [A]
time = 0.41, size = 32, normalized size = 1.00

$$-\frac{\left(b \cos \left(d x + c\right)\right)^{m} A \cos \left(d x + c\right) \sin \left(d x + c\right)}{d m + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="fri
cas")
```

```
[Out] -(b*cos(d*x + c))^m*A*cos(d*x + c)*sin(d*x + c)/(d*m + d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(27) = 54.

time = 20.51, size = 272, normalized size = 8.50

$$\begin{cases} \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} - \frac{2A\left(-\frac{b\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}+\frac{b}{\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1}\right)^m\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{dm\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2dm\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+dm+d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+d} & \text{for } d \neq 0 \\ x(b\cos(c))^m\left(A-\frac{A(m+2)\cos^2(c)}{m+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**m*(A-A*(2+m)*cos(d*x+c)**2/(1+m)),x)
```

```
[Out] Piecewise((2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(c
/2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)**3/(d*m*tan(c/2 + d*x/2)**4 + 2*d*
m*tan(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/2)*
*2 + d) - 2*A*(-b*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + b/(tan(c/
2 + d*x/2)**2 + 1))**m*tan(c/2 + d*x/2)/(d*m*tan(c/2 + d*x/2)**4 + 2*d*m*ta
n(c/2 + d*x/2)**2 + d*m + d*tan(c/2 + d*x/2)**4 + 2*d*tan(c/2 + d*x/2)**2 +
 d), Ne(d, 0)), (x*(b*cos(c))**m*(A - A*(m + 2)*cos(c)**2/(m + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2489 vs. $2(32) = 64$.
time = 12.00, size = 2489, normalized size = 77.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="gia
c")
```

```
[Out] -2*(A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 + 1))
^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^
2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c))
+ 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m
*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*
b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x
+ 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*
c)^3 - A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
1))^m*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*
c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)
) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c
)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + p
i*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/
4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sg
```

```
n(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d
*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1
/2*c) - A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(tan(1/2*d*x + 1/2*c)^2 +
 1))^m*tan(1/2*d*x + 1/2*c)^3 + A*(abs(tan(1/2*d*x + 1/2*c)^2 - 1)*abs(b)/(
tan(1/2*d*x + 1/2*c)^2 + 1))^m*tan(1/2*d*x + 1/2*c))/(d*m*tan(-1/4*pi*m*sgn
(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*t
an(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x +
 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2
*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^
2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1
/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/4*pi
*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn
(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn
(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/
2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^4 + 2*d*m*t
an(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 +
2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4
*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/
4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*flo
or(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*s
gn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(t
an(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/
2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2
 + d*m*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2
*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*
sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4
- 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*
sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(
tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x
 + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*m*sgn(tan(1/2*d*
x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + d*m*tan(
-1/4*pi*m*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b
)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi
*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*p
```

```
i*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + pi*m*floor(
1/4*sgn(2*b*tan(1/2*d*x + 1/2*c)^4 - 4*b*tan(1/2*d*x + 1/2*c)^2 + 2*b)*sgn(
tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)*sgn(b)*sgn(tan(1/2*d*x ...
```

Mupad [B]
time = 0.99, size = 30, normalized size = 0.94

$$-\frac{A\,\sin\left(2\,c+2\,d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^m}{2\,d\,\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^m*(A - (A*cos(c + d*x)^2*(m + 2))/(m + 1)),x)
```

```
[Out] -(A*sin(2*c + 2*d*x)*(b*cos(c + d*x))^m)/(2*d*(m + 1))
```

3.37 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=112

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}+\frac{2C(b\cos(c+dx))^7}{9b^3d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b^3/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*
x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[C
os[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d
) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{(9A+7C)\int$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.26, size = 88, normalized size = 0.79

$$\frac{\sqrt{b\cos(c+dx)}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))\right)}{180d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Co
s[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*d*
Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(124) = 248.
time = 0.37, size = 322, normalized size = 2.88

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-160*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*
C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x
+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 122, normalized size = 1.09

$$-\frac{3\sqrt{2}\,(-9i\,A-7i\,C)\sqrt{b}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\sqrt{2}\,(9i\,A+7i\,C)\sqrt{b}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(5\,C\cos(dx+c)^3+(9\,A+7\,C)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.38 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=110

$$\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx))^{5/2}}{7b^2d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/21*b*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$

$$=\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}+\frac{(7A+5C)\int(}{}$$

$$=\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b}{}$$

$$=\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b}{}$$

$$=\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2}{}$$

Mathematica [A]
time = 0.34, size = 89, normalized size = 0.81

$$\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)\right)}{42bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*b*d*Co
s[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(122)=244$.
time = 0.35, size = 294, normalized size = 2.67

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 106, normalized size = 0.96

$$\frac{\sqrt{2}\,(-7iA-5iC)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{2}\,(7iA+5iC)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3C\cos(dx+c)^2+7A+5C\right)\sqrt{b\cos(dx+c)}\ \sin(dx+c)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.39 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3093, 2721, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{1}{5}(5A+3C)\int \sqrt{b\cos(c$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{\left((5A+3C)\sqrt{b\cos(c+}{5\sqrt{\cos(}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+}{}$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 0.91

$$\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(93) = 186$.
time = 0.00, size = 261, normalized size = 3.39

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{5\sqrt{-b\left(2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 101, normalized size = 1.31

$$\frac{2\sqrt{b\cos\left(dx+c\right)}\,C\cos\left(dx+c\right)\sin\left(dx+c\right)+\sqrt{2}\,(5i\,A+3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2), x)
```

3.40 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3093, 2721, 2720}

$$\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec(c+dx)\,dx &= b\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}(b(3A+C))\int\\ &= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{\left(b(3A+C)\sqrt{c\right.}}{}\\ &= \frac{2b(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 59, normalized size = 0.81

$$\frac{b\left(2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
[Out] (b*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c +
 d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(89) = 178$.
time = 0.38, size = 237, normalized size = 3.25

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(s\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 89, normalized size = 1.22

$$\frac{\sqrt{2}\left(-3i\,A-i\,C\right)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\left(3i\,A+i\,C\right)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,C\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(c+dx\right)}\,\left(A+C\cos^{2}\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

3.41 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=69

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2719}

$$\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + (-A+C)\int \sqrt{b\cos(c+dx}$$
$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left((-A+C)\sqrt{b\cos(c+dx)}\right.}{\sqrt{\cos(c+}}$$
$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2}{d}$$

Mathematica [A]
time = 0.23, size = 55, normalized size = 0.80

$$\frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d
*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(89) = 178.
time = 0.42, size = 214, normalized size = 3.10

method	result
default	$\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 114, normalized size = 1.65

$$\frac{\sqrt{2}\,(-i\,A+i\,C)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,(i\,A-i\,C)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*
x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

3.42 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=76

$$\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b*(A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2720}

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}(b(A+3C))\int \frac{1}{\sqrt{b\cos(c}}$$
$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b(A+3C)\sqrt{\cos(c+dx)}\right.}{3\sqrt{b\cos(c}}$$
$$= \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2}{3d}$$

Mathematica [A]
time = 0.22, size = 56, normalized size = 0.74

$$\frac{2b\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
[Out] (2*b*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(92) = 184$.
time = 0.41, size = 292, normalized size = 3.84

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 113, normalized size = 1.49

$$\frac{\sqrt{2}\,(-i\,A-3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,A+3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

3.43 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=110

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b*(3*A+5*C)*sin(d*x+c)/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*
(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(b^2(3A+5C)\right)\int \frac{}{(b\cos(}$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} -$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} -$$
$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.26, size = 84, normalized size = 0.76

$$-\frac{\sqrt{b\cos(c+dx)}\,\sec^2(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2A\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -1/5*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)
*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x
]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(122) = 244$.
time = 0.77, size = 598, normalized size = 5.44

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*
c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-
20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-40*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b
*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

<u>Fricas</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 136, normalized size = 1.24

$$\frac{\sqrt{2}\left(-3i\,A-5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(3i\,A+5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(\left(3\,A+5\,C\right)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

3.44 $\int \sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=113

$$\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^2*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = b^5\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(b^3(5A+7C)\right)\int \frac{}{(b\cos(}$$
$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$
$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$
$$= \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{}{7\ldots}$$

Mathematica [A]
time = 0.52, size = 83, normalized size = 0.73

$$\frac{\sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C)\sin(2(c+dx))+6A\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*Elli
pticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(
21*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(125) = 250$.
time = 0.71, size = 411, normalized size = 3.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{d}{}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 132, normalized size = 1.17

$$\frac{\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4 \,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4 \,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left((5\,A+7\,C)\cos\left(dx+c\right)^2+3\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
```

```
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5,x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^5, x)
```

3.45 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=110

$$\frac{2b(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx)}{9b}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/2)
*sin(d*x+c)/b^2/d+2/15*b*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*
x+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{2b(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt
[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d
) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{(9A+7C)\int$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C($$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2C($$

$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.20, size = 91, normalized size = 0.83

$$\frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))\right)}{180bd\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
b*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(122) = 244.
time = 0.35, size = 324, normalized size = 2.95

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 124, normalized size = 1.13

$$\frac{3i\sqrt{2}\,(9A+7C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(9A+7C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(5Cb\cos(dx+c)^3+(9A+7C)b\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(3
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b*cos(d*x + c)^3 + (9*A + 7*C)*b*cos(d*x + c))*sqrt(b*c
os(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.46 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=113

$$\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx)}{7b}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3093, 2715, 2721, 2720}

$$\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{1}{7}(7A+5C)\int (b\cos(c+$$
$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx}{7b}$$
$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx}{7b}$$
$$=\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C}{}$$

Mathematica [A]
time = 0.08, size = 86, normalized size = 0.76

$$\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2\sqrt{\cos(c+dx)}\,(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)\right)}{42d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[C
os[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[
c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(125)=250$.
time = 0.00, size = 296, normalized size = 2.62

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 112, normalized size = 0.99

$$\frac{-i\sqrt{2}\,(7\,A+5\,C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,(7\,A+5\,C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,(3\,Cb\cos(dx+c)^2+(7\,A+5\,C)b)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b*cos(d*x + c)^2 + (7*A + 5*C)*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2), x)
```

3.47 $\int (b\cos(c+dx))^{3/2} \left(A+C\cos^2(c+dx)\right) \sec(c+dx)\,dx$

Optimal. Leaf size=75

$$\frac{2b(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*b*(5*A+3*C)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3093, 2721, 2719}

$$\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(b(5A+3C$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{(b(5A+3C)}{}$$
$$= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.95

$$\frac{b\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[C
os[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(91) = 182.
time = 0.35, size = 263, normalized size = 3.51

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{5\sqrt{-b\left(2\left(\mathrm{s}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(8*C*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 104, normalized size = 1.39

$$\frac{2\sqrt{b\cos(dx+c)}\,Cb\cos(dx+c)\sin(dx+c)+i\sqrt{2}\,(5A+3C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-i\sqrt{2}\,(5A+3C)b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*b*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*(5*A
+ 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - I*sqrt(2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

3.48 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=76

$$\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*C*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2720}

$$\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}(b^2(3A+C$$
$$= \frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(b^2(3A+C)\right.}{}$$
$$= \frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + $$

Mathematica [A]
time = 0.09, size = 61, normalized size = 0.80

$$\frac{b^2\left(2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
[Out] (b^2*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c
 + d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(92) = 184.
time = 0.34, size = 239, normalized size = 3.14

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 88, normalized size = 1.16

$$\frac{-i\sqrt{2}\,(3\,A+C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}\,(3\,A+C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})+2\,\sqrt{b\cos{(dx+c)}}\,Cb\sin{(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*b*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

3.49 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{2b(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b*(A-C)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.121$, Rules used = {16, 3091, 2721, 2719}

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c
 + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - (b(A-C))\int \sqrt{b\cos(c+a}$$
$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b(A-C)\sqrt{b\cos(c+dx)}\right.}{\sqrt{\cos(c+}}$$
$$= -\frac{2b(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.16, size = 57, normalized size = 0.79

$$\frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c +
 d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(92) = 184.
time = 0.41, size = 216, normalized size = 3.00

method	result
default	$\frac{2b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 113, normalized size = 1.57

$$\frac{-i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Ab\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(3/2)
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x
+ c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

3.50 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$,
Rules used = {16, 3091, 2721, 2720}

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}\left(b^2(A+3C)\right)\int \frac{}{\sqrt{b\,\text{co}}}$$
$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b^2(A+3C)\sqrt{\cos(c+d}\right.}{3\sqrt{b\cos(}}$$
$$= \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{}{3}$$

Mathematica [A]
time = 0.20, size = 58, normalized size = 0.74

$$\frac{2b^2\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(94) = 188$.
time = 0.46, size = 294, normalized size = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 1.44

$$\frac{-i\sqrt{2}\,(A+3\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(A+3\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,Ab\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(
d*x + c))*A*b*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

3.51 $\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=113

$$-\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^2*(3*A+5*C)*sin(d*x+c)/d/
(b*cos(d*x+c))^(1/2)-2/5*b*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (-2*b*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt
[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
b^2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx &= b^5\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}\left(b^3(3A+5C)\right)\int \frac{}{(b\cos}\\ &= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx}{5d\sqrt{b\cos(c+dx)}}\\ &= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx}{5d\sqrt{b\cos(c+dx)}}\\ &= -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.23, size = 84, normalized size = 0.74

$$-\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(2(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-(3A+5C)\sin(2(c+dx))-2A\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] -1/5*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(2*(3*A + 5*C)*Cos[c + d*x]^(3/
2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d
*x]))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(125) = 250$.
time = 0.74, size = 599, normalized size = 5.30

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-4
0*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 141, normalized size = 1.25

$$\frac{-i\sqrt{2}\,(3\,A+5\,C)b^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(3\,A+5\,C)b^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left((3\,A+5\,C)b\cos(dx+c)^2+Ab\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{5\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b*cos(d*x + c)^2 + A*
b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

3.52 $\int (b\cos(c+dx))^{3/2} (A+C\cos^2(c+dx)) \sec^6(c+dx)\, dx$

Optimal. Leaf size=115

$$\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^3*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b^2*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*c
os(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (
2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = b^6\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(b^4(5A+7C)\right)\int \frac{}{(b\cos}$$
$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx}{21d(b\cos(c+dx))^{3/2}}$$
$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3(5A+7C)\sin(c+dx}{21d(b\cos(c+dx))^{3/2}}$$
$$= \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A]
time = 0.34, size = 83, normalized size = 0.72

$$\frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C)\sin(2(c+dx))+6A\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*El
lipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))
/(21*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(127) = 254.
time = 0.72, size = 413, normalized size = 3.59

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 136, normalized size = 1.18

$$\frac{-i\sqrt{2}\,(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left((5A+7C)b\cos(dx+c)^2+3Ab\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x
```

```
+ c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b*cos(d*x + c)^2 + 3*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^6, x)
```

3.53 $\int (b\cos(c+dx))^{5/2} (A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=113

$$\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+dx$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/
2)*sin(d*x+c)/b/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3093, 2715, 2721, 2719}

$$\frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{1}{9}(9A+7C)\int (b\cos(c+$$

$$=\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+d}{9}$$

$$=\frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2C(b\cos(c+d}{9}$$

$$=\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7}{}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.78

$$\frac{(b\cos(c+dx))^{5/2}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(18A+19C+5C\cos(2(c+dx)))\sin(2(c+dx))\right)}{180d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[
Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*
d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(125) = 250$.
time = 0.00, size = 324, normalized size = 2.87

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{d}{}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+13
6*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 128, normalized size = 1.13

$$\frac{3i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(5Cb^2\cos(dx+c)^3+(9A+7C)b^2\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/45*(3*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(9*A + 7*C)*b^(5
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(5*C*b^2*cos(d*x + c)^3 + (9*A + 7*C)*b^2*cos(d*x + c))*sqrt
(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2), x)
```

3.54 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2C(b\cos(c+dx$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/21*b^3*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d
```

Rubi [A]

time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(
21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(b(7A+5C)$$

$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2C(}{}$$

$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2C(}{}$$

$$= \frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + $$

Mathematica [A]
time = 0.09, size = 87, normalized size = 0.78

$$\frac{b(b\cos(c+dx))^{3/2}\left(4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sqrt{\cos(c+dx)}\,(14A+13C+3C\cos(2(c+dx)))\sin(c+dx)\right)}{42d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (b*(b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt
[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Co
s[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(124) = 248.
time = 0.38, size = 296, normalized size = 2.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3 \left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 116, normalized size = 1.04

$$\frac{-i\sqrt{2}\,(7A+5C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(7A+5C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3Cb^2\cos(dx+c)^2+(7A+5C)b^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*b^2*cos(d*x + c)^2 + (7*A + 5*C
)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

3.55 $\int (b\cos(c+dx))^{5/2} (A+C\cos^2(c+dx)) \sec^2(c+dx)\, dx$

Optimal. Leaf size=78

$$\frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}\, E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*b^2*(5*A+3*C)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2719}

$$\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|\, 2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqr
t[Cos[c + d*x]]) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(b^2(5A+\right.$$
$$= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(b^2(5A+3\right.}{}$$
$$= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 0.94

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+C\sqrt{\cos(c+dx)}\,\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt
[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(94) = 188$.
time = 0.38, size = 263, normalized size = 3.37

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{5\sqrt{-b\left(2\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(8*C*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 106, normalized size = 1.36

$$\frac{2\sqrt{b\cos(dx+c)}\,Cb^2\cos(dx+c)\sin(dx+c) + i\sqrt{2}\,(5A+3C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}\,(5A+3C)b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*b^2*cos(d*x + c)*sin(d*x + c) + I*sqrt(2)*(5*
A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) - I*sqrt(2)*(5*A + 3*C)*b^(5/2)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

3.56 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*
C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.121$, Rules used = {16, 3093, 2721, 2720}

$$\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}\left(b^3(3A+\right.$$
$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(b^3(3A+C\right.}{}$$
$$= \frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}} + $$

Mathematica [A]
time = 0.18, size = 65, normalized size = 0.83

$$\frac{2(b\cos(c+dx))^{5/2}\left((3A+C)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+C\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*((3*A + C)*EllipticF[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(94) = 188.
time = 0.37, size = 239, normalized size = 3.06

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 90, normalized size = 1.15

$$\frac{-i\sqrt{2}\,(3\,A+C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(3\,A+C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,Cb^{2}\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*b^2*sin(d*x + c))/
d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

3.57 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2719}

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b^2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx &= b^4\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \left(b^2(A-C)\right)\int \sqrt{b\cos(c+ \\
&= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2(A-C)\sqrt{b\cos(c+dx)}\right.}{\sqrt{\cos(c+}} \\
&= -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} +
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 57, normalized size = 0.77

$$\frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c +
 d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.
time = 0.40, size = 216, normalized size = 2.92

method	result
default	$\frac{2b^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 115, normalized size = 1.55

$$\frac{-i\sqrt{2}(A-C)b^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}(A-C)b^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Ab^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(A - C)*b^(5/2)
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*
x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

3.58 $\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$

Optimal. Leaf size=78

$$\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^3*(A+3*C)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {16, 3091, 2721, 2720}

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = b^5\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}\left(b^3(A+3C)\right)\int \frac{}{\sqrt{b\cos}}$$
$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left(b^3(A+3C)\sqrt{\cos(c+d}\right.}{3\sqrt{b\cos(}}$$
$$= \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{}{3}$$

Mathematica [A]
time = 0.20, size = 58, normalized size = 0.74

$$\frac{2b^3\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+A\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]
[Out] (2*b^3*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c +
d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(94) = 188$.
time = 0.39, size = 294, normalized size = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*b^3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 114, normalized size = 1.46

$$\frac{-i\sqrt{2}\,(A+3\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,(A+3\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,Ab^{2}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(
d*x + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

3.59 $\int (b\cos(c+dx))^{5/2} \left(A + C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$

Optimal. Leaf size=115

$$-\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*b^3*(3*A+5*C)*sin(d*x+c)/d/
(b*cos(d*x+c))^(1/2)-2/5*b^2*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/co
s(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2b^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (-2*b^2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (
2*b^3*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = b^6\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(b^4(3A+5C)\right)\int \frac{}{(b\cos}$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx}{5d\sqrt{b\cos(c+dx)}}$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3(3A+5C)\sin(c+dx}{5d\sqrt{b\cos(c+dx)}}$$
$$= -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.28, size = 80, normalized size = 0.70

$$-\frac{2b^4\left((3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{1}{2}(3A+5C)\sin(2(c+dx))-A\tan(c+dx)\right)}{5d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]
```

```
[Out] (-2*b^4*((3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - ((3*A +
 5*C)*Sin[2*(c + d*x)])/2 - A*Tan[c + d*x]))/(5*d*(b*Cos[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(127) = 254.
time = 0.78, size = 601, normalized size = 5.23

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2
-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 145, normalized size = 1.26

$$\frac{-i\sqrt{2}\,(3A+5C)b^{\frac{5}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+i\sqrt{2}\,(3A+5C)b^{\frac{5}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left((3A+5C)b^2\cos(dx+c)^2+Ab^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{5\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/5*(-I*sqrt(2)*(3*A + 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*(3*A
+ 5*C)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*b^2*cos(d*x + c)^2 +
A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

3.60 $\int (b\cos(c+dx))^{5/2} (A + C\cos^2(c+dx)) \sec^7(c+dx)\, dx$

Optimal. Leaf size=115

$$\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b^4*(5*A+7*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(3/2)+2/21*b^3*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*c
os(d*x+c))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (
2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx &= b^7\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx\\ &= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(b^5(5A+7C)\right)\int \frac{}{(b\cos}\\ &= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx}{21d(b\cos(c+dx))^{3/2}}\\ &= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx}{21d(b\cos(c+dx))^{3/2}}\\ &= \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\end{aligned}$$

Mathematica [**A**]
time = 0.44, size = 83, normalized size = 0.72

$$\frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(2(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C)\sin(2(c+dx))+6A\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*El
lipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))
/(21*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(127) = 254$.
time = 0.74, size = 413, normalized size = 3.59

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 140, normalized size = 1.22

$$\frac{-i\sqrt{2}\,(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left((5A+7C)b^2\cos(dx+c)^2+3Ab^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="fricas")
```

```
[Out] 1/21*(-I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(-4,
 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x
```

```
+ c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((5*A
+ 7*C)*b^2*cos(d*x + c)^2 + 3*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^7, x)
```

3.61 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{10(11A+9C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{b\cos(c+dx)}}+\frac{10(11A+9C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{231bd}+\frac{2(11A+9C)(b}{}$$

```
[Out] 2/77*(11*A+9*C)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/11*C*(b*cos(d*x+c))
^(9/2)*sin(d*x+c)/b^5/d+10/231*(11*A+9*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*
cos(d*x+c))^(1/2)+10/231*(11*A+9*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2(11A+9C)\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^3d}+\frac{10(11A+9C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{231bd}+\frac{10(11A+9C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[
b*Cos[c + d*x]]) + (10*(11*A + 9*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(231
*b*d) + (2*(11*A + 9*C)*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*b^3*d) + (
2*C*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{(11A+9C)\int (b\cos(c+dx))^{7}}{11b^4}$$

$$= \frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2C(b\cos(c+dx))^{9/}}{11b^5d}$$

$$= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{231bd} + \frac{2(11A+9C)(b\cos(}{7}$$

$$= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{231bd} + \frac{2(11A+9C)(b\cos(}{7}$$

$$= \frac{10(11A+9C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{b\cos(c+dx)}} + \frac{10(11A+9C)\sqrt{}}{}$$

Mathematica [A]
time = 0.41, size = 94, normalized size = 0.64

$$\frac{80(11A+9C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(572A+531C+12(11A+16C)\cos(2(c+dx))+21C\cos(4(c+dx)))\sin(2(c+dx))}{1848d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (80*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (572*A + 53
1*C + 12*(11*A + 16*C)*Cos[2*(c + d*x)] + 21*C*Cos[4*(c + d*x)])*Sin[2*(c +
 d*x)])/(1848*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(155) = 310$.
time = 0.36, size = 349, normalized size = 2.37

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(1344C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3360C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/231*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1344*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-3360*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^10+(528*A+3792*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-792*A-
2328*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(616*A+924*C)*sin(1/2*d*x+1
/2*c)^4*cos(1/2*d*x+1/2*c)+(-176*A-186*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+55*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 128, normalized size = 0.87

$$-\frac{5\sqrt{2}\,(11i\,A+9i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-11i\,A-9i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-2\left(21\,C\cos(dx+c)^4+3\,(11\,A+9\,C)\cos(dx+c)^2+55\,A+45\,C\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{231\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/231*(5*sqrt(2)*(11*I*A + 9*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-11*I*A - 9*I*C)*sqrt(b)*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 2*(21*C*cos(d*x + c)^4 + 3
*(11*A + 9*C)*cos(d*x + c)^2 + 55*A + 45*C)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7320 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.62 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}+\frac{2C(b\cos(c+dx)}{9b^4}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^4/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/co
s(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt
[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b
^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3}\\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}+\frac{(9A+7C)\int (b\cos(c+dx))^{5/2}}{9b^3}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}+\frac{2C(b\cos(c+dx))^{7/2}}{9b^4d}\\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}+\frac{2C(b\cos(c+dx))^{7/2}}{9b^4d}\\
&= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos}{}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 83, normalized size = 0.72

$$\frac{6(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+19C+5C\cos(2(c+dx)))\sin(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(127) = 254.
time = 0.41, size = 321, normalized size = 2.79

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{d}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

<u>Fricas</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 125, normalized size = 1.09

$$-\frac{3\sqrt{2}\left(-9iA-7iC\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(9iA+7iC\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(5C\cos\left(dx+c\right)^3+\left(9A+7C\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{45bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.63 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=112

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd}+\frac{2C(b\cos(c+dx))^{5/}}{7b^3d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))
^(1/2)/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{(7A+5C)\int (b\cos(c+dx))^{3/2}}{7b^2}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\,\mathrm{s}}{7b^3d}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2C(b\cos(c+dx))^{5/2}\,\mathrm{s}}{7b^3d}$$

$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos}}{2}$$

Mathematica [A]
time = 0.25, size = 77, normalized size = 0.69

$$\frac{4(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(14A+13C+3C\cos(2(c+dx)))\sin(2(c+dx))}{42d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
+ 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(124) = 248.
time = 0.35, size = 293, normalized size = 2.62

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 109, normalized size = 0.97

$$\frac{\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(3\,C\cos(dx+c)^2+7\,A+5\,C\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.64 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3093, 2721, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[
Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}\ d$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right)}{5b\sqrt{\cos(c+d}}$$
$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{}}{5b^2d}$$

Mathematica [A]
time = 0.08, size = 73, normalized size = 0.91

$$\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos
[c + d*x]]*Sin[2*(c + d*x)]))/(5*b*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(96) = 192.
time = 0.38, size = 260, normalized size = 3.25

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5A\right.}{5\sqrt{-b\left(2\left(\mathrm{si}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3
*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 104, normalized size = 1.30

$$\frac{2\sqrt{b\cos(dx+c)}\,C\cos(dx+c)\sin(dx+c)+\sqrt{2}\,(5i\,A+3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2), x)
```

3.65 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3093, 2721, 2720}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}+\frac{1}{3}(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}+\frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.77

$$
\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.
time = 0.00, size = 236, normalized size = 3.15

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\mathrm{si}\right.}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d
*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 92, normalized size = 1.23

$$\frac{\sqrt{2}\,(-3i\,A-i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(3i\,A+i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,C\sin\left(dx+c\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.82, size = 94, normalized size = 1.25

$$\frac{2\,C\,\sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{3\,b\,d}+\frac{2\,A\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}+\frac{2\,C\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(c + d*x
)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.66 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=71

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3091, 2721, 2719}

$$\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
 + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b}\\ &= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}}{b\sqrt{\cos(c+dx)}}\\ &= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.35, size = 200, normalized size = 2.82

$$-\frac{\csc(c)\left(-6A\cos(dx)+3C\cos(dx)+3C\cos(2c+dx)+3(A-C)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}+(A-C)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))\sqrt{1+\cos(2(c+dx))+i\sin(2(c+dx))}\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] -1/3*(Csc[c]*(-6*A*Cos[d*x] + 3*C*Cos[d*x] + 3*C*Cos[2*c + d*x] + 3*(A - C)
*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*
(Cos[d*x] - I*Sin[d*x])*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + (
A - C)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*(Cos[d*x] + I*Sin[d*x])*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]
]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(91) = 182.
time = 0.40, size = 213, normalized size = 3.00

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d
*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 117, normalized size = 1.65

$$\frac{\sqrt{2}\left(-i A+i C\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(i A-i C\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(
d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+C\cos^2\left(c+dx\right)\right)\sec\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

3.67 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3091, 2721, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A + C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^2 \int \frac{A + C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.51, size = 141, normalized size = 1.93

$$-\frac{4b(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))-A\sqrt{\csc^2(c)}\sin(c+dx)\right)}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+dx)))\sqrt{\csc^2(c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-4*b*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTa
n[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*
d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(89) = 178.
time = 0.40, size = 291, normalized size = 3.99

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A+}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 116, normalized size = 1.59

$$\frac{\sqrt{2}\left(-i\,A-3i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\left(i\,A+3i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{3\,bd\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + C\cos^{2}\left(c + dx\right)\right)\sec^{2}\left(c + dx\right)}{\sqrt{b\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

3.68 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=112

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt
[Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}(b(3A+5C))\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{(3A+5C)\int \sqrt{\cdots}}{\cdots}\\ &= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{\left((3A+5C)\sqrt{\cdots}\right.}{\cdots}\\ &= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sin(c+\cdots}{5d(b\cos(c+dx\cdots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.32, size = 522, normalized size = 4.66

$$b\left(-\frac{i(3A+5C)\cos^{\frac{7}{2}}(c+dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)(C+A\sec^2(c+dx))\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{1+e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}-\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\sqrt{1+e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)}}{-id\left(1+e^{2idx}\right)\cos(c)+d\left(-1+e^{2idx}\right)\sin(c)}\right)}{10(b\cos(c+dx))^{3/2}(2A+C+C\cos(2c+2dx))}+\frac{\cos^4(c+dx)(C+A\sec^2(c+dx))\left(\frac{4(3A+5C)\csc(c)\sec(c)}{5d}+\frac{4A\sec(c)\sec^3(c+dx)\sin(dx)}{5d}+\frac{4\sec(c)\sec(c+dx)(3A\sin(dx)+5C\sin(dx))}{5d}+\frac{4A\sec^2(c+dx)\tan(c)}{5d}\right)}{(b\cos(c+dx))^{3/2}(2A+C+C\cos(2c+2dx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] b*(((-1/10*I)*(3*A + 5*C)*Cos[c + d*x]^(7/2)*Csc[c/2]*Sec[c/2]*(C + A*Sec[c
 + d*x]^2)*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*
```

```
I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Co
s[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2
*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d
*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*
Sin[c])))/((b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c
+ d*x]^4*(C + A*Sec[c + d*x]^2)*((4*(3*A + 5*C)*Csc[c]*Sec[c])/(5*d) + (4*A
*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]*(3*A*Sin[d*
x] + 5*C*Sin[d*x]))/(5*d) + (4*A*Sec[c + d*x]^2*Tan[c])/(5*d)))/((b*Cos[c +
 d*x])^(3/2)*(2*A + C + C*Cos[2*c + 2*d*x])))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(124) = 248$.
time = 0.84, size = 601, normalized size = 5.37

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-4
0*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 139, normalized size = 1.24

$$\frac{\sqrt{2}\,(-3i\,A-5i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,(3i\,A+5i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\,((3\,A+5\,C)\cos\left(dx+c\right)^{2}+A)\,\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\,bd\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

3.69 $\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=110

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*b*(5*A+7*C)*sin(d*x+c)/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x
+c))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*
(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^4\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx\\ &= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(b^2(5A+7C)\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{1}{21}(5A+7C)\\ &= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left((5A+7C)\right.}{}\\ &= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx}{7d(b\cos(c+dx))}\end{aligned}$$

Mathematica [**A**]
time = 0.41, size = 74, normalized size = 0.67

$$\frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(122) = 244.
time = 0.70, size = 412, normalized size = 3.75

method	result
default	$-\frac{\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*cos(1
/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 135, normalized size = 1.23

$$\frac{\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left((5\,A+7\,C)\cos(dx+c)^2+3\,A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,bd\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
```

```
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)), x)
```

3.70 $\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{2(7A+9C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{2(7A+9C}{15d\sqrt{b\,c}}$$

```
[Out] 2/9*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+2/45*b^2*(7*A+9*C)*sin(d*x+c)/d
/(b*cos(d*x+c))^(5/2)+2/15*(7*A+9*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2/15
*(7*A+9*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$\frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{2(7A+9C)\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{2(7A+9C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*(7*A + 9*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqr
t[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (2
*b^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (2*(7*A + 9*
C)*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^5\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(b^3(7A+9C)\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}(b(7A+9$$
$$= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)}{15d\sqrt{b\cos}}$$
$$= \frac{2Ab^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{2b^2(7A+9C)\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{2(7A+9C)}{15d\sqrt{b\cos}}$$
$$= -\frac{2(7A+9C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2Ab^4\sin(c+}{9d(b\cos(c+d}$$

Mathematica [A]
time = 0.83, size = 97, normalized size = 0.66

$$\frac{-6(7A+9C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+6(7A+9C)\sin(c+dx)+2\sec(c+dx)\left(7A+9C+5A\sec^2(c+dx)\right)\tan(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*(7*A + 9*C
)*Sin[c + d*x] + 2*Sec[c + d*x]*(7*A + 9*C + 5*A*Sec[c + d*x]^2)*Tan[c + d*
x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(155) = 310$.
time = 1.19, size = 729, normalized size = 4.96

method	result	size
default	Expression too large to display	729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/144*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1
/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))))+2/5*C/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(
1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x
+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-
2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2))/sin(1/2*d*x+1/2*c)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 160, normalized size = 1.09

$$-\frac{3 \sqrt{2} (7i A + 9i C)\sqrt{b} \cos(dx+c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i \sin(dx+c))) + 3\sqrt{2}(-7i A - 9i C)\sqrt{b} \cos(dx+c)^5 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i \sin(dx+c))) - 2\left(3(7A+9C)\cos(dx+c)^4 + (7A+9C)\cos(dx+c)^2 + 5A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{45 bd \cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(7*I*A + 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*
(-7*I*A - 9*I*C)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(7*A + 9*C)*cos(d*x +
 c)^4 + (7*A + 9*C)*cos(d*x + c)^2 + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)
)/(b*d*cos(d*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^5\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)), x)
```

3.71 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2C(b\cos(c+dx))^{7}}{9b^5d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^5/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/
cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45
*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}\\ &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}+\frac{(9A+7C)\int (b\cos(c+dx))^5\\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2C(b\cos(c+dx))^{7/}}{9b^5d}\\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2C(b\cos(c+dx))^{7/}}{9b^5d}\\ &= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\,\mathrm{co}}{}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 86, normalized size = 0.75

$$\frac{6(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+19C+5C\cos(2(c+dx)))\sin(c+dx)}{45bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b*d*Sqrt[b*Cos[c +
 d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(127) = 254$.
time = 0.39, size = 324, normalized size = 2.82

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-160*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*
C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x
+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 125, normalized size = 1.09

$$-\frac{3\sqrt{2}\left(-9iA-7iC\right)\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\sqrt{2}\left(9iA+7iC\right)\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(5C\cos(dx+c)^3+(9A+7C)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{45b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C
)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b
*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.72 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d}+\frac{2C(b\cos(c+dx))^{5/2}\,\text{s}}{7b^4d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c
))^(1/2)/b^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3}$$
$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{(7A+5C)\int (b\cos(c+dx))^3}{7b^3}$$
$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2C(b\cos(c+dx))^{5/2}}{7b^4d}$$
$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2C(b\cos(c+dx))^{5/2}}{7b^4d}$$
$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\,c}}{}$$

Mathematica [A]
time = 0.28, size = 80, normalized size = 0.70

$$\frac{4(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+(14A+13C+3C\cos(2(c+dx)))\sin(2(c+dx))}{42bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
+ 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(127) = 254.
time = 0.40, size = 296, normalized size = 2.57

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 109, normalized size = 0.95

$$\frac{\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\ \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\ \text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,(3\,C\cos(dx+c)^2+7\,A+5\,C)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.73 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqr
t[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right.}{5b^2\sqrt{\cos(c-}$$
$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx}{5b}$$

Mathematica [A]
time = 0.18, size = 69, normalized size = 0.86

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\cos(c+dx)\sin(2(c+dx))}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x
]*Sin[2*(c + d*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(96) = 192$.
time = 0.38, size = 263, normalized size = 3.29

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\right.}{5b\sqrt{-b\left(2\left(\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(8*C*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 104, normalized size = 1.30

$$\frac{2\sqrt{b\cos\left(dx+c\right)}\,C\cos\left(dx+c\right)\sin\left(dx+c\right)+\sqrt{2}\left(5i\,A+3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\left(-5i\,A-3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.74 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

Rubi [A]

time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {16, 3093, 2721, 2720}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,d}{3b}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int}{3b\sqrt{b\cos(c+d}}$$

$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}}{3b^2d}$$

Mathematica [A]
time = 0.14, size = 61, normalized size = 0.78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(94) = 188.
time = 0.34, size = 239, normalized size = 3.06

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2(}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 92, normalized size = 1.18

$$\frac{\sqrt{2}\,(-3i\,A - i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) + i\,\sin\left(dx + c\right)) + \sqrt{2}\,(3i\,A + i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4, 0, \cos\left(dx + c\right) - i\,\sin\left(dx + c\right)) + 2\,\sqrt{b\cos\left(dx + c\right)}\,C\sin\left(dx + c\right)}{3\,b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.75 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3091, 2721, 2719}

$$\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int\sqrt{b\cos(c+dx)}\,dx}{b^2}$$
$$= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}$$
$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 57, normalized size = 0.77

$$\frac{-2(A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.
time = 0.00, size = 216, normalized size = 2.92

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 117, normalized size = 1.58

$$\frac{\sqrt{2}\,(-i\,A+i\,C)\sqrt{b}\,\cos\left(dx+c\right){\rm weierstrassZeta}(-4,0,{\rm weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,(i\,A-i\,C)\sqrt{b}\,\cos\left(dx+c\right){\rm weierstrassZeta}(-4,0,{\rm weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{b^2d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*co
s(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.76 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3091, 2721, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} \\
&= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{3b\sqrt{b\cos(c+dx)}} \\
&= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.44, size = 140, normalized size = 1.87

$$
-\frac{4(A+C\cos^2(c+dx))\left((A+3C)\cos^2(c+dx)\sqrt{\cos^2(dx-\mathrm{ArcTan}(\cot(c)))}\,\csc(c)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\mathrm{ArcTan}(\cot(c)))\right)\sec(dx-\mathrm{ArcTan}(\cot(c)))-A\sqrt{\csc^2(c)}\,\sin(c+dx)\right)}{3d(b\cos(c+dx))^{3/2}(2A+C+C\cos(2(c+dx)))\sqrt{\csc^2(c)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-4*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan[
Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Co
t[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*d*
(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(91) = 182$.
time = 0.40, size = 294, normalized size = 3.92

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A\ldots}{3\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 116, normalized size = 1.55

$$\frac{\sqrt{2}\,(-i\,A-3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,A+3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{3\,b^{2}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + C\cos^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(b\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

3.77 $\int \frac{\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*
x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}-\frac{(3A+5C)\int}{}\\ &= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}-\frac{\left((3A+5C)\sqrt{}\right.}{}\\ &= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+}{5d(b\cos(c+d}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 81, normalized size = 0.72

$$\frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\sec(c+dx)\tan(c+dx)\right)}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b*d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(125) = 250.
time = 0.77, size = 601, normalized size = 5.32

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2
-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 139, normalized size = 1.23

$$\frac{\sqrt{2}\,(-3i\,A-5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(3i\,A+5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left((3\,A+5\,C)\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{5\,b^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{(b\cos{(c + dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos{(c+d\,x)}^2 + A}{\cos{(c+d\,x)}^2\,(b\,\cos{(c+d\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

3.78 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/d/(b*
cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x
+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(b(5A+7C))\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{(5A+7C)\int \cdots}{\cdots}\\ &= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left((5A+7C)\sqrt{\cdots}\right.}{\cdots}\\ &= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx}{7d(b\cos(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 77, normalized size = 0.69

$$\frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2),x]
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(124) = 248$.
time = 0.72, size = 413, normalized size = 3.69

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(C*(-1/6*cos
(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 135, normalized size = 1.21

$$\frac{\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left((5\,A+7\,C)\cos\left(dx+c\right)^2+3\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,b^2d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
```

```
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*
cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)), x)
```

3.79 $\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}+\frac{2C(b\cos(c+dx))^{7}}{9b^6d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/9*C*(b*cos(d*x+c))^(
7/2)*sin(d*x+c)/b^6/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/
cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2719}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45
*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^5}$$
$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{(9A+7C)\int (b\cos(c+dx))^5}{9b^5}$$
$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/}}{9b^6d}$$
$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/}}{9b^6d}$$
$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\,\mathrm{c}}{}$$

Mathematica [A]
time = 0.44, size = 86, normalized size = 0.75

$$\frac{6(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(18A+19C+5C\cos(2(c+dx)))\sin(c+dx)}{45b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^
2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(127) = 254$.
time = 0.40, size = 324, normalized size = 2.82

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-160C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+320C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 125, normalized size = 1.09

$$-\frac{3\sqrt{2}\left(-9iA-7iC\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(9iA+7iC\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-2\left(5C\cos\left(dx+c\right)^3+\left(9A+7C\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{45b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/45*(3*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*C*cos(d*x + c)^3 + (9*A + 7*C)*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^5*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.80 $\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d}+\frac{2C(b\cos(c+dx))^{5/2}\,\sin(c+dx)}{7b^5d}$$

```
[Out] 2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3093, 2715, 2721, 2720}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b
^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{(7A+5C)\int (b\cos(c+dx))^{3}}{7b^4}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}}{7b^5d}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}}{7b^5d}$$

$$= \frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\,c}}{}$$

Mathematica [A]
time = 0.27, size = 80, normalized size = 0.70

$$\frac{4(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(14A+13C+3C\cos(2(c+dx)))\sin(2(c+dx))}{42b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C
+ 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(127) = 254$.
time = 0.39, size = 296, normalized size = 2.57

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-72*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^6+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 109, normalized size = 0.95

$$\frac{\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left(3\,C\cos(dx+c)^2+7\,A+5\,C\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*C*cos(d*x + c)^2 + 7*A + 5*C)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.81 $\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/5*(5*A+3*C)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos
(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2719}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqr
t[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx}{b^3}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)}$$
$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{\left((5A+3C)\sqrt{b\cos(c+dx)}\right.}{5b^3\sqrt{\cos(c-}$$
$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx}{5b}$$

Mathematica [A]
time = 0.18, size = 69, normalized size = 0.86

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\cos(c+dx)\sin(2(c+dx))}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x
]*Sin[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(96) = 192$.
time = 0.38, size = 263, normalized size = 3.29

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\right.}{5b^2\sqrt{-b\left(2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(8*C*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^2+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 104, normalized size = 1.30

$$\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,C\cos\left(dx+c\right)\sin\left(dx+c\right)+\sqrt{2}\,\left(5i\,A+3i\,C\right)\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\,\left(-5i\,A-3i\,C\right)\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*C*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(5*I*A +
3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.82 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*C*si
n(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3093, 2721, 2720}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int}{3b^2\sqrt{b\cos(c+}}$$

$$= \frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}}{3b^3d}$$

Mathematica [A]
time = 0.15, size = 61, normalized size = 0.78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*
x)])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(94) = 188.
time = 0.34, size = 239, normalized size = 3.06

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\right.}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 92, normalized size = 1.18

$$\frac{\sqrt{2}\,(-3i\,A-i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}\,(3i\,A+i\,C)\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})+2\,\sqrt{b\cos{(dx+c)}}\,C\sin{(dx+c)}}{3\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/
(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.83 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))
^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3091, 2721, 2719}

$$\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos
[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
```

```
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{(A-C)\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$
$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left((A-C)\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+d}}{b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx}}$$

Mathematica [**A**]
time = 0.15, size = 57, normalized size = 0.77

$$\frac{-2(A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x]
)/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(94) = 188.
time = 0.42, size = 216, normalized size = 2.92

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\right.}\right.}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 117, normalized size = 1.58

$$\frac{\sqrt{2}\,(-i\,A+i\,C)\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(i\,A-i\,C)\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,\sqrt{b\cos(dx+c)}\,A\sin(dx+c)}{b^3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] (sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*A - I*C)*sqr
t(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*co
s(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.84 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3091, 2721, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{(A+3C)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$
$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{\left((A+3C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Mathematica [**A**]
time = 0.22, size = 58, normalized size = 0.74

$$\frac{2\left((A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+A\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]
))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(94) = 188.
time = 0.00, size = 294, normalized size = 3.77

method	result
default	$-\frac{2\left(-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)(A}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 116, normalized size = 1.49

$$\frac{\sqrt{2}\,(-i\,A-3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,A+3i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{3\,b^{3}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*
cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c+dx)^2+A}{(b\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.85 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^2/d/(b*c
os(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*
x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3091, 2716, 2721, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3
*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b}$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int \sqrt{\cdots}}{\cdots}$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{\left((3A+5C)\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+d\cdots}{5d(b\cos(c+dx\cdots}$$

Mathematica [A]
time = 0.11, size = 81, normalized size = 0.72

$$\frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\sec(c+dx)\tan(c+dx)\right)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^2*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(124) = 248$.
time = 0.76, size = 601, normalized size = 5.37

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2
-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 139, normalized size = 1.24

$$\frac{\sqrt{2}\left(-3i\,A-5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{2}\left(3i\,A+5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(\left(3\,A+5\,C\right)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\,b^{3}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

3.86 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b/d/(b*
cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3091, 2716, 2721, 2720}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{(5A+7C)\int \cdots}{\cdots}\\ &= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{\left((5A+7C)\sqrt{\cdots}\right.}{\cdots}\\ &= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))\cdots}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 77, normalized size = 0.68

$$\frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5A+7C+3A\sec^2(c+dx))\tan(c+dx)\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(125) = 250.
time = 0.72, size = 413, normalized size = 3.65

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 135, normalized size = 1.19

$$\frac{\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\left((5\,A+7\,C)\cos(dx+c)^2+3\,A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,b^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
```

```
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*
cos(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)), x)
```

3.87 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b
*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3091, 2716, 2721, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*
(3*A + 5*C)*Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{(3A+5C)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{(3A+5C)\int \sqrt{b\cos(c+dx}}{5b^4}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{\left((3A+5C)\sqrt{b\cos(c+dx)}\right.}{5b^4\sqrt{\cos(c}}$$
$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2(3A}{5b^3}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.70

$$\frac{2\left(-\left((3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+(3A+5C)\sin(c+dx)+A\sec(c+dx)\tan(c+dx)\right)}{5b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*
C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(127) = 254$.
time = 0.00, size = 601, normalized size = 5.23

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*A*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4+40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^6-20*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2
-40*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))+10*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*C*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 139, normalized size = 1.21

$$\frac{\sqrt{2}\,(-3i\,A-5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(3i\,A+5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left((3\,A+5\,C)\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{5\,b^4d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(3*I*
A + 5*I*C)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*((3*A + 5*C)*cos(d*x + c)^2 +
A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.88 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=115

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(7/2)+2/21*(5*A+7*C)*sin(d*x+c)/b^3/d/(
b*cos(d*x+c))^(3/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^4/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3091, 2716, 2721, 2720}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}}+\frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt
[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*
(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx = \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{(5A+7C)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{7b^2}$$

$$= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{(5A+7C)\int \frac{1}{\sqrt{b\cos(c+dx}}}{21b^4}$$

$$= \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{\left((5A+7C)\sqrt{\cos(c+dx)}\right.}{21b^4\sqrt{b\cos(c}}$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} + \frac{2(5A+}{21b^3d(b}$$

Mathematica [**A**]
time = 0.11, size = 77, normalized size = 0.67

$$\frac{2\left((5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(5A+7C+3A\sec^2(c+dx)\right)\tan(c+dx)\right)}{21b^4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(127) = 254$.
time = 0.00, size = 413, normalized size = 3.59

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(C\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{6b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 135, normalized size = 1.17

$$\frac{\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left((5\,A+7\,C)\cos\left(dx+c\right)^2+3\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,b^5d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(
d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*((
5*A + 7*C)*cos(d*x + c)^2 + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^5*d*
cos(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2),x)
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(9/2), x)
```

3.89 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=116

$$\frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{(A+2C)\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*(A+2*C)*sin(d*
x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*C*sin(d*x+c)^5*(b*cos(d*
x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3092, 380}

$$-\frac{(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((A +
2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (C*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\left(A+C\cos^2(c+\right.}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int (1-x^2)\left(A+C-C\right.\right.}{d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int \left(A(1+\frac{C}{A})-(A+2\right.\right.}{d\sqrt{\cos(c+d}}$$

$$= \frac{(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(A+2C}{}$$

Mathematica [A]
time = 0.24, size = 70, normalized size = 0.60

$$\frac{\sqrt{b\cos(c+dx)}\,(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(c+dx)}{120d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C*
Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 2.38, size = 70, normalized size = 0.60

method	result
default	$\frac{\left(3C\left(\cos^4(dx+c)\right)+5A\left(\cos^2(dx+c)\right)+4C\left(\cos^2(dx+c)\right)+10A+8C\right)\sin(dx+c)\sqrt{b\cos(dx+c)}}{15d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}(6A+5C)}{8\left(e^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.63, size = 111, normalized size = 0.96

$$\frac{C\sqrt{b}\left(3\sin(5dx+5c)+25\sin\left(\frac{3}{5}\arctan(\sin(5dx+5c),\cos(5dx+5c))\right)+150\sin\left(\frac{1}{5}\arctan(\sin(5dx+5c),\cos(5dx+5c))\right)\right)+20A\sqrt{b}\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/240*(C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c
)))) + 20*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))))/d
```

Fricas [**A**]
time = 0.38, size = 63, normalized size = 0.54

$$\frac{\left(3\,C\cos\left(dx+c\right)^{4}+\left(5\,A+4\,C\right)\cos\left(dx+c\right)^{2}+10\,A+8\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sqrt(b*
cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.47, size = 97, normalized size = 0.84

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+175\,C\,\sin\left(2\,c+2\,d\,x\right)+28\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)\right)}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*A*s
in(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*sin(
6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.90 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=113

$$\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos}$$

```
[Out] 1/4*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*(4*A+3*C)*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/
2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*
Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\sqrt{b\cos(c+dx)}\int\cos^2(c+dx)\left(A+C\cos^2(c+\right.}{\sqrt{\cos(c+dx)}}$$
$$=\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d}+$$
$$=\frac{(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c}{8d}$$
$$=\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sqrt{co}}{}$$

Mathematica [A]
time = 0.19, size = 67, normalized size = 0.59

$$\frac{\sqrt{b\cos(c+dx)}\,(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)]
 + C*Sin[4*(c + d*x)]))/(32*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.50, size = 88, normalized size = 0.78

method	result
default	$\frac{\sqrt{b\cos(dx+c)}\,\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)e^{i(dx+c)}(8A+6C)x}{8\,e^{2i(dx+c)}+8}-\frac{i\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)e^{5i(dx+c)}C}{32\left(e^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b\,c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(
d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.62, size = 75, normalized size = 0.66

$$\frac{8\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)C\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqr
t(b))/d
```

Fricas [A]
time = 0.46, size = 200, normalized size = 1.77

$$\left[\frac{2\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{16\,d},\frac{\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{b}\cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*C
*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0(4*sqrt(sageVAR
```

Mupad [B]
time = 2.25, size = 112, normalized size = 0.99

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c+3\,d\,x\right)+C\,\sin\left(5\,c+5\,d\,x\right)+32\,A\,d\,x\,\cos\left(c+d\,x\right)+24\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) + 1)
)
```

3.91 $\int \sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))$

Optimal. Leaf size=74

$$\frac{(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{C\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*C*sin(d*x+c)^3
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {17, 3092}

$$\frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (C*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\sqrt{b\cos(c+dx)}\,\int\cos(c+dx)\,(A+C\cos^2(c+}{\sqrt{\cos(c+dx)}}$$

$$=-\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,\right.}{d\sqrt{\cos(c+dx)}}$$

$$=\frac{(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{C\sqrt{b\,c}}{}$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 0.70

$$\frac{\sqrt{b\cos(c+dx)}\,(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*S
qrt[Cos[c + d*x]])
```

Maple [A]
time = 0.30, size = 47, normalized size = 0.64

method	result
default	$\frac{\left(C\left(\cos^2(dx+c)\right)+3A+2C\right)\sin(dx+c)\sqrt{b\cos\left(dx+c\right)}}{3d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{4i(dx+c)}C}{12\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{2i(dx+c)}(4A+3C)}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2)
```

Maxima [A]
time = 0.62, size = 57, normalized size = 0.77

$$\frac{C\sqrt{b}\,\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)+12\,A\sqrt{b}\,\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
[Out] 1/12*(C*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))) + 12*A*sqrt(b)*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.62

$$\frac{\left(C\cos\left(dx+c\right)^{2}+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqr
t(cos(d*x + c)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(68) = 136$.
time = 40.81, size = 139, normalized size = 1.88

$$\begin{cases} 0 & \text{for } c = -dx + \frac{\pi}{2} \vee c = -dx + \frac{3\pi}{2} \\ x\sqrt{b\cos(c)}\,(A + C\cos^2(c))\sqrt{\cos(c)} & \text{for } d = 0 \\ \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (x*sqrt(b*cos(c))
*(A + C*cos(c)**2)*sqrt(cos(c)), Eq(d, 0)), (A*sqrt(b*cos(c + d*x))*sin(c +
 d*x)/(d*sqrt(cos(c + d*x))) + 2*C*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*
d*sqrt(cos(c + d*x))) + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x)**(
3/2)/d, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79987 vs. $2(64) = 128$.
time = 6.58, size = 79987, normalized size = 1080.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/96*(3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*
C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
+ 1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*
tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*C*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^4*tan(1/3*c)^4*tan(c)^2 - 18*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
```

n(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*ta
n(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*C*sqrt(b)*d*
x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*t
an(1/3*c)^6 + 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c
)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x +
1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*C
*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^4*tan(1/3*c)^6 - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c) - 72*C*sqrt(b)*d
*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*
tan(1/3*c)^4*tan(c) + 24*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*x^4
*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(
1/3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1
/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) + 144*C*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c) - 72*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 24*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
6*tan(c) + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*
tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^4*
tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^
6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 + 27*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^
4*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)
^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/
2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan(c)^2 + 48*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 + 108*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^2*tan(1/3*c)^6*tan(c)^2 - 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1
/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 48*C*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^3*tan(1/3*c)^6*tan(c)^2 + 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*C*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 + 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/
2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*C*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1
/2*c)^2*tan(1/3*c)^4 + 144*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d

```
*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 27*C*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*
c)^4 + 54*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 - 3*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 54*C*sqr
t(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2
*c)^2*tan(1/3*c)^6 - 108*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x
 + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 144*C*sqrt(b)*d*x^4*tan(
1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c
)^6 - 48*C*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*t...
```

Mupad [B]
time = 0.95, size = 72, normalized size = 0.97

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.92 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=90

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos(
d*x+c)^(1/2)+1/2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 2715, 8}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/
(2*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+C\cos^2(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(C\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c}{2d}$$

$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}}{}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.58

$$\frac{\sqrt{b\cos(c+dx)}\,(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sq
rt[Cos[c + d*x]])
```

Maple [A]
time = 0.32, size = 54, normalized size = 0.60

method	result	size
default	$\frac{\sqrt{b\cos(dx+c)}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	54
risch	$\frac{\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}} + \frac{\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/
cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.58, size = 52, normalized size = 0.58

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})C\sqrt{b}+8\,A\sqrt{b}\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*
x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.46, size = 162, normalized size = 1.80

$$\left[\frac{2\,\sqrt{b\cos{(dx+c)}}\,C\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}+(2\,A+C)\sqrt{-b}\,\log\left(2\,b\cos{(dx+c)}^2-2\,\sqrt{b\cos{(dx+c)}}\,\sqrt{-b}\,\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}-b\right)}{4\,d}, \frac{\sqrt{b\cos{(dx+c)}}\,C\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}+(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos{(dx+c)}}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [A]
time = 19.13, size = 146, normalized size = 1.62

$$\begin{cases} \frac{Ax\sqrt{b\cos{(c+dx)}}}{\sqrt{\cos{(c+dx)}}}+\frac{Cx\sqrt{b\cos{(c+dx)}}\,\sin^2{(c+dx)}}{2\sqrt{\cos{(c+dx)}}}+\frac{Cx\sqrt{b\cos{(c+dx)}}\,\cos^{\frac{3}{2}}{(c+dx)}}{2}+\frac{C\sqrt{b\cos{(c+dx)}}\,\sin{(c+dx)}\sqrt{\cos{(c+dx)}}}{2d} & \text{for } d\neq 0 \\ \frac{x\sqrt{b\cos{(c)}}\,(A+C\cos^2{(c)})}{\sqrt{\cos{(c)}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + C*x*sqrt(b*cos(c +
 d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*co
s(c + d*x)**(3/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x)
)/(2*d), Ne(d, 0)), (x*sqrt(b*cos(c))*(A + C*cos(c)**2)/sqrt(cos(c)), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 0.43, size = 45, normalized size = 0.50

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*cos(
c + d*x)^(1/2))
```

3.93 $\int \frac{\sqrt{b \cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+C*sin(d*x+c)*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3093, 3855}

$$\frac{A\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{C\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\,\text{si}}{d\sqrt{\cos(c+d}}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.65

$$\frac{\sqrt{b\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),
x]
[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[C
os[c + d*x]])
```

Maple [A]
time = 0.30, size = 55, normalized size = 0.81

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{i\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{b\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+\sqrt{b\,\text{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^
(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.64, size = 80, normalized size = 1.18

$$\frac{A\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)+2C\sqrt{b}\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 2*C*sqrt(b)*s
in(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 201, normalized size = 2.96

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqr
t(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos\left(c+dx\right)}\left(A+C\cos^{2}\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x)
```

3.94 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+C*x*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.086$, Rules used = {17, 3091, 8}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(C\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.76

$$\frac{\sqrt{b\cos(c+dx)}\,(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x
]^(3/2))
```

Maple [A]
time = 0.27, size = 45, normalized size = 0.76

method	result	size
default	$\frac{\sqrt{b\cos(dx+c)}\,(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{3}{2}}}$	45
risch	$\frac{Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}} + \frac{2i\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/
2)
```

Maxima [A]
time = 0.59, size = 80, normalized size = 1.36

$$\frac{2\left(C\sqrt{b}\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + \frac{A\sqrt{b}\,\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(C*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*sqrt(b)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Fricas [A]
time = 0.43, size = 185, normalized size = 3.14

$$\left[\frac{C\sqrt{-b}\cos\left(dx+c\right)^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)^{2}}, \frac{C\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^{2}+\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))
*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*sqrt(b)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x
)
```

Mupad [B]
time = 1.39, size = 81, normalized size = 1.37

$$\frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(A\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x)
+ 1))
```

3.95 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*(A+2*C)*arctan
h(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3091, 3855}

$$\frac{(A+2C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*
x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left((A+2C)\sqrt{b\cos(c+dx)}\right)}{2\sqrt{\cos(c+dx}}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c-}}{2d\,\mathrm{co}}$$

Mathematica [**A**]
time = 0.11, size = 59, normalized size = 0.76

$$\frac{\sqrt{b\cos(c+dx)}\,\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*S
in[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(66) = 132$.
time = 0.32, size = 134, normalized size = 1.72

method	result
default	$\frac{\left(-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx\cdot}{\sin(dx+c}\right.\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,A\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b\cos(dx+c)}\,(A+}{2\sqrt{\cos(dx\cdot}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(66) = 132$.
time = 0.65, size = 728, normalized size = 9.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.43, size = 213, normalized size = 2.73

$$\left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+c)}{4d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt
```

```
(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x +
 c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2), x)
```

3.96 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{(2A+3C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*(2*A+3*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + ((2*A + 3*
C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\Big((2A+3C)\sqrt{b\cos(c+dx)}}{3\sqrt{\cos(c+d}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} - \frac{\Big((2A+3C)\sqrt{b\cos(c+dx)}}{3d\sqrt{co}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{(2A+3C)\sqrt{b\cos(c+dx)}\,\mathrm{si}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.23, size = 51, normalized size = 0.65

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)\,(3(A+C)+A\tan^2(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),
x]
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Cos
[c + d*x]^(3/2))
```

Maple [A]
time = 0.30, size = 54, normalized size = 0.68

method	result	size
default	$\frac{(2A(\cos^2(dx+c))+3C(\cos^2(dx+c))+A)\sin(dx+c)\sqrt{b\cos(dx+c)}}{3d\cos(dx+c)^{\frac{7}{2}}}$	54
risch	$\frac{2i\sqrt{b\cos(dx+c)}\,(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)
/cos(d*x+c)^(7/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(67) = 134$.
time = 0.63, size = 350, normalized size = 4.43

$$\frac{2\left(\frac{2\left((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c)\right)A\sqrt{b}}{2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3\cos(2dx+2c)+1)\cos(4dx+4c)+9\cos(4dx+4c)^2+9\cos(2dx+2c)^2+6(\sin(4dx+4c)+\sin(2dx+2c))\sin(6dx+6c)+\sin(6dx+6c)^2+9\sin(4dx+4c)^2+18\sin(4dx+4c)\sin(2dx+2c)+9\sin(2dx+2c)^2+6\cos(2dx+2c)+1}+\frac{3C\sqrt{b}\sin(2dx+2c)}{\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) +
 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x +
4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c
) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*
x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(
4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^
2 + 6*cos(2*d*x + 2*c) + 1) + 3*C*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [**A**]
time = 0.37, size = 47, normalized size = 0.59

$$\frac{\left((2A+3C)\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*c
os(d*x + c)^(7/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x
)
```

Mupad [B]
time = 3.32, size = 217, normalized size = 2.75

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20{}\mathrm{i}+C\,30{}\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30{}\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12{}\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2{}\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45{}\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18{}\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3{}\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x)
 + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.97 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{(3A+4C)\sqrt{b\cos(c+dx}}{8d\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] 1/4*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*(3*A+4*C)*arctanh(sin(d*
x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left((3A+4C)\sqrt{b\cos(c+dx)}\right.}{4\sqrt{\cos(c+d}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b\cos(c+dx)}\,\mathrm{s}}{8d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c}}{4d\,\mathrm{c}}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 0.66

$$\frac{\sqrt{b\cos(c+dx)}\,\left((3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (
2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.
time = 0.32, size = 214, normalized size = 1.75

method	result
default	$\frac{\left(-3A(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A(\cos^4(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c}{\sin(d}\right.\right.}{}$

risch	$-\frac{i\sqrt{b\cos(dx+c)}\,(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(104) = 208$.
time = 0.71, size = 2318, normalized size = 19.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
```

```
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*
c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d
*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(co
s(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]

time = 0.46, size = 255, normalized size = 2.09

$$\left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16d\cos(dx+c)^5}, -\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqr
t(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c
))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(11/2), x)
```

3.98 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=119

$$\frac{b(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b(A+2C)\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\,\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] b*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*(A+2*C)*si
n(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b*C*sin(d*x+c)^5*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3092, 380}

$$-\frac{b(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*(
A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b
*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^3(c+dx)\left(A+C\cos\right.}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1-x^2)\left(A+\right.\right.}{d\sqrt{\cos(c+dx}}$$

$$= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(A\left(1+\frac{C}{A}\right)-\right.\right.}{d\sqrt{\cos(c}}$$

$$= \frac{b(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b(A\cdot}{}$$

Mathematica [A]
time = 0.25, size = 70, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{3/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(c+dx)}{120d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*
C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.34, size = 70, normalized size = 0.59

method	result
default	$\frac{\left(3C\left(\cos^4(dx+c)\right)+5A\left(\cos^2(dx+c)\right)+4C\left(\cos^2(dx+c)\right)+10A+8C\right)(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{15d\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d} - \frac{ib\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}(6A+5C)}{8\left(e^{2i(dx+c)}+1\right)d} + \frac{ib}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(b*cos
(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.64, size = 117, normalized size = 0.98

$$\frac{20\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+\left(3\,b\sin\left(5\,dx+5\,c\right)+25\,b\sin\left(\frac{3}{5}\,\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,b\sin\left(\frac{1}{5}\,\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)\right)C\sqrt{b}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="maxima")
```

```
[Out] 1/240*(20*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*A*sqrt(b) + (3*b*sin(5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin
(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b*sin(1/5*arctan2(sin(5*d*x + 5*c),
 cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.39, size = 69, normalized size = 0.58

$$\frac{\left(3\,Cb\cos\left(dx+c\right)^{4}+\left(5\,A+4\,C\right)b\cos\left(dx+c\right)^{2}+2\left(5\,A+4\,C\right)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*b*cos(d*x + c)^4 + (5*A + 4*C)*b*cos(d*x + c)^2 + 2*(5*A + 4*C)*b
)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.24, size = 98, normalized size = 0.82

$$\frac{b\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(200\,A\,\sin\left(2\,c+2\,d\,x\right)+20\,A\,\sin\left(4\,c+4\,d\,x\right)+175\,C\,\sin\left(2\,c+2\,d\,x\right)+28\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)\right)}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20*A
*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*si
n(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.99 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))$

Optimal. Leaf size=116

$$\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d}+\frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b}$$

```
[Out] 1/4*b*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b*(4*A+3*C)*
x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b*(4*A+3*C)*sin(d*x+c)*cos(d*x+
c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{bx(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}+\frac{bC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3
*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[
c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,(A+C\,}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d}$$

$$= \frac{b(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,}{8d}$$

$$= \frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(4A+3C}{}$$

Mathematica [A]
time = 0.20, size = 67, normalized size = 0.58

$$\frac{(b\cos(c+dx))^{3/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x
)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.45, size = 88, normalized size = 0.76

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{b\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{i(dx+c)}(8A+6C)x}{8\,\mathrm{e}^{2i(dx+c)}+8} - \frac{ib\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d} + \frac{ib\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(3/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(
d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [**A**]
time = 0.63, size = 82, normalized size = 0.71

$$\frac{8\,(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right))A\sqrt{b}+\left(12\,(dx+c)b+b\sin\left(4\,dx+4\,c\right)+8\,b\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)C\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*b +
b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*C*sqrt(b))/d
```

Fricas [**A**]
time = 0.47, size = 209, normalized size = 1.80

$$\left[\frac{(4\,A+3\,C)\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(2\,Cb\cos\left(dx+c\right)^2+(4\,A+3\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{16\,d},\ \frac{(4\,A+3\,C)b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)+\left(2\,Cb\cos\left(dx+c\right)^2+(4\,A+3\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x + c)^2
+ (4*A + 3*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1
/8*((4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2))) + (2*C*b*cos(d*x + c)^2 + (4*A + 3*C)*b)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0sageVARb*(4*sqr
```

Mupad [B]
time = 1.94, size = 113, normalized size = 0.97

$$\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c+3\,d\,x\right)+C\,\sin\left(5\,c+5\,d\,x\right)+32\,A\,d\,x\,\cos\left(c+d\,x\right)+24\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c
+ d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) +
 32*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) +
1))
```

3.100 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{b(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{bC\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*C*sin(d*x+
c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3092}

$$\frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{bC\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*C
*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int\left(A+C-Cx^2\right)dx,x,-\sin\right.}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{bC\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c}}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.70

$$\frac{b\sqrt{b\cos(c+dx)}\,(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d
*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.29, size = 47, normalized size = 0.62

method	result	size
default	$\frac{\left(C\left(\cos^2(dx+c)\right)+3A+2C\right)(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{3d\cos(dx+c)^{\frac{3}{2}}}$	47
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(
3/2)
```

Maxima [A]
time = 0.66, size = 60, normalized size = 0.79

$$\frac{12\,Ab^{\frac{3}{2}}\sin(dx+c)+\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)C\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.36, size = 50, normalized size = 0.66

$$\frac{(Cb\cos{(dx+c)}^2+(3A+2C)b)\sqrt{b\cos{(dx+c)}}\sin{(dx+c)}}{3d\sqrt{\cos{(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*b*cos(d*x + c)^2 + (3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/
(d*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)),
 x)
```

Mupad [B]
time = 0.60, size = 54, normalized size = 0.71

$$\frac{b\sqrt{b\cos{(c+d\,x)}}\,(12\,A\sin{(c+d\,x)}+9\,C\sin{(c+d\,x)}+C\sin{(3\,c+3\,d\,x)})}{12\,d\sqrt{\cos{(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(3*c
 + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

3.101 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)/
cos(d*x+c)^(1/2)+1/2*b*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x
]])/(2*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*S
in[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+d}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\ \mathrm{s}}{2d}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{\cos(c}}{}$$

Mathematica [A]
time = 0.10, size = 52, normalized size = 0.56

$$\frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*
Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.27, size = 54, normalized size = 0.58

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\cos(dx+c)^{\frac{3}{2}}}$	54
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}} + \frac{b\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/
cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.59, size = 55, normalized size = 0.59

$$\frac{8\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(2\,(dx+c)b+b\sin\left(2\,dx+2\,c\right)\right)C\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b +
 b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Fricas [A]
time = 0.41, size = 165, normalized size = 1.77

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,Cb\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(2\,A+C\right)\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,Cb\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(2\,A+C\right)b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C
)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(
cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b*sqrt(cos(
d*x + c))*sin(d*x + c) + (2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2),
 x)
```

Mupad [B]
time = 1.04, size = 46, normalized size = 0.49

$$\frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*co
s(c + d*x)^(1/2))
```

3.102 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*sin(d*x
+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3093, 3855}

$$\frac{Ab\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (
b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\,(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int s}{\sqrt{\cos(c+dx}}$$
$$= \frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b\cos(c+d}}{d\sqrt{\cos(c}}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 0.63

$$\frac{(b\cos(c+dx))^{3/2}\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos[
c + d*x]^(3/2))
```

Maple [A]
time = 0.27, size = 55, normalized size = 0.79

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)(b\cos(dx+c))^{\frac{3}{2}}}{d\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{ib\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{ib\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos\left(dx+c\right)}\,d}-\frac{b\sqrt{b\cos\left(dx+c\right)}\,A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^
(3/2)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.66, size = 83, normalized size = 1.19

$$\frac{2\,Cb^{\frac{3}{2}}\sin\left(dx+c\right)+\left(b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)A\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(2*C*b^(3/2)*sin(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*
sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1))*A*sqrt(b))/d
```

Fricas [**A**]
time = 0.43, size = 204, normalized size = 2.91

$$\left[\frac{Ab^{\frac{3}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}Cb\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)}, -\frac{A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}Cb\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)), -(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x)
```

3.103 $\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*C*x*(b*cos(d*x+c))
^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3091, 8}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(bC\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.74

$$\frac{(b\cos(c+dx))^{3/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d
*x]^(5/2))
```

Maple [A]
time = 0.24, size = 45, normalized size = 0.74

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{5}{2}}}$	45
risch	$\frac{bCx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}} + \frac{2ib\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(5/
2)
```

Maxima [A]
time = 0.61, size = 80, normalized size = 1.31

$$\frac{2\left(Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{Ab^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2*(C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(3/2)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Fricas [A]
time = 0.41, size = 188, normalized size = 3.08

$$\left[\frac{C\sqrt{-b}\, b\cos\left(dx+c\right)^{2}\log\left(2\, b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,Ab\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\, d\cos\left(dx+c\right)^{2}}, \frac{Cb^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^{2}+\sqrt{b\cos\left(dx+c\right)}\,Ab\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(3/2)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c)^2 + sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2),
 x)
```

Mupad [B]
time = 1.23, size = 82, normalized size = 1.34

$$\frac{b\sqrt{b\cos\left(c+dx\right)}\,\left(A\sin\left(2c+2dx\right)+C\,d\,x+C\,d\,x\,\cos\left(2c+2dx\right)+A\,1\mathrm{i}+A\cos\left(2c+2dx\right)\,1\mathrm{i}\right)}{d\sqrt{\cos\left(c+dx\right)}\,\left(\cos\left(2c+2dx\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x
) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x
) + 1))
```

3.104 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b*(A+2*C)*ar
ctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.086$, Rules used = {17, 3091, 3855}

$$\frac{b(A+2C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c +
d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b(A+2C)\sqrt{b\cos(c+dx)}\right.}{2\sqrt{\cos(c}}$$

$$= \frac{b(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos}}{2}$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 0.74

$$\frac{(b\cos(c+dx))^{3/2}\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A
*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.28, size = 134, normalized size = 1.68

method	result
default	$\frac{\left(-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(}{\sin(dx}\right.\right.}{2d\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,A\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(68) = 136$.
time = 0.66, size = 761, normalized size = 9.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log
(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b*s
in(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*c
os(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4
*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x
+ 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) +
b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*
c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*
(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*
x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.44, size = 216, normalized size = 2.70

$$\left[\frac{(A+2C)b^{\frac{3}{2}}\cos(dx+c)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos(dx+c)}Ab\sqrt{\cos(dx+c)}\sin(dx+c)}{4d\cos(dx+c)^{3}}, -\frac{(A+2C)\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^{3}-\sqrt{b\cos(dx+c)}Ab\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*
```

```
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2), x)
```

3.105 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b(2A+3C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*b*(2*A+3*C)*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {17, 3091, 3852, 8}

$$\frac{b(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*(2*A
+ 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b(2A+3C)\sqrt{b\cos(c+\right.}{3\sqrt{\cos(c}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} - \frac{\left(b(2A+3C)\sqrt{b\cos(c+\right.}{3d}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b(2A+3C)\sqrt{b\cos(c+d}}{3d\cos^{\frac{3}{2}}(c+d}$$

Mathematica [A]
time = 0.16, size = 52, normalized size = 0.64

$$\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*C
os[c + d*x]^(3/2))
```

Maple [A]
time = 0.25, size = 54, normalized size = 0.67

method	result	size
default	$\frac{\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)\sin(dx+c)(b\cos(dx+c))^{\frac{3}{2}}}{3d\cos(dx+c)^{\frac{9}{2}}}$	54
risch	$\frac{2ib\sqrt{b\cos\left(dx+c\right)}\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(3/2)
/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(69) = 138$.
time = 0.61, size = 355, normalized size = 4.38

$$\frac{2\left(\frac{3\,Cb^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1}-\frac{2\,\left(3\,b\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(6\,dx+6\,c\right)-3\,\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(4\,dx+4\,c\right)\right)A\sqrt{b}}{2\,\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\,\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^{2}+9\,\cos\left(2\,dx+2\,c\right)^{2}+6\,\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^{2}+9\,\sin\left(4\,dx+4\,c\right)^{2}+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^{2}+6\,\cos\left(2\,dx+2\,c\right)+1}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 2/3*(3*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - 2*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*
cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x +
6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4
*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*c
os(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c
) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.35, size = 50, normalized size = 0.62

$$\frac{\left((2\,A+3\,C)b\cos\left(dx+c\right)^{2}+Ab\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*b*cos(d*x + c)^2 + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2)
, x)
```

Mupad [B]
time = 2.55, size = 218, normalized size = 2.69

$$\frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20\mathrm{i}+C\,30\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4
*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*
c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4
*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c +
 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*
x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.106 $\int \frac{(b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+d}{8d\cos^{\frac{5}{2}}(c+c}$$

[Out] 1/4*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*b*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3853, 3855}

$$\frac{b(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]

[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b(3A+4C)\sqrt{b\cos(c+\right.}{4\sqrt{\cos(c}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C)\sqrt{b\cos(c+d}}{8d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b}}{}$$

Mathematica [A]
time = 0.22, size = 81, normalized size = 0.65

$$\frac{b\sqrt{b\cos(c+dx)}\left((3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\left(2A+(3A+4C)\cos^2(c+dx)\right)\sin(c+dx)\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
 (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.30, size = 214, normalized size = 1.71

method	result
default	$\frac{\left(-3A\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A\left(\cos^4(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx}{\sin}\right.\right.}{}$

risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(3A\,e^{7i(dx+c)}+4C\,e^{7i(dx+c)}+11A\,e^{5i(dx+c)}+4C\,e^{5i(dx+c)}-11A\,e^{3i(dx+c)}-4C\,e^{3i(dx+c)}-3A\,e^{i(dx+c)}-4C}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2434 vs. $2(107) = 214$.
time = 0.71, size = 2434, normalized size = 19.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
+ 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*
b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*
sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2
*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
 + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x +
4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) +
2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
```

```
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12
*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*
b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1) + 4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*s
in(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*
cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x
 + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x +
 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 +
2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*l
og(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) +
 b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x +
 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4
*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [**A**]
time = 0.43, size = 260, normalized size = 2.08

$$\left[\frac{(3A+4C)b^{\frac{3}{2}}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)b\cos(dx+c)^2+2Ab\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16d\cos(dx+c)^5}, -\frac{(3A+4C)\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)b\cos(dx+c)^2+2Ab\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d
*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(3/2))/cos(c + d*x)^(13/2), x)
```

3.107 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\,(A+C\cos^2(c+dx$

Optimal. Leaf size=125

$$\frac{b^2(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\,\sin^5}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*(A+2*C
)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b^2*C*sin(d*x+c)
^5*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3092, 380}

$$-\frac{b^2(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b
^2*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
+ (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,(A+C\,\mathrm{c}}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\mathrm{Subst}\left(\int (1-x^2)\,(A-\right.}{d\sqrt{\cos(c+d}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\mathrm{Subst}\left(\int \left(A(1+\frac{C}{A})-\right.\right.}{d\sqrt{\cos(}}$$

$$= \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2(}{}$$

Mathematica [A]
time = 0.29, size = 70, normalized size = 0.56

$$\frac{(b\cos(c+dx))^{5/2}(100A+89C+4(5A+7C)\cos(2(c+dx))+3C\cos(4(c+dx)))\sin(c+dx)}{120d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*
C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.36, size = 70, normalized size = 0.56

method	result
default	$\frac{(3C(\cos^4(dx+c))+5A(\cos^2(dx+c))+4C(\cos^2(dx+c))+10A+8C)(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{15d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{6i(dx+c)}C}{80(\mathrm{e}^{2i(dx+c)}+1)d} - \frac{ib^2\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{2i(dx+c)}(6A+5C)}{8(\mathrm{e}^{2i(dx+c)}+1)d} + \frac{ib^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(b*cos
(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.62, size = 127, normalized size = 1.02

$$\frac{20\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+\left(3\,b^2\sin\left(5\,dx+5\,c\right)+25\,b^2\sin\left(\frac{3}{5}\,\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,b^2\sin\left(\frac{1}{5}\,\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)\right)C\sqrt{b}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/240*(20*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))))*A*sqrt(b) + (3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arc
tan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d
*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.37, size = 75, normalized size = 0.60

$$\frac{\left(3\,Cb^2\cos\left(dx+c\right)^4+\left(5\,A+4\,C\right)b^2\cos\left(dx+c\right)^2+2\,\left(5\,A+4\,C\right)b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(3*C*b^2*cos(d*x + c)^4 + (5*A + 4*C)*b^2*cos(d*x + c)^2 + 2*(5*A + 4*
C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.16, size = 100, normalized size = 0.80

$$\frac{b^2 \sqrt{\cos(c+dx)} \sqrt{b\cos(c+dx)} \left(200 A \sin(2c+2dx) + 20 A \sin(4c+4dx) + 175 C \sin(2c+2dx) + 28 C \sin(4c+4dx) + 3 C \sin(6c+6dx)\right)}{240 d \left(\cos(2c+2dx)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(200*A*sin(2*c + 2*d*x) + 20
*A*sin(4*c + 4*d*x) + 175*C*sin(2*c + 2*d*x) + 28*C*sin(4*c + 4*d*x) + 3*C*
sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.108 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d}+\frac{b^2C\cos^{\frac{5}{2}}(c+dx)}{}$$

```
[Out] 1/4*b^2*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b^2*(4*A+3
*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C)*sin(d*x+c)*co
s(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{b^2x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}+\frac{b^2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(4*A
 + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*
C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(b^2(4A+3C\right.}{}$$

$$= \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d} + \frac{b^2}{}$$

$$= \frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C)\sqrt{\cos(c+dx)}}{}$$

Mathematica [A]
time = 0.20, size = 67, normalized size = 0.55

$$\frac{(b\cos(c+dx))^{5/2}(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x
)] + C*Sin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.47, size = 88, normalized size = 0.72

method	result
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(8A+6C)x}{16\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,(A+C)\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(5/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(
d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.62, size = 92, normalized size = 0.75

$$\frac{8\left(2\left(dx+c\right)b^{2}+b^{2}\sin\left(2\,dx+2\,c\right)\right)A\sqrt{b}+\left(12\left(dx+c\right)b^{2}+b^{2}\sin\left(4\,dx+4\,c\right)+8\,b^{2}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)C\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*
b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.46, size = 219, normalized size = 1.80

$$\left[\frac{\left(4\,A+3\,C\right)\sqrt{-b}\,b^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(2\,Cb^{2}\cos\left(dx+c\right)^{2}+\left(4\,A+3\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{16\,d},\ \frac{\left(4\,A+3\,C\right)b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)+\left(2\,Cb^{2}\cos\left(dx+c\right)^{2}+\left(4\,A+3\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*x + c
)^2 + (4*A + 3*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/d, 1/8*((4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*cos(d*x + c)^(3/2))) + (2*C*b^2*cos(d*x + c)^2 + (4*A + 3*C)*b^2)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)),
 x)
```

Mupad [B]
time = 0.80, size = 72, normalized size = 0.59

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)+16\,A\,d\,x+12\,C\,d\,x\right)}{32\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(8*A*sin(2*c + 2*d*x) + 8*C*sin(2*c + 2*d*x) +
C*sin(4*c + 4*d*x) + 16*A*d*x + 12*C*d*x))/(32*d*cos(c + d*x)^(1/2))
```

3.109 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{b^2(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2C\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*(A+C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*C*sin(
d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3092}

$$\frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b^2C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b
^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int \left(A+C-Cx^2\right)\,dx, x, -\sin(\right.}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(A+C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2C\sqrt{b\cos(c+dx}}{3d\sqrt{\cos(c}}$$

Mathematica [A]
time = 0.15, size = 52, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{5/2}(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d
*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.26, size = 47, normalized size = 0.59

method	result	size
default	$\frac{\left(C\left(\cos^2(dx+c)\right)+3A+2C\right)\sin(dx+c)(b\cos(dx+c))^{\frac{5}{2}}}{3d\cos(dx+c)^{\frac{5}{2}}}$	47
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
5/2)
```

Maxima [A]
time = 0.60, size = 64, normalized size = 0.80

$$\frac{12\,Ab^{\frac{5}{2}}\sin(dx+c)+\left(b^2\sin(3\,dx+3\,c)+9\,b^2\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)C\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Fricas [**A**]
time = 0.37, size = 54, normalized size = 0.68

$$\frac{\left(Cb^2\cos\left(dx+c\right)^2+\left(3\,A+2\,C\right)b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*b^2*cos(d*x + c)^2 + (3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2),
 x)
```

Mupad [**B**]
time = 0.56, size = 56, normalized size = 0.70

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)\right)}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + C*sin(3
*c + 3*d*x)))/(12*d*cos(c + d*x)^(1/2))
```

3.110 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(1
/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c +
 d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+d$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{\cos(}$$

Mathematica [A]
time = 0.12, size = 52, normalized size = 0.53

$$\frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*
Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.27, size = 54, normalized size = 0.55

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\cos(dx+c)^{\frac{5}{2}}}$	54
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}} + \frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/
cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.58, size = 59, normalized size = 0.60

$$\frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+(2\,(dx+c)b^2+b^2\sin\left(2\,dx+2\,c\right))C\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b^2
 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Fricas [A]
time = 0.40, size = 171, normalized size = 1.73

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,Cb^2\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+(2\,A+C)\sqrt{-b}\,b^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,Cb^2\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+(2\,A+C)b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A +
 C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*s
qrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b^2*sqr
t(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2),
 x)
```

Mupad [B]
time = 1.04, size = 48, normalized size = 0.48

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(C*sin(2*c + 2*d*x) + 4*A*d*x + 2*C*d*x))/(4*d*
cos(c + d*x)^(1/2))
```

3.111 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] A*b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*sin
(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3093, 3855}

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) +
 (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+dx}}$$

$$= \frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+}}{d\sqrt{\cos(c}}$$

Mathematica [A]
time = 0.09, size = 44, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{5/2}\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Cos[
c + d*x]^(5/2))
```

Maple [A]
time = 0.27, size = 55, normalized size = 0.74

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)(b\cos(dx+c))^{\frac{5}{2}}}{d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{ib^2\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,A\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*(b*cos(d*x+c))^
(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.61, size = 87, normalized size = 1.18

$$\frac{2\,Cb^{\frac{5}{2}}\sin(dx+c)+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)A\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(2*C*b^(5/2)*sin(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 +
2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x +
 c) + 1))*A*sqrt(b))/d
```

Fricas [**A**]
time = 0.41, size = 210, normalized size = 2.84

$$\left[\frac{Ab^{\frac{5}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}Cb^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)},-\frac{A\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}Cb^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), -(A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b^2*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x)
```

3.112 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*C*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3091, 8}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2C\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{5/2}(Cdx\cos(c+dx)+A\sin(c+dx))}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2
),x]
[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d
*x]^(7/2))
```

Maple [A]
time = 0.24, size = 45, normalized size = 0.69

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}(C\cos(dx+c)(dx+c)+A\sin(dx+c))}{d\cos(dx+c)^{\frac{7}{2}}}$	45
risch	$\frac{b^2Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}} + \frac{2ib^2\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
[Out] 1/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(7/
2)
```

Maxima [A]
time = 0.58, size = 80, normalized size = 1.23

$$\frac{2\left(Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + \frac{Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2*(C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(5/2)*sin(2*d*x
+ 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/
d
```

Fricas [A]
time = 0.44, size = 194, normalized size = 2.98

$$\left[\frac{C\sqrt{-b}\,b^2\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\,\sqrt{b\cos(dx+c)}\,Ab^2\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)^2}, \frac{Cb^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)^2+\sqrt{b\cos(dx+c)}\,Ab^2\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x +
 c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(5/2)*
arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(
d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2),
 x)
```

Mupad [B]
time = 1.19, size = 84, normalized size = 1.29

$$\frac{b^2 \sqrt{b \cos(c+dx)}\ (A \sin(2c+2dx) + C\,d\,x + C\,d\,x \cos(2c+2dx) + A\,1\mathrm{i} + A \cos(2c+2dx)\ 1\mathrm{i})}{d \sqrt{\cos(c+dx)}\ (\cos(2c+2dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d
*x) + C*d*x + C*d*x*cos(2*c + 2*d*x)))/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

3.113 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{b^2(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b^2*(A+2*C
)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3091, 3855}

$$\frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c
+ d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2
))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2(A+2C)\sqrt{b\cos(c+\right.}{2\sqrt{\cos(c}}$$

$$= \frac{b^2(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\,c}}{2}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.70

$$\frac{(b\cos(c+dx))^{5/2}\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A
*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.29, size = 134, normalized size = 1.60

method	result
default	$\frac{\left(-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx}{\sin(dx+c}\right.\right.}{2d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,A\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{b^2\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c}}{2\sqrt{\cos(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(72) = 144$.
time = 0.65, size = 821, normalized size = 9.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 1/4*(2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*
(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x +
4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*
c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*s
in(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d
*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^
2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c)
+ 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.42, size = 222, normalized size = 2.64

$$\left[\frac{(A+2C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}Ab^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4d\cos\left(dx+c\right)^{3}}, -\frac{(A+2C)\sqrt{-b}\,b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}Ab^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c)
```

```
)/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*c
os(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2), x)
```

3.114 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2(2A+3C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/3*b^2*(2*A+3
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3852, 8}

$$\frac{b^2(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*(
2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b^2(2A+3C)\sqrt{b\cos(c+\right.}{3\sqrt{\cos(c}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} - \frac{\left(b^2(2A+3C)\sqrt{b\cos(c+\right.}{3d}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2(2A+3C)\sqrt{b\cos(c+}{3d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A]
time = 0.26, size = 51, normalized size = 0.60

$$\frac{(b\cos(c+dx))^{5/2}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*C
os[c + d*x]^(7/2))
```

Maple [A]
time = 0.25, size = 54, normalized size = 0.64

method	result	size
default	$\frac{\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)\sin(dx+c)(b\cos(dx+c))^{\frac{5}{2}}}{3d\cos(dx+c)^{\frac{11}{2}}}$	54
risch	$\frac{2ib^2\sqrt{b\cos\left(dx+c\right)}\,\left(3C\,e^{4i(dx+c)}+6A\,e^{2i(dx+c)}+6C\,e^{2i(dx+c)}+2A+3C\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)
/cos(d*x+c)^(11/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(73) = 146$.
time = 0.64, size = 367, normalized size = 4.32

$$\frac{2\left(\frac{3\,Cb^{\frac{5}{2}}\sin\left(2\,dx+2\,c\right)}{\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1}-\frac{2\,\left(3\,b^{2}\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b^{2}\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b^{2}\cos\left(2\,dx+2\,c\right)+b^{2}\right)\sin\left(6\,dx+6\,c\right)-3\,\left(3\,b^{2}\cos\left(2\,dx+2\,c\right)+b^{2}\right)\sin\left(4\,dx+4\,c\right)\right)A\sqrt{b}}{2\,\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\,\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^{2}+9\,\cos\left(2\,dx+2\,c\right)^{2}+6\,\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^{2}+9\,\sin\left(4\,dx+4\,c\right)^{2}+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^{2}+6\,\cos\left(2\,dx+2\,c\right)+1}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 2/3*(3*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - 2*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*
b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(
6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)
/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*
d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x +
4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1))/d
```

Fricas [**A**]
time = 0.37, size = 54, normalized size = 0.64

$$\frac{\left((2\,A+3\,C)b^{2}\cos\left(dx+c\right)^{2}+Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*b^2*cos(d*x + c)^2 + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x +
 c)/(d*cos(d*x + c)^(7/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2)
, x)
```

Mupad [B]
time = 2.36, size = 220, normalized size = 2.59

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20\mathrm{i}+C\,30\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos
(4*c + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(
4*c + 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin
(4*c + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c
 + 4*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.115 $\int \frac{(b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{b^2(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c}{8d\cos^{\frac{5}{2}}(c}$$

```
[Out] 1/4*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b^2*(3*A+4
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/8*b^2*(3*A+4*C)*ar
ctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3091, 3853, 3855}

$$\frac{b^2(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2),x]
```

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[
c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9
/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \left(A+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b^2(3A+4C)\sqrt{b\cos(c+\right.}{4\sqrt{\cos(c}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A+4C)\sqrt{b\cos(c+}{8d\cos^{\frac{5}{2}}(c+}$$
$$= \frac{b^2(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b}}{}$$

Mathematica [A]
time = 0.19, size = 80, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left((3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\left(2A+(3A+4C)\cos^2(c+dx)\right)\sin(c+dx)\right)}{8d\cos^{\frac{13}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/
2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
 (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2)
)
```

Maple [A]
time = 0.29, size = 214, normalized size = 1.63

method	result
default	$\frac{\left(-3A\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A\left(\cos^4(dx+c)\right)\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c}{\sin(d}\right.\right.}{}$

risch	$-\frac{i b^{2} \sqrt{b \cos \left(d x+c\right)}\, \left(3 A \,{\mathrm e}^{7 i \left(d x+c\right)}+4 C \,{\mathrm e}^{7 i \left(d x+c\right)}+11 A \,{\mathrm e}^{5 i \left(d x+c\right)}+4 C \,{\mathrm e}^{5 i \left(d x+c\right)}-11 A \,{\mathrm e}^{3 i \left(d x+c\right)}-4 C \,{\mathrm e}^{3 i \left(d x+c\right)}-3 A \,{\mathrm e}^{i \left(d x+c\right)}-}{4 \sqrt{\cos \left(d x+c\right)}\, d \left({\mathrm e}^{2 i \left(d x+c\right)}+1\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. $2(113) = 226$.
time = 0.70, size = 2662, normalized size = 20.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="maxima")
```

```
[Out] -1/16*((12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x
 + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin
(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b
^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c)
 + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*
x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*si
n(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 +
48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^
2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*
c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x
 + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d
*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x
+ 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*
```

```
d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*
sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c
) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x +
8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*
cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*
cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4
*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*c
os(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x +
 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*
x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x +
4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c)
 + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 +
 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*
x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*s
in(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(
b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)
^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b
^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x +
 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x
 + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x +
 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x +
4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
```

```
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4...
```

Fricas [A]

time = 0.47, size = 270, normalized size = 2.06

$$\left[\frac{(3A+4C)b^{\frac{5}{2}}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)b^2\cos(dx+c)^2+2Ab^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16d\cos(dx+c)^5}, -\frac{(3A+4C)\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)b^2\cos(dx+c)^2+2Ab^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A +
4*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt
(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2)
, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos(c+dx)^2+A\right)\left(b\cos(c+dx)\right)^{5/2}}{\cos(c+dx)^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(5/2))/cos(c + d*x)^(15/2), x)
```

3.116 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(
d*x+c)^(7/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+c)^(
1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*
x]^(7/2)*Sin[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \text{co}}{4\sqrt{b\cos(c+dx)}}$$
$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+d}{4d\sqrt{b\cos(c+dx)}}$$
$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+a}{8d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 67, normalized size = 0.59

$$\frac{\sqrt{\cos(c+dx)}\,(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],
x]
[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.49, size = 88, normalized size = 0.78

method	result	size
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d\sqrt{b\cos\left(dx+c\right)}}$	88
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(8A+6C)x}{16\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(4dx+4c)}{32\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)(A+C)\sin(2dx+2c)}{4\sqrt{b\cos\left(dx+c\right)}\,d}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*x+
c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.62, size = 75, normalized size = 0.66

$$\frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{\sqrt{b}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/sqr
t(b))/d
```

Fricas [A]
time = 0.43, size = 207, normalized size = 1.83

$$\left[\frac{2\,(2\,C\cos(dx+c)^{2}+4\,A+3\,C)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-(4\,A+3\,C)\sqrt{-b}\,\log\left(2\,b\cos(dx+c)^{2}+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)}{16\,bd},\ \frac{(2\,C\cos(dx+c)^{2}+4\,A+3\,C)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+(4\,A+3\,C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{8\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(
(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(
sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B]
time = 1.93, size = 115, normalized size = 1.02

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\cos(c+d\,x)}\,\left(8\,A\sin(c+d\,x)+8\,C\sin(c+d\,x)+8\,A\sin(3\,c+3\,d\,x)+9\,C\sin(3\,c+3\,d\,x)+C\sin(5\,c+5\,d\,x)+32\,A\,d\,x\cos(c+d\,x)+24\,C\,d\,x\cos(c+d\,x)\right)}{32\,b\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b*d*(cos(2*c + 2*d*x) +
1))
```

3.117 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=74

$$\frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+c)^3
*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3092}

$$\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (C*Sqr
t[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\,(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqr
t[b*Cos[c + d*x]])
```

Maple [A]
time = 0.32, size = 47, normalized size = 0.64

method	result	size
default	$\frac{\left(C\left(\cos^2(dx+c)\right)+3A+2C\right)\left(\sqrt{\cos}\,(dx+c)\right)\sin(dx+c)}{3d\sqrt{b\cos\left(dx+c\right)}}$	47
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+3C)\sin(dx+c)}{4\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(3dx+3c)}{12\sqrt{b\cos\left(dx+c\right)}\,d}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)*sin(d*x+c)/(b*cos(d*x+c))^(
1/2)
```

Maxima [A]
time = 0.62, size = 57, normalized size = 0.77

$$\frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/sqrt(b) + 12*A*sin(d*x + c)/sqrt(b))/d
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.66

$$\frac{\left(C\cos\left(dx+c\right)^{2}+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*s
qrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B]
time = 0.95, size = 75, normalized size = 1.01

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.118 $\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=90

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {17, 2715, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2
*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Co
s[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(\right.}{2\sqrt{b\,c}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(}{2d\sqrt{b\cos(c+}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}\,(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.32, size = 54, normalized size = 0.60

method	result	size
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d\sqrt{b\cos\left(dx+c\right)}}$	54
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+2C)x}{4\sqrt{b\cos\left(dx+c\right)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(2dx+2c)}{4\sqrt{b\cos\left(dx+c\right)}\,d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.59, size = 52, normalized size = 0.58

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 169, normalized size = 1.88

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-(2\,A+C)\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,bd},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*
x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [A]
time = 23.83, size = 146, normalized size = 1.62

$$\begin{cases} \frac{Ax\sqrt{\cos\left(c+dx\right)}}{\sqrt{b\cos\left(c+dx\right)}}+\frac{Cx\sin^{2}\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{2\sqrt{b\cos\left(c+dx\right)}}+\frac{Cx\cos^{\frac{5}{2}}\left(c+dx\right)}{2\sqrt{b\cos\left(c+dx\right)}}+\frac{C\sin\left(c+dx\right)\cos^{\frac{3}{2}}\left(c+dx\right)}{2d\sqrt{b\cos\left(c+dx\right)}} & \text{for}\ d\neq 0 \\ \frac{x\left(A+C\cos^{2}\left(c\right)\right)\sqrt{\cos\left(c\right)}}{\sqrt{b\cos\left(c\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + C*x*sin(c + d*x)**
2*sqrt(cos(c + d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(2*
sqrt(b*cos(c + d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(
c + d*x))), Ne(d, 0)), (x*(A + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c)), Tr
ue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x
)
```

Mupad [B]
time = 1.49, size = 81, normalized size = 0.90

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(C\,\sin(c+d\,x)+C\,\sin(3\,c+3\,d\,x)+8\,A\,d\,x\,\cos(c+d\,x)+4\,C\,d\,x\,\cos(c+d\,x)\right)}{4\,b\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*
x) + 1))
```

3.119 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*
cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3093, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,d$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+d}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.65

$$\frac{\sqrt{\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[b*C
os[c + d*x]])
```

Maple [A]
time = 0.32, size = 55, normalized size = 0.81

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)\left(\sqrt{\cos}\,(dx+c)\right)}{d\sqrt{b\cos\left(dx+c\right)}}$
risch	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{C\sin(2dx+2c)}{2d\sqrt{\cos\left(dx+c\right)}\sqrt{b\cos\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(1/2
)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.60, size = 80, normalized size = 1.18

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{2\,C\sin(dx+c)}{\sqrt{b}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x +
 c)/sqrt(b))/d
```

Fricas [A]
time = 0.44, size = 207, normalized size = 3.04

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos\left(dx+c\right)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)},\ -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
 + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+C\cos^{2}\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.120 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {18, 3091, 8}

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*Sin[c + d*x])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.76

$$\frac{Cdx\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] (C*d*x*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c +
d*x]])
```

Maple [A]
time = 0.29, size = 45, normalized size = 0.76

method	result	size
default	$\frac{C\cos(dx+c)(dx+c)+A\sin(dx+c)}{d\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}}$	45
risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{\sqrt{b\cos(dx+c)}} + \frac{i\mathrm{e}^{-i(dx+c)}A}{\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/
2)
```

Maxima [A]
time = 0.57, size = 85, normalized size = 1.44

$$\frac{2\left(\frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + A*sqrt(b)*sin(2*d*x
+ 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c)
+ b))/d
```

Fricas [A]
time = 0.43, size = 191, normalized size = 3.24

$$\left[-\frac{C\sqrt{-b}\cos\left(dx+c\right)^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)^{2}}, \frac{C\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^{2}+\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2), (C*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x +
 c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^{2}\left(c + dx\right)}{\sqrt{b\cos\left(c + dx\right)}\,\cos^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)),
 x)
```

Mupad [B]
time = 1.25, size = 84, normalized size = 1.42

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x
) + 1))
```

3.121 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctan
h(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3091, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*
x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int \sec(}{2\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+d}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\,c}}$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 0.76

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Cos[
c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.
time = 0.32, size = 134, normalized size = 1.72

method	result
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d} - \frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}(dx+c\right.}{2\sqrt{b\,c}}$
default	$\frac{-A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A(\cos^2(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{-1+\cos(dx}{\sin(dx+c}\right.}{2d\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(66) = 132$.
time = 0.62, size = 728, normalized size = 9.33

$$\frac{\frac{2\,C\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)}{\sqrt{b}}-\frac{\left(4\left(\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)\right)\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)-4\left(\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)\right)\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)-\left(2\left(2\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+\cos\left(4\,dx+4\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)^2+\sin\left(4\,dx+4\,c\right)^2+4\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+4\,\sin\left(2\,dx+2\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)+1\right)\log\left(\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)^2+\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)^2+2\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)+1\right)+\left(2\left(2\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+\cos\left(4\,dx+4\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)^2+\sin\left(4\,dx+4\,c\right)^2+4\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+4\,\sin\left(2\,dx+2\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)+1\right)\log\left(\cos\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)^2+\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)^2-2\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)+1\right)-4\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)+4\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)\right)A}{\left(2\left(2\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+\cos\left(4\,dx+4\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)^2+\sin\left(4\,dx+4\,c\right)^2+4\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+4\,\sin\left(2\,dx+2\,c\right)^2+4\,\cos\left(2\,dx+2\,c\right)+1\right)\sqrt{b}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [A]
time = 0.42, size = 219, normalized size = 2.81

$$\left[\frac{\left(A+2\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^3\log\left(-\frac{b\cos\left(dx+c\right)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\,bd\cos\left(dx+c\right)^3},\ -\frac{\left(A+2\,C\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^3-\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sq
```

```
rt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x
 + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.122 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=79

$$\frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*
C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int \text{sec}}{3\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\text{Subs}}{3d\sqrt{b\cos(c-}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 51, normalized size = 0.65

$$\frac{\sin(c+dx)\,(3(A+C)+A\tan^2(c+dx))}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),
x]
[Out] (Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]])
```

Maple [A]
time = 0.30, size = 54, normalized size = 0.68

method	result	size
default	$\frac{\sin(dx+c)(2A(\cos^2(dx+c))+3C(\cos^2(dx+c))+A)}{3d\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{5}{2}}}$	54
risch	$\frac{i(3C\,e^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c))}{3\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)^2d}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(1/2)
/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(67) = 134.
time = 0.62, size = 355, normalized size = 4.49

$$\frac{2\left(\frac{3\,C\sqrt{b}\,\sin\left(2\,dx+2\,c\right)}{b\cos\left(2\,dx+2\,c\right)^{2}+b\sin\left(2\,dx+2\,c\right)^{2}+2\,b\cos\left(2\,dx+2\,c\right)+b}+\frac{2\left(\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(6\,dx+6\,c\right)+3\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(4\,dx+4\,c\right)-3\,\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)-9\,\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)\right)A}{\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^{2}+9\,\cos\left(2\,dx+2\,c\right)^{2}+6\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^{2}+9\,\sin\left(4\,dx+4\,c\right)^{2}+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^{2}+6\,\cos\left(2\,dx+2\,c\right)+1\right)\sqrt{b}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c
)^2 + 2*b*cos(2*d*x + 2*c) + b) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6
*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(
2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*
c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6
*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [A]
time = 0.36, size = 50, normalized size = 0.63

$$\frac{\left((2\,A+3\,C)\cos\left(dx+c\right)^{2}+A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,bd\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d
*cos(d*x + c)^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)),
 x)
```

Mupad [B]
time = 2.83, size = 220, normalized size = 2.78

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20\mathrm{i}+C\,30\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3\mathrm{i}\right)}{3\,b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*
x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.123 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*
x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (
(3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int \sec$$
$$= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$
$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b}$$

Mathematica [A]
time = 0.19, size = 80, normalized size = 0.66

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),
x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.
time = 0.33, size = 214, normalized size = 1.75

method	result
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d} + \frac{\left(\sqrt{\cos}(dx+c)\right)(3A+4}{8\sqrt{b\cos(dx}$
default	$\frac{-3A(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A(\cos^4(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+}{\sin(}\right.}{8d\sqrt{b\cos(dx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(104) = 208$.
time = 0.67, size = 2318, normalized size = 19.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
```

```
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*sqrt(b)) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(
cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [A]
time = 0.41, size = 261, normalized size = 2.14

$$\left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16bd\cos(dx+c)^5}, -\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8bd\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*s
qrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x +
 c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\cos\left(c+d\,x\right)}^{9/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.124 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*co
s(d*x+c)^(7/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c
+ d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int\cos^2}{4b\sqrt{b\cos(c+dx)}}$$
$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{4bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{8bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 67, normalized size = 0.55

$$\frac{\cos^{\frac{3}{2}}(c+dx)(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.45, size = 88, normalized size = 0.72

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d(b\cos(dx+c))^{\frac{3}{2}}}$	88
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(8A+6C)x}{16b\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(4dx+4c)}{32b\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)(A+C)\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*cos(d*x+c)^(3/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*x+
c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.65, size = 75, normalized size = 0.61

$$\frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{b^{\frac{3}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(
3/2))/d
```

Fricas [A]
time = 0.42, size = 207, normalized size = 1.70

$$\left[\frac{2\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(4\,A+3\,C\right)\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{16\,b^{2}d},\ \frac{\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{8\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8
*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [B]
time = 1.96, size = 115, normalized size = 0.94

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c+3\,d\,x\right)+C\,\sin\left(5\,c+5\,d\,x\right)+32\,A\,d\,x\,\cos\left(c+d\,x\right)+24\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^2*d*(cos(2*c + 2*d*x)
+ 1))
```

3.125 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+c)
^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3092}

$$\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (C*S
qrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}\\
&= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int\left(A+C-Cx^2\right)\,dx,x,-\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\
&= \frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.65

$$\frac{\cos^{\frac{3}{2}}(c+dx)(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*
Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.28, size = 47, normalized size = 0.59

method	result	size
default	$\frac{\left(C\left(\cos^{2}(dx+c)\right)+3A+2C\right)\sin(dx+c)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{3d(b\cos(dx+c))^{\frac{3}{2}}}$	47
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+3C)\sin(dx+c)}{4b\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(3dx+3c)}{12b\sqrt{b\cos\left(dx+c\right)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2)
```

Maxima [A]
time = 0.62, size = 57, normalized size = 0.71

$$\frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{3}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/b^(3/2) + 12*A*sin(d*x + c)/b^(3/2))/d
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.61

$$\frac{\left(C\cos\left(dx+c\right)^{2}+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,b^{2}d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d
*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [**B**]
time = 0.84, size = 75, normalized size = 0.94

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.126 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(
1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2715, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]]
)/(2*b*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*S
qrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c\cdots\right.}{2b\sqrt{b\cos\cdots}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c\cdots}{2bd\sqrt{b\cos(c+d\cdots}}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.53

$$\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*(b*C
os[c + d*x])^(3/2))
```

Maple [A]
time = 0.28, size = 54, normalized size = 0.55

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d(b\cos(dx+c))^{\frac{3}{2}}}$	54
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+2C)x}{4b\sqrt{b\cos\left(dx+c\right)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(2dx+2c)}{4b\sqrt{b\cos\left(dx+c\right)}\,d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*c
os(d*x+c))^(3/2)
```

Maxima [A]
time = 0.58, size = 52, normalized size = 0.53

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(3/2))/d
```

Fricas [A]
time = 0.50, size = 169, normalized size = 1.71

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(2\,A+C\right)\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,b^{2}d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(2\,A+C\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(
d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [B]
time = 0.70, size = 81, normalized size = 0.82

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\left(c+d\,x\right)+4\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*
d*x) + 1))
```

3.127 $\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c
)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3093, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (
C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int(A+C\cos^2(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec(c+}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\,\sin(}{bd\sqrt{b\cos(c+dx}}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*(b*Cos[c
 + d*x])^(3/2))
```

Maple [A]
time = 0.29, size = 55, normalized size = 0.74

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$
risch	$-\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{i(dx+c)}}{2b\sqrt{b\cos(dx+c)}\,d}+\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{-i(dx+c)}}{2b\sqrt{b\cos(dx+c)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln(\mathrm{e}^{i(dx+c)}-i)}{b\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)}{b\sqrt{b\cos(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(3/2
)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.60, size = 80, normalized size = 1.08

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{2\,C\sin(dx+c)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x +
 c)/b^(3/2))/d
```

Fricas [**A**]
time = 0.43, size = 207, normalized size = 2.80

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2b^2d\cos\left(dx+c\right)}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^2d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(3/2), x)
```

3.128 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)
/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = $\{18, 3091, 8\}$

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*S
qrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\,(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*
x])^(3/2))
```

Maple [A]
time = 0.27, size = 45, normalized size = 0.69

method	result	size
default	$\frac{(C\cos(dx+c)(dx+c)+A\sin(dx+c))\left(\sqrt{\cos}\,(dx+c)\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$	45
risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{b\sqrt{b\cos\left(dx+c\right)}} + \frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\,d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/
2)
```

Maxima [A]
time = 0.57, size = 93, normalized size = 1.43

$$\frac{2\left(\frac{A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2} + \frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c
)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + C*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/b^(3/2))/d
```

Fricas [**A**]
time = 0.45, size = 191, normalized size = 2.94

$$\left[-\frac{C \sqrt{-b} \cos\left(dx+c\right)^{2} \log\left(2 \, b \cos\left(dx+c\right)^{2} + 2 \, \sqrt{b \cos\left(dx+c\right)} \sqrt{-b} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) - b\right) - 2 \, \sqrt{b \cos\left(dx+c\right)} A \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{2 \, b^{2} d \cos\left(dx+c\right)^{2}}, \frac{C \sqrt{b} \arctan\left(\frac{\sqrt{b \cos\left(dx+c\right)} \sin\left(dx+c\right)}{\sqrt{b} \cos\left(dx+c\right)^{\frac{3}{2}}}\right) \cos\left(dx+c\right)^{2} + \sqrt{b \cos\left(dx+c\right)} A \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{b^{2} d \cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2), (C*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C \cos^{2}\left(c + dx\right)}{\left(b \cos\left(c + dx\right)\right)^{\frac{3}{2}} \sqrt{\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)
)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))
), x)
```

Mupad [B]
time = 1.24, size = 84, normalized size = 1.29

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(A\,\sin\left(2\,c+2\,d\,x\right)+C\,d\,x+C\,d\,x\,\cos\left(2\,c+2\,d\,x\right)+A\,1\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*1i + A*cos(2*c + 2*d*x)*1i + A*sin(2*c + 2*d*x)
+ C*d*x + C*d*x*cos(2*c + 2*d*x)))/(b^2*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d
*x) + 1))
```

3.129 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arct
anh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3091, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A + C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)\int \text{se}}{2b\sqrt{b\cos(c+dx)}}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.70

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx) + A\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Sqrt
[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.28, size = 134, normalized size = 1.60

method	result
default	$\frac{-A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A(\cos^2(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^2(dx+c))\,\text{arctanh}\left(\frac{-1+\cos(dx}{\sin(dx+}\right.}{2d(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d} - \frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln(e^{i(dx+c)}-i)}{2b\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}(dx+\right.}{2b\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(72) = 144$.
time = 0.63, size = 736, normalized size = 8.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2
*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*
sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*sqrt(b)) - 2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2
*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/b^(3/2))/d
```

Fricas [A]
time = 0.41, size = 219, normalized size = 2.61

$$\left[\frac{(A+2C)\sqrt{b}\,\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\,A\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4b^2d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}\,A\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2b^2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*
```

```
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.130 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A +
3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int s}{3b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\mathrm{Su}}{3bd\sqrt{b\cos(}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+d}}$$

Mathematica [A]
time = 0.15, size = 51, normalized size = 0.60

$$\frac{\sqrt{\cos(c+dx)}\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Co
s[c + d*x])^(3/2))
```

Maple [A]
time = 0.26, size = 54, normalized size = 0.64

method	result	size
default	$\frac{\sin(dx+c)\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)}{3d(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}$	54
risch	$\frac{i\left(3C\,e^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c)\right)}{3b\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\left(e^{2i(dx+c)}+1\right)^2 d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(3/2)
/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(73) = 146$.
time = 0.62, size = 380, normalized size = 4.47

$$\frac{2\left(\frac{3C\sqrt{b}\sin(2dx+2c)}{b^2\cos(2dx+2c)^2+b^2\sin(2dx+2c)^2+2b^2\cos(2dx+2c)+b^2}+\frac{2((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c))A}{\left(b\cos(6dx+6c)^2+9b\cos(4dx+4c)^2+9b\cos(2dx+2c)^2+b\sin(6dx+6c)^2+9b\sin(4dx+4c)^2+18b\sin(4dx+4c)\sin(2dx+2c)+9b\sin(2dx+2c)^2+2(3b\cos(4dx+4c)+3b\cos(2dx+2c)+b)\cos(6dx+6c)+6(3b\cos(2dx+2c)+b)\cos(4dx+4c)+6b\cos(2dx+2c)+6(b\sin(4dx+4c)+b\sin(2dx+2c))\sin(6dx+6c)+b\right)\sqrt{b}}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6
*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x
 + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x +
 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) +
b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*c
os(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + b)*sqrt(b)))/d
```

Fricas [A]
time = 0.37, size = 50, normalized size = 0.59

$$\frac{\left((2A+3C)\cos(dx+c)^2+A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{3b^2d\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)
), x)
```

Mupad [B]
time = 2.42, size = 220, normalized size = 2.59

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20\mathrm{i}+C\,30\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3\mathrm{i}\right)}{3\,b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^2*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.131 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+d}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(}$$

```
[Out] 1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(si
n(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c
 + d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
 + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int s}{4b\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx}}$$
$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c-}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{}}$$

Mathematica [A]
time = 0.13, size = 80, normalized size = 0.61

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.28, size = 214, normalized size = 1.63

method	result
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d} - \frac{\left(\sqrt{\cos}\,(dx+c)\right)(3A+4}{8b\sqrt{b\cos(dx}}$

default	$\frac{-3A(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A(\cos^4(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx}\right.}{8d(b\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2350 vs. $2(113) = 226$.
time = 0.65, size = 2350, normalized size = 17.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
```

```
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x
+ 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x
+ 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c)
 + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(
4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x +
 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) + 4*(4*(sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
+ 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2
*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d
*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*C/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*
d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)
^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) +
 b)*sqrt(b)))/d
```

Fricas [**A**]

time = 0.44, size = 261, normalized size = 1.99

$$\left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16b^2d\cos(dx+c)^5}, -\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8b^2d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^{7/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.132 $\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*
cos(d*x+c)^(7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(
d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3093, 2715, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3
*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Co
s[c + d*x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right)\int \cos^2$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{4b^2d\sqrt{b\cos(c+dx)}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{8b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.16, size = 70, normalized size = 0.57

$$\frac{\sqrt{\cos(c+dx)}\,(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] +
 C*Sin[4*(c + d*x)]))/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.45, size = 88, normalized size = 0.72

method	result	size
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(2C\left(\cos^3(dx+c)\right)\sin(dx+c)+4A\sin(dx+c)\cos(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)\right)}{8d(b\cos(dx+c))^{\frac{5}{2}}}$	88
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(8A+6C)x}{16b^2\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(4dx+4c)}{32b^2\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)(A+C)\sin(2dx+2c)}{4b^2\sqrt{b\cos(dx+c)}\,d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*cos(d*x+c)^(5/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*sin(d*x+c)*cos(d*x+
c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.60, size = 75, normalized size = 0.61

$$\frac{\frac{8\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{b^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + (12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(
5/2))/d
```

Fricas [A]
time = 0.41, size = 207, normalized size = 1.70

$$\left[\frac{2\,\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(4\,A+3\,C\right)\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{16\,b^{3}d},\ \frac{\left(2\,C\cos\left(dx+c\right)^{2}+4\,A+3\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(4\,A+3\,C\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{8\,b^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8
*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B]
time = 1.97, size = 115, normalized size = 0.94

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,A\,\sin\left(c+d\,x\right)+8\,C\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(3\,c+3\,d\,x\right)+9\,C\,\sin\left(3\,c+3\,d\,x\right)+C\,\sin\left(5\,c+5\,d\,x\right)+32\,A\,d\,x\,\cos\left(c+d\,x\right)+24\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(9/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*A*sin(c + d*x) + 8*C*sin(c +
d*x) + 8*A*sin(3*c + 3*d*x) + 9*C*sin(3*c + 3*d*x) + C*sin(5*c + 5*d*x) + 3
2*A*d*x*cos(c + d*x) + 24*C*d*x*cos(c + d*x)))/(32*b^3*d*(cos(2*c + 2*d*x)
+ 1))
```

3.133 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A+C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-1/3*C*sin(d*x+
c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3092}

$$\frac{(A+C)\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}-\frac{C\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (C
*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int \cos(c+dx)\,(A+C\cos^2(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int (A+C-Cx^2)\,dx,x,-\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}\\ &= \frac{(A+C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{C\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\,(6A+5C+C\cos(2(c+dx)))\sin(c+dx)}{6b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d
*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.28, size = 47, normalized size = 0.59

method	result	size
default	$\frac{\left(C\left(\cos^2(dx+c)\right)+3A+2C\right)\sin(dx+c)\left(\cos^{\frac{5}{2}}(dx+c)\right)}{3d(b\cos(dx+c))^{\frac{5}{2}}}$	47
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+3C)\sin(dx+c)}{4b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(3dx+3c)}{12b^2\sqrt{b\cos(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2)
```

Maxima [A]
time = 0.62, size = 57, normalized size = 0.71

$$\frac{\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))))/b^(5/2) + 12*A*sin(d*x + c)/b^(5/2))/d
```

Fricas [A]
time = 0.39, size = 49, normalized size = 0.61

$$\frac{\left(C\cos\left(dx+c\right)^2+3\,A+2\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,b^3d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d
*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [**B**]
time = 0.84, size = 75, normalized size = 0.94

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(2\,c+2\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(12*A*sin(2*c + 2*d*x) + 10*C*si
n(2*c + 2*d*x) + C*sin(4*c + 4*d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.134 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)
^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = $\{17, 2715, 8\}$

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x
]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c\right.}}{2b^2\sqrt{b\,c}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(}{2b^2d\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\,(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.29, size = 54, normalized size = 0.55

method	result	size
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c))}{2d(b\cos(dx+c))^{\frac{5}{2}}}$	54
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+2C)x}{4b^2\sqrt{b\cos\left(dx+c\right)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(2dx+2c)}{4b^2\sqrt{b\cos\left(dx+c\right)}\,d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*c
os(d*x+c))^(5/2)
```

Maxima [A]
time = 0.60, size = 52, normalized size = 0.53

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(5/2))/d
```

Fricas [A]
time = 0.46, size = 169, normalized size = 1.71

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left(2\,A+C\right)\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,b^{3}d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(2\,A+C\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,b^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(
d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B]
time = 0.72, size = 81, normalized size = 0.82

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(C\,\sin(c+d\,x)+C\,\sin(3\,c+3\,d\,x)+8\,A\,d\,x\,\cos(c+d\,x)+4\,C\,d\,x\,\cos(c+d\,x)\right)}{4\,b^3\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + C*sin(3*c + 3*
d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*
d*x) + 1))
```

3.135 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x
+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3093, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 0.64

$$\frac{\sqrt{\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+C\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(b^2*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [A]
time = 0.29, size = 55, normalized size = 0.74

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-C\sin(dx+c)\right)\left(\cos^{\frac{5}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$-\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{i(dx+c)}}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{-i(dx+c)}}{2b^2\sqrt{b\cos(dx+c)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln(\mathrm{e}^{i(dx+c)}-i)}{b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)}{b^2\sqrt{b\cos(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-C*sin(d*x+c))*cos(d*x+c)^(5/2
)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.60, size = 80, normalized size = 1.08

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{2\,C\sin(dx+c)}{b^{\frac{5}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x +
 c)/b^(5/2))/d
```

Fricas [A]
time = 0.43, size = 207, normalized size = 2.80

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2b^{3}d\cos\left(dx+c\right)}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{3}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^{3/2}\left(C\cos(c+dx)^2+A\right)}{\left(b\cos(c+dx)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2), x)
```

3.136 $\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/
2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3091, 8}

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2
*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+C\cos^2(c+dx))\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}} + \frac{\left(C\sqrt{\cos(c+dx)}\right)\int 1\,}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.69

$$\frac{\cos^{\frac{3}{2}}(c+dx)(Cdx\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*
x])^(5/2))
```

Maple [A]
time = 0.27, size = 45, normalized size = 0.69

method	result	size
default	$\frac{(C\cos(dx+c)(dx+c)+A\sin(dx+c))\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$	45
risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{b^2\sqrt{b\cos(dx+c)}} + \frac{2i\left(\sqrt{\cos}\,(dx+c)\right)A}{b^2\sqrt{b\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/
2)
```

Maxima [A]
time = 0.58, size = 93, normalized size = 1.43

$$\frac{2\left(\frac{A\sqrt{b}\,\sin(2\,dx+2\,c)}{b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3} + \frac{C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c
)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + C*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/b^(5/2))/d
```

Fricas [**A**]
time = 0.40, size = 191, normalized size = 2.94

$$\left[-\frac{C\sqrt{-b}\cos\left(dx+c\right)^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{3}d\cos\left(dx+c\right)^{2}},\ \frac{C\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^{2}+\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{3}d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c)
)*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2), (C*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
 + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2),
 x)
```

Mupad [B]
time = 1.90, size = 117, normalized size = 1.80

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\left(A\sin(c+dx)+A\sin(3c+3dx)+C\,d\,x\cos(3c+3dx)+3\,C\,d\,x\cos(c+dx)+A\cos(c+dx)\,3\mathrm{i}+A\cos(3c+3dx)\,1\mathrm{i}\right)}{b^3\,d\,\left(4\cos(2c+2dx)+\cos(4c+4dx)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A*cos(c + d*x)*3i + A*sin(c +
 d*x) + A*cos(3*c + 3*d*x)*1i + A*sin(3*c + 3*d*x) + C*d*x*cos(3*c + 3*d*x)
 + 3*C*d*x*cos(c + d*x)))/(b^3*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3
))
```

3.137 $\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*ar
ctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3091, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((A+2C)\sqrt{\cos(c+dx)}\right)}{2b^2\sqrt{b\cos(c+d}}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c}{2b^2d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+A\sin(c+dx)\right)}{2d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin
[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.31, size = 135, normalized size = 1.61

method	result
default	$-\frac{\left(A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A(\cos^2(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+4C(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{-1+\cos(}{\sin(dx}\right.\right.}{2d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$-\frac{iA(e^{2i(dx+c)}-1)}{2b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d}-\frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln(e^{i(dx+c)}-i)}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+\right.}{2b^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1
+cos(d*x+c))/sin(d*x+c))-A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(72) = 144$.

time = 0.63, size = 754, normalized size = 8.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x
 + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(
2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)) - 2*C*(log(cos(d*x + c)^2 +
sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2
- 2*sin(d*x + c) + 1))/b^(5/2))/d
```

Fricas [A]

time = 0.41, size = 219, normalized size = 2.61

$$\left[\frac{(A+2C)\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4b^{3}d\cos\left(dx+c\right)^{3}}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2b^{3}d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b
^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*
```

```
sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d
*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A
+ 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\int}{3b^2\sqrt{b\cos(c+dx}}$$
$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} - \frac{\left((2A+3C)\sqrt{\cos(c+dx)}\right)\mathrm{S}}{3b^2d\sqrt{b\cos}}$$
$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.14, size = 51, normalized size = 0.60

$$\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)\left(3(A+C)+A\tan^2(c+dx)\right)}{3d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Co
s[c + d*x])^(5/2))
```

Maple [A]
time = 0.26, size = 54, normalized size = 0.64

method	result	size
default	$\frac{\sin(dx+c)\left(2A\left(\cos^2(dx+c)\right)+3C\left(\cos^2(dx+c)\right)+A\right)}{3d(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}$	54
risch	$\frac{i\left(3C\,\mathrm{e}^{3i(dx+c)}+(9C+8A)\cos(dx+c)+i(4A+3C)\sin(dx+c)\right)}{3b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(\mathrm{e}^{2i(dx+c)}+1\right)^2 d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(5/2)
/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(73) = 146$.
time = 0.62, size = 412, normalized size = 4.85

$$\frac{2\left(\frac{3\,C\sqrt{b}\sin\left(2\,dx+2\,c\right)}{b^3\cos\left(2\,dx+2\,c\right)^2+b^3\sin\left(2\,dx+2\,c\right)^2+2\,b^3\cos\left(2\,dx+2\,c\right)+b^3}+\frac{2\left(\left(3\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(6\,dx+6\,c\right)+3\left(3\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(4\,dx+4\,c\right)-3\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)-9\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)\right)A}{\left(b^2\cos\left(6\,dx+6\,c\right)^2+9\,b^2\cos\left(4\,dx+4\,c\right)^2+9\,b^2\cos\left(2\,dx+2\,c\right)^2+b^2\sin\left(6\,dx+6\,c\right)^2+9\,b^2\sin\left(4\,dx+4\,c\right)^2+18\,b^2\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b^2\sin\left(2\,dx+2\,c\right)^2+6\,b^2\cos\left(2\,dx+2\,c\right)+b^2+2\left(3\,b^2\cos\left(4\,dx+4\,c\right)+3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\cos\left(6\,dx+6\,c\right)+6\left(3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\cos\left(4\,dx+4\,c\right)+6\left(b^2\sin\left(4\,dx+4\,c\right)+b^2\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)\right)\sqrt{b}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6
*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d
*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin
(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3
*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*
(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) +
 b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)))/d
```

Fricas [A]
time = 0.40, size = 50, normalized size = 0.59

$$\frac{\left(\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,b^3 d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)
), x)
```

Mupad [B]
time = 2.49, size = 220, normalized size = 2.59

$$\frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(18\,A\,\sin\left(2\,c+2\,d\,x\right)+12\,A\,\sin\left(4\,c+4\,d\,x\right)+2\,A\,\sin\left(6\,c+6\,d\,x\right)+15\,C\,\sin\left(2\,c+2\,d\,x\right)+12\,C\,\sin\left(4\,c+4\,d\,x\right)+3\,C\,\sin\left(6\,c+6\,d\,x\right)+A\,20{}\mathrm{i}+C\,30{}\mathrm{i}+A\,\cos\left(2\,c+2\,d\,x\right)\,30{}\mathrm{i}+A\,\cos\left(4\,c+4\,d\,x\right)\,12{}\mathrm{i}+A\,\cos\left(6\,c+6\,d\,x\right)\,2{}\mathrm{i}+C\,\cos\left(2\,c+2\,d\,x\right)\,45{}\mathrm{i}+C\,\cos\left(4\,c+4\,d\,x\right)\,18{}\mathrm{i}+C\,\cos\left(6\,c+6\,d\,x\right)\,3{}\mathrm{i}\right)}{3\,b^3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(A*20i + C*30i + A*cos(2*c + 2*d*x)*30i + A*cos(4*c
 + 4*d*x)*12i + A*cos(6*c + 6*d*x)*2i + C*cos(2*c + 2*d*x)*45i + C*cos(4*c
+ 4*d*x)*18i + C*cos(6*c + 6*d*x)*3i + 18*A*sin(2*c + 2*d*x) + 12*A*sin(4*c
 + 4*d*x) + 2*A*sin(6*c + 6*d*x) + 15*C*sin(2*c + 2*d*x) + 12*C*sin(4*c + 4
*d*x) + 3*C*sin(6*c + 6*d*x)))/(3*b^3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*
d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.139 $\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=131

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctan
h(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3091, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c
+ d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+C\cos^2(c+dx))\sec^5(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left((3A+4C)\sqrt{\cos(c+dx)}\right)\int}{4b^2\sqrt{b\cos(c+dx}}$$
$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx}}$$
$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c}{4b^2d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A]
time = 0.17, size = 80, normalized size = 0.61

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2A+(3A+4C)\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)
),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[
c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.29, size = 214, normalized size = 1.63

method	result
risch	$-\frac{i(3A\,e^{6i(dx+c)}+4C\,e^{6i(dx+c)}+11A\,e^{4i(dx+c)}+4C\,e^{4i(dx+c)}-11A\,e^{2i(dx+c)}-4C\,e^{2i(dx+c)}-3A-4C)}{8b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d} - \frac{\left(\sqrt{\cos}\,(dx+c)\right)(3A+4}{8b^2\sqrt{b\cos(d}}$

default	$\frac{-3A(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3A(\cos^4(dx+c))\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-4C(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)}{\sin(dx}\right.}{8d(b\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos
(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1
+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d
*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^2*sin(d*x+c)+4*C*sin(d*x+c)*cos(d*x+c)^2+
2*A*sin(d*x+c))/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(113) = 226$.
time = 0.65, size = 2418, normalized size = 18.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44
*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*
x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
```

```
 + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(
4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^
2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2
 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x
 + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((b^2*co
s(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(
2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqr
t(b)))/d
```

Fricas [A]
time = 0.45, size = 261, normalized size = 1.99

$$\left[\frac{(3A+4C)\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{16\,b^3d\cos(dx+c)^5},\ -\frac{(3A+4C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\left((3A+4C)\cos(dx+c)^2+2A\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{8\,b^3d\cos(dx+c)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b
*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/
cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)
*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
 + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.140 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,d$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^3/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^3d}-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{130b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^3*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{7/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d} + \frac{(13A+10C)}{}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^3d} - \frac{3(13A+10C}{}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 1.01

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(8A\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+5C\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(8*A*Cos[c + d*x]^2*Hypergeometric2
F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^4*Hypergeometric2F1[1/
2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*d)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.141 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}-\frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{70b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^2/d-3/70*(10*A+7*C)*(b*cos(d*x+c))
^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$,
Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10b^2d}-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{70b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^2*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx &= \frac{\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\
&= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}+\frac{(10A+7C)\int}{}\\
&= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^2d}-\frac{3(10A+7C)}{}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 91, normalized size = 0.96

$$
-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{91bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*b*d)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$
\int \cos\left(dx+c\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.142 $\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{28bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/
3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7bd}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos[c
 + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(28*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{1}{7}(7A+4C)\int \sqrt[3]{b\cos(c+$$
$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3(7A+4C)(b\cos(c+dx)}{28b}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.93

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.143 $\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=87

$$\frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d}-\frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)*hy
pergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d}-\frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*(4*A + C)*(b*Cos[c + d
*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}(b(4A+C))$$
$$= \frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d} - \frac{3(4A+C)\sqrt[3]{b}}{}$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 1.01

$$-\frac{3b\cot(c+dx)\left(7A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x), x)
```

3.144 $\int \sqrt[3]{b \cos(c+dx)} \, (A + C\cos^2(c+dx)) \sec^2(c+dx) \, dx$

Optimal. Leaf size=91

$$\frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)*
hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}} + \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{1}{2}(-A+2C)\int \sqrt[3]{b\cos(c\cdots}$$
$$= \frac{3Ab\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{\cdots}}{8b\cdots}$$

Mathematica [A]
time = 0.18, size = 88, normalized size = 0.97

$$-\frac{3b\csc(c+dx)\left(-2A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
[Out] (-3*b*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^2, x)
```

3.145 $\int \sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=92

$$\frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}-\frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(1
/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}-\frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d
*x])/(5*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{1}{5}(b(2A+5C))\int \frac{}{(b\cos(}$$
$$= \frac{3Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+}}{5}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 1.04

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\csc(c+dx)\left(-A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2, 1
/6, Cos[c + d*x]^2]) + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6,
Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3))/cos(c + d*x)^3, x)
```

3.146 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d}-\frac{3(14A+11C)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\sin(c+dx)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

[Out] 3/14*C*(b*cos(d*x+c))^(11/3)*sin(d*x+c)/b^3/d-3/154*(14*A+11*C)*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)

Rubi [A]

time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{11/3}}{14b^3d}-\frac{3(14A+11C)\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{154b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]

[Out] (3*C*(b*Cos[c + d*x])^(11/3)*Sin[c + d*x])/(14*b^3*d) - (3*(14*A + 11*C)*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(154*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{8/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d} + \frac{(14A+11C}{}$$
$$= \frac{3C(b\cos(c+dx))^{11/3}\sin(c+dx)}{14b^3d} - \frac{3(14A+11}{}$$

Mathematica [A]
time = 0.12, size = 96, normalized size = 1.01

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(17A\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)+11C\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{17}{6};\frac{23}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{187d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(17*A*Cos[c + d*x]^2*Hypergeometric
2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2] + 11*C*Cos[c + d*x]^4*Hypergeometric2F
1[1/2, 17/6, 23/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(187*d)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.147 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^2d}-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{5/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d} + \frac{(11A+8C)}{}$$
$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d} - \frac{3(11A+8C}{}$$

Mathematica [A]
time = 0.18, size = 91, normalized size = 0.96

$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)+4C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{3};\frac{10}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{56bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/3, 10/3, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*b*d)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(b\cos(dx+c)\right)^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.148 $\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5/
3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c
 + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(40*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{1}{8}(8A+5C)\int (b\cos(c+$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3(8A+5C)(b\cos(c+dx}{4}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.93

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{55d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 1
1/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.149 $\int (b\cos(c+dx))^{2/3} (A + C\cos^2(c + dx)) \sec(c+ dx)\, dx$

Optimal. Leaf size=89

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3)
*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5d}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(10*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{1}{5}(b(5A+2C$$
$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d} - \frac{3(5A+2C)}{}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 0.99

$$-\frac{3b\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{8d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x), x)
```

3.150 $\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hy
pergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d
*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + (-2A+C)\int (b\cos(c+dx$$
$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/}}{5bd}$$

Mathematica [A]
time = 0.18, size = 88, normalized size = 0.97

$$-\frac{3b\csc(c+dx)\left(-5A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{5d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*Csc[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^2\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^2, x)
```

3.151 $\int (b\cos(c+dx))^{2/3} (A + C\cos^2(c+dx)) \sec^3(c+dx)\, dx$

Optimal. Leaf size=90

$$\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3
)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$
$$= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{1}{4}(b(A+4C))\int \frac{}{\sqrt[3]{b\cos}}$$
$$= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx}{}$$

Mathematica [A]
time = 0.14, size = 96, normalized size = 1.07

$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1
/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3,
Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3))/cos(c + d*x)^3, x)
```

3.152 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right) d$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d}-\frac{3(16A+13C)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin(c+dx)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/16*C*(b*cos(d*x+c))^(13/3)*sin(d*x+c)/b^3/d-3/208*(16*A+13*C)*(b*cos(d*x+
c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin
(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{13/3}}{16b^3d}-\frac{3(16A+13C)\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(13/3)*Sin[c + d*x])/(16*b^3*d) - (3*(16*A + 13*C)*(b
*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Si
n[c + d*x])/(208*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{\int(b\cos(c+dx))^{10/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$=\frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d}+\frac{(16A+13C}{}$$
$$=\frac{3C(b\cos(c+dx))^{13/3}\sin(c+dx)}{16b^3d}-\frac{3(16A+13}{}$$

Mathematica [A]
time = 0.27, size = 96, normalized size = 1.01

$$-\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\left(19A\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)+13C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{19}{6};\frac{25}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{247d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(19*A*Hypergeometric
2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*C*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, 19/6, 25/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(247*d)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^5 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.153 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^2d}-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{7/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} + \frac{(13A+10}{}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d} - \frac{3(13A+10}{}$$

Mathematica [A]
time = 0.17, size = 91, normalized size = 0.96

$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\left(8A\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{80bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/3, 8/
3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 8/3, 11/3, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*b*d)
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.154 $\int (b\cos(c+dx))^{4/3} \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^(
7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd} - \frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos
[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}+\frac{1}{10}(10A+7C)\int (b\cos(c$$
$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}-\frac{3(10A+7C)(b\cos(c+d}{7}$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 0.93

$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{91d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d)
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.155 $\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=89

$$\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3)
*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7d}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(28*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d} + \frac{1}{7}(b(7A+4C$$
$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d} - \frac{3(7A+4C)}{}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 1.00

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\left(5A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3,
5/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

3.156 $\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=89

$$\frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d}-\frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d}-\frac{3b(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{1}{4}\left(b^2(4A+C\right.$$
$$= \frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d} - \frac{3b(4A+C)\sqrt[3]{}}{}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 1.01

$$-\frac{3b^2\cot(c+dx)\left(7A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
+ C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^2, x)
```

3.157 $\int (b\cos(c+dx))^{4/3} (A + C\cos^2(c+dx)) \sec^3(c+dx)\, dx$

Optimal. Leaf size=90

$$\frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3
)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c
 + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}-\frac{1}{2}(b(A-2C))\int \sqrt[3]{b\cos}\\ &= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}+\frac{3(A-2C)(b\cos(c+dx}{}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 90, normalized size = 1.00

$$-\frac{3b^2\csc(c+dx)\left(-2A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2
] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3))/cos(c + d*x)^3, x)
```

3.158 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^3d}-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{5/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}\\ &= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}+\frac{(11A+8C)\int (b\cos(c+dx))^{5/\cdots}}{11b^2}\\ &= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/\cdots}}{88b^3d\cdots}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 96, normalized size = 1.01

$$-\frac{3\cot(c+dx)\left(7A\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)+4C\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{3};\frac{10}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{56d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(7*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c
 + d*x]^2] + 4*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 7/3, 10/3, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(dx+c\right)\right)\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.159 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^2d}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}+\frac{(8A+5C)\int (b\cos(c+dx))^{2/}}{8b}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}}{40b^2d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 91, normalized size = 0.96

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\left(11A\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{55bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 1
1/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*b*d)
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)\left(A+C(\cos^2\left(dx+c\right))\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.160 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/
3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(10*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{1}{5}(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$$
$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos\right.}{10bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 0.92

$$-\frac{3\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{8d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{A+C\left(\cos^2(dx+c)\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

3.161 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=90

$$\frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hype
rgeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3091, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x
])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$
$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{(2A-C)\int (b\cos(c+dx))^{2/3}\,dx}{b}$$
$$= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos\right.}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.83, size = 283, normalized size = 3.14

$$-\frac{3e^{-idx}\sqrt[3]{\cos(c+dx)}\csc(c)(\cos(dx)+i\sin(dx))\left(-8A\cos(dx)+2C\cos(dx)+2C\cos(2c+dx)+2(2A-C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))\sqrt[3]{1+\cos(2(c+dx))+i\sin(2(c+dx))}+(2A-C)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))\sqrt[3]{1+\cos(2(c+dx))+i\sin(2(c+dx))}\right)}{4\,2^{2/3}d\sqrt[3]{b\cos(c+dx)}\sqrt[3]{e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1/3)*Csc[c]*(Cos[d*x] + I*Sin[d*x])*(-8*A*Cos[d*x] + 2*C*
Cos[d*x] + 2*C*Cos[2*c + d*x] + 2*(2*A - C)*Hypergeometric2F1[-1/3, 1/3, 2/
3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*(1 + Cos
[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(1/3) + (2*A - C)*Hypergeometric2F1[1/3
, 2/3, 5/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])
*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(1/3)))/(4*2^(2/3)*d*E^(I*d*x)
*(b*Cos[c + d*x])^(1/3)*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x))^(1/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(A+C(\cos^2(dx+c)))\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(
d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + A}{\cos(c + d\,x)\,\left(b\,\cos(c + d\,x)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)), x)
```

3.162 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*
hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{1}{4}(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\text{co}\right.}{8bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.84, size = 101, normalized size = 1.11

$$\frac{-((A+4C)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\cos^2(dx-\text{ArcTan}(\cot(c)))\right)\sin(2dx-2\text{ArcTan}(\cot(c))))+6A\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}\,\tan(c+dx)}{8d\sqrt[3]{b\cos(c+dx)}\,\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-((A + 4*C)*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*
Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)*Ta
n[c + d*x])/(8*d*(b*Cos[c + d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)
)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(A+C(\cos^2(dx+c)))\,(\sec^2(dx+c))}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*co
s(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{\sqrt[3]{b\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(1/3), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c + d\,x\right)^2 + A}{\cos\left(c + d\,x\right)^2\,\left(b\,\cos\left(c + d\,x\right)\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)), x)
```

3.163 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6,
1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3(4A+7C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hyperg
eometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{1}{7}(b(4A+7C))\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx \\
&= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\,\mathrm{s}}{7d\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.31, size = 481, normalized size = 5.23

$$
b\left(-\frac{i(4A+7C)\cos^{\frac{10}{3}}(c+dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(C+A\sec^2(c+dx)\right)\left(-\frac{3ie^{-idx}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt[3]{1+e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)}}{2^{2/3}d\sqrt[3]{e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)}}-\frac{3ie^{idx}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt[3]{1+e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)}}{2\,2^{2/3}d\sqrt[3]{e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)}}\right)}{7(b\cos(c+dx))^{4/3}(2A+C+C\cos(2c+2dx))}+\frac{\cos^4(c+dx)\left(C+A\sec^2(c+dx)\right)\left(\frac{6(4A+7C)\csc(c)\sec(c)}{7d}+\frac{6A\sec(c)\sec^3(c+dx)\sin(dx)}{7d}+\frac{6\sec(c)\sec(c+dx)(4A\sin(dx)+7C\sin(dx))}{7d}+\frac{6A\sec^2(c+dx)\tan(c)}{7d}\right)}{(b\cos(c+dx))^{4/3}(2A+C+C\cos(2c+2dx))}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] b*(((-1/7*I)*(4*A + 7*C)*Cos[c + d*x]^(10/3)*Csc[c/2]*Sec[c/2]*(C + A*Sec[c
 + d*x]^2)*(((-3*I)*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*(1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c])^
(1/3))/(2^(2/3)*d*E^(I*d*x)*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x))^(1/3)) - (((3*I)/2)*E^(I*d*x)*Hypergeometric2F1[1
/3, 2/3, 5/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(1 + E^((2*I)*d*x)*Co
s[2*c] + I*E^((2*I)*d*x)*Sin[2*c])^(1/3))/(2^(2/3)*d*(((1 + E^((2*I)*d*x))*
Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(1/3))))/((b*Cos[c + d*x
])^(4/3)*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^4*(C + A*Sec[c + d
*x]^2)*((6*(4*A + 7*C)*Csc[c]*Sec[c])/(7*d) + (6*A*Sec[c]*Sec[c + d*x]^3*Si
n[d*x])/(7*d) + (6*Sec[c]*Sec[c + d*x]*(4*A*Sin[d*x] + 7*C*Sin[d*x]))/(7*d)
 + (6*A*Sec[c + d*x]^2*Tan[c])/(7*d)))/((b*Cos[c + d*x])^(4/3)*(2*A + C + C
*Cos[2*c + 2*d*x])))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$
\int \frac{\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^3\left(dx+c\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*co
s(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)), x)
```

3.164 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d}-\frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b^3/d-3/70*(10*A+7*C)*(b*cos(d*x+c))
^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10b^3d}-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{70b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^3*d) - (3*(10*A + 7*C)*(b*C
os[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(70*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int (b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} + \frac{(10A+7C)\int(b\cos(c+dx)}{10b^2}$$
$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} - \frac{3(10A+7C)(b\cos(c+dx))}{70b^3}$$

Mathematica [A]
time = 0.14, size = 96, normalized size = 1.01

$$-\frac{3\cot(c+dx)\left(13A\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7C\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{91d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(13*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2] + 7*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c
+ d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(dx+c)\right)\left(A+C\left(\cos^2(dx+c)\right)\right)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.165 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{28b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/b^2/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(
4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7b^2d}-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b^2*d) - (3*(7*A + 4*C)*(b*Cos
[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(28*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$
$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d} + \frac{(7A+4C)\int \sqrt[3]{b\cos(c+dx)}\,d}{7b}$$
$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,{}_2}{28b^2d\sqrt{}}$$

Mathematica [**A**]
time = 0.13, size = 91, normalized size = 0.96

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\cot(c+dx)\left(5A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+2C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{20bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*b*d)
```

Maple [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(A+C(\cos^2(dx+c))\right)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(C\cos(c+d\,x)^2+A\right)}{\left(b\,\cos(c+d\,x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.166 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4bd}-\frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*(4*A+C)*(b*cos(d*x+c))^(1/3)*
hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]

time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3093, 2722}

$$\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4bd}-\frac{3(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos[c +
 d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{1}{4}(4A+C)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3C\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{3(4A+C)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(\right.}{4bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.94

$$-\frac{3\cot(c+dx)\left(7A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
[Out] (-3*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{A+C(\cos^2(dx+c))}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(2/3), x)
```

3.167 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}+\frac{3(A-2C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)+3/8*(A-2*C)*(b*cos(d*x+c))^(4/3)*hy
pergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3091, 2722}

$$\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d
*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} - \frac{(A-2C)\int \sqrt[3]{b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{3A\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}} + \frac{3(A-2C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos\right.}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.98, size = 277, normalized size = 3.08

$$-\frac{3e^{-idx}\cos^{\frac{2}{3}}(c+dx)\csc(c)(\cos(dx)+i\sin(dx))\left(10((-A+C)\cos(dx)+C\cos(2c+dx))+5(A-2C)\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)-i\sin(dx))(1+\cos(2(c+dx))+i\sin(2(c+dx)))^{2/3}+(A-2C)\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2idx}(\cos(c)+i\sin(c))^2\right)(\cos(dx)+i\sin(dx))(1+\cos(2(c+dx))+i\sin(2(c+dx)))^{2/3}\right)}{10\sqrt[3]{2}\,d(b\cos(c+dx))^{2/3}\left(e^{-idx}\left((1+e^{2idx})\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2/3)*Csc[c]*(Cos[d*x] + I*Sin[d*x])*(10*((-A + C)*Cos[d*x
] + C*Cos[2*c + d*x]) + 5*(A - 2*C)*Hypergeometric2F1[-1/6, 2/3, 5/6, -(E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*(1 + Cos[2*(c +
d*x)] + I*Sin[2*(c + d*x)])^(2/3) + (A - 2*C)*Hypergeometric2F1[2/3, 5/6, 1
1/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*(1 + C
os[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(2/3)))/(10*2^(1/3)*d*E^(I*d*x)*(b*Co
s[c + d*x])^(2/3)*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x))^(2/3))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(
d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + A}{\cos(c + d\,x)\,(b\,\cos(c + d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

3.168 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)-3/5*(2*A+5*C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{1}{5}(2A+5C)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};c\right.}{5bd\sqrt{\sin^2(c+dx}}$$

Mathematica [A]
time = 0.80, size = 103, normalized size = 1.11

$$\frac{-\left((2A+5C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\cos^2(dx-\text{ArcTan}(\cot(c)))\right)\sin(2dx-2\text{ArcTan}(\cot(c)))\right)+6A\sqrt[6]{\sin^2(dx-\text{ArcTan}(\cot(c)))}\,\tan(c+dx)}{10d(b\cos(c+dx))^{2/3}\sqrt[6]{\sin^2(dx-\text{ArcTan}(\cot(c)))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-((2*A + 5*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2
]*Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/6)*
Tan[c + d*x])/(10*d*(b*Cos[c + d*x])^(2/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1
/6))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(A+C(\cos^2(dx+c)))\,(\sec^2(dx+c))}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*co
s(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2(c + dx))\sec^2(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + A}{\cos(c + d\,x)^2\,(b\,\cos(c + d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

3.169 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=92

$$\frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{3(5A+8C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{16d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(8/3)+3/16*(5*A+8*C)*hypergeom([-1/3,
 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{3(5A+8C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{16d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(8*d*(b*Cos[c + d*x])^(8/3)) + (3*(5*A + 8*C)*Hyperg
eometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16*d*(b*Cos[c +
d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{11/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{1}{8}(b(5A+8C))\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}}+\frac{3(5A+8C)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin}{16d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.30, size = 473, normalized size = 5.14

$$b\left(-\frac{i(5A+8C)\cos^{\frac{11}{3}}(c+dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(C+A\sec^2(c+dx)\right)\left(-\frac{3ie^{-idx}\,{}_2F_1\left(-\frac{1}{6},\frac{2}{3};\frac{5}{6};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\left(2+2e^{2idx}\cos(2c)+2ie^{2idx}\sin(2c)\right)^{2/3}}{d\left(e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\right)^{2/3}}-\frac{3ie^{idx}\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\left(2+2e^{2idx}\cos(2c)+2ie^{2idx}\sin(2c)\right)^{2/3}}{5d\left(e^{-idx}\left(\left(1+e^{2idx}\right)\cos(c)+i\left(-1+e^{2idx}\right)\sin(c)\right)\right)^{2/3}}\right)}{32(b\cos(c+dx))^{5/3}(2A+C+C\cos(2c+2dx))}+\frac{\cos^4(c+dx)\left(C+A\sec^2(c+dx)\right)\left(\frac{3(5A+8C)\csc(c)\sec(c)}{8d}+\frac{3A\sec(c)\sec^3(c+dx)\sin(dx)}{4d}+\frac{3\sec(c)\sec(c+dx)(5A\sin(dx)+8C\sin(dx))}{8d}+\frac{3A\sec^2(c+dx)\tan(c)}{4d}\right)}{(b\cos(c+dx))^{5/3}(2A+C+C\cos(2c+2dx))}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] b*(((-1/32*I)*(5*A + 8*C)*Cos[c + d*x]^(11/3)*Csc[c/2]*Sec[c/2]*(C + A*Sec[
c + d*x]^2)*(((-3*I)*Hypergeometric2F1[-1/6, 2/3, 5/6, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*(2 + 2*E^((2*I)*d*x)*Cos[2*c] + (2*I)*E^((2*I)*d*x)*Sin
[2*c])^(2/3))/(d*E^(I*d*x)*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x))^(2/3)) - (((3*I)/5)*E^(I*d*x)*Hypergeometric2F1[2/
3, 5/6, 11/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(2 + 2*E^((2*I)*d*x)*
Cos[2*c] + (2*I)*E^((2*I)*d*x)*Sin[2*c])^(2/3))/(d*(((1 + E^((2*I)*d*x))*Co
s[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(2/3))))/((b*Cos[c + d*x])
^(5/3)*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c + d*x]^4*(C + A*Sec[c + d*x
]^2)*((3*(5*A + 8*C)*Csc[c]*Sec[c])/(8*d) + (3*A*Sec[c]*Sec[c + d*x]^3*Sin[
d*x])/(4*d) + (3*Sec[c]*Sec[c + d*x]*(5*A*Sin[d*x] + 8*C*Sin[d*x]))/(8*d) +
 (3*A*Sec[c + d*x]^2*Tan[c])/(4*d)))/((b*Cos[c + d*x])^(5/3)*(2*A + C + C*C
os[2*c + 2*d*x])))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(A+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*co
s(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

3.170 $\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^3d}-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int (b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$
$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d} + \frac{(8A+5C)\int (b\cos(c+dx))^{2/3}}{8b^2}$$
$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3}}{40b^3d\sqrt{}}$$

Mathematica [A]
time = 0.21, size = 96, normalized size = 1.01

$$-\frac{3\cos^2(c+dx)\cot(c+dx)\left(11A\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{55d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos
[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c
+ d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(dx+c)\right)\left(A+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
[Out] int((cos(c + d*x)^2*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.171 $\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=95

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(
2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5b^2d}-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos
[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d
*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{(5A+2C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}$$
$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3}}{10b^2d}$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.95

$$-\frac{3\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{8bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*
Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(8*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)\left(A+C\left(\cos^2(dx+c)\right)\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)),
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.172 $\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hy
pergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3091, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d
*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/
(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} - \frac{(2A-C)\int(b\cos(c+dx))^{2/3}\,dx}{b^2}$$
$$= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}} + \frac{3(2A-C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 87, normalized size = 0.94

$$-\frac{3\cot(c+dx)\left(-5A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)+C\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{5d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Si
n[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{A+C(\cos^2(dx+c))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

3.173 $\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)-3/8*(A+4*C)*(b*cos(d*x+c))^(2/3)*hy
pergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2
)
```

Rubi [A]

time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3091, 2722}

$$\frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d
*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$
$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{(A+4C)\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{4b}$$
$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} - \frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos\right.}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.65, size = 104, normalized size = 1.16

$$\frac{-\left((A+4C)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\cos^2(dx-\text{ArcTan}(\cot(c)))\right)\sin(2dx-2\text{ArcTan}(\cot(c)))\right)+6A\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}\tan(c+dx)}{8bd\sqrt[3]{b\cos(c+dx)}\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-((A + 4*C)*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*
Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)*Ta
n[c + d*x])/(8*b*d*(b*Cos[c + d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/
3))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(A+C(\cos^2(dx+c)))\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

3.174 $\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/7*(4*A+7*C)*hypergeom([-1/6, 1/
2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {16, 3091, 2722}

$$\frac{3(4A+7C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}+\frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeo
metric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b^2\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{1}{7}(4A+7C)\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$$
$$= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin}{7bd\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.23, size = 90, normalized size = 0.97

$$\frac{3b^2\cot(c+dx)\left(A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7C\cos^2(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] +
 7*C*Cos[c + d*x]^2*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{(A+C(\cos^2(dx+c)))(\sec^2(dx+c))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

3.175 $\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=92

$$\frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{3(7A+10C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(10/3)+3/40*(7*A+10*C)*hypergeom([-2
/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3091, 2722}

$$\frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{3(7A+10C)\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{40d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(10*d*(b*Cos[c + d*x])^(10/3)) + (3*(7*A + 10*C)*Hyp
ergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(40*d*(b*Cos[c
 + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^3\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{13/3}}\,dx\\
&= \frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{1}{10}(b(7A+10C))\int \frac{1}{(b\cos(c+dx))^{7/3}}\\
&= \frac{3Ab^2\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}}+\frac{3(7A+10C)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx\right.}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+}}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 91, normalized size = 0.99

$$
\frac{3b^2\csc(c+dx)\left(2A\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)+5C\cos^2(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{20d(b\cos(c+dx))^{10/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
[Out] (3*b^2*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]
 + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(20*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$
\int \frac{(A+C(\cos^2(dx+c)))(\sec^3(dx+c))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int((A + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

3.176 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)$

Optimal. Leaf size=148

$$\frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{d(10+3m)}-\frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+}}{d(7+3m)(10+}$$

```
[Out] 3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(7
+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6
+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+51*m+70)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)}{d(3m+10)}-\frac{3b\left(\frac{A}{3m+7}+\frac{C}{3m+10}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3
*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\left(A+C\right.}{\sqrt[3]{\cos(c+dx)}}$$
$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx}{d(10+3m)}$$
$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx}{d(10+3m)}$$

Mathematica [A]
time = 0.31, size = 142, normalized size = 0.96

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(A(13+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)+C(7+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(13+3m);\frac{1}{6}(19+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(7+3m)(13+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(A*(13 + 3*m)*
Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] + C*(7 +
3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(13 + 3*m))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)(b\cos(dx+c))^{\frac{4}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,\left(C\cos(c+d\,x)^2+A\right)\,\left(b\,\cos(c+d\,x)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(4/3), x)
```

3.177 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right) d$

Optimal. Leaf size=146

$$\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))^{2}}{d(5+3m)(8+3m}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m)
+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m
],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+39*m+40)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)}{d(3m+8)}-\frac{3\left(\frac{A}{3m+5}+\frac{C}{3m+8}\right)\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{2/3}\int\cos^{\frac{2}{3}+m}(c+dx)\left(A+C\,\mathrm{co}\right.}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+d}{d(8+3m)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+d}{d(8+3m)}$$

Mathematica [A]
time = 0.24, size = 142, normalized size = 0.97

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(A(11+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)+C(5+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(11+3m);\frac{1}{6}(17+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(5+3m)(11+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(11 + 3*m)*
Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 +
3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[
c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(11 + 3*m))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(2/3), x)
```

3.178 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+C\cos^2(c+dx))$

Optimal. Leaf size=146

$$\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{d(7+3m)}-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}}{d(4+3m)(7+3m}$$

[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m)+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+33*m+28)/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)}{d(3m+7)}-\frac{3\left(\frac{A}{3m+4}+\frac{C}{3m+7}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]

[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)) - (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)\left(A+C\cos^2(c\right.}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)} +$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)}{d(7+3m)} -$$

Mathematica [A]
time = 0.33, size = 142, normalized size = 0.97

$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\left(C(4+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2};\frac{8}{3}+\frac{m}{2};\cos^2(c+dx)\right)+A(10+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(4+3m)(10+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(4 + 3*m)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2]
+ A*(10 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(10 + 3*m))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos{\left(c+dx \right)}}\left(A+C\cos^{2}{\left(c+dx \right)}\right)\cos^{m}{\left(c+dx \right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + C*cos(c + d*x)**2)*cos(c + d*x)**m, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(c+d\,x\right)}^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^m*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^(1/3), x)
```

3.179 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=146

$$\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\mathrm{co}\ldots\right.}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m)
+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(9*m^2+21*m+10)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b\cos(c+dx)}}-\frac{3\left(\frac{A}{3m+2}+\frac{C}{3m+5}\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)
) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1
/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{\sqrt[3]{b\cos(c+dx)}}\\ &= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}}+\frac{\left(\left(C\left(\frac{2}{3}+m\right)+A\left(\frac{5}{3}+m\right)\right)\sqrt[3]{}\right.}{\left(\frac{5}{3}+m\right)}\\ &= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(2+3m)+A(5+3m))\,c}{d(2+}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 142, normalized size = 0.97

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(8+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)+C(2+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8+3m);\frac{7}{3}+\frac{m}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+3m)(8+3m)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(8 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(2 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^m(dx+c)\right)\left(A+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*co
s(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2{(c + dx)})\cos^m{(c + dx)}}{\sqrt[3]{b\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(1/3), x)
```

3.180 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=144

$$\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}-\frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos\right.}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+A
*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c
)^2)*sin(d*x+c)/d/(9*m^2+15*m+4)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+4)(b\cos(c+dx))^{2/3}}-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 +
 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)\,dx}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} + \frac{\left(\left(C\left(\frac{1}{3}+m\right)+A\left(\frac{4}{3}+m\right)\right)\cos \cdots}{\left(\frac{4}{3}+m\right)(b \cdots}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm+A(4+3m))\cos \cdots}{d(1+3 \cdots}$$

Mathematica [A]
time = 0.27, size = 142, normalized size = 0.99

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(7+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)+C(1+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+3m)(7+3m)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(7 + 3*m)*Hypergeometric2F1[1/2, (
1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + C*(1 + 3*m)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(1 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^m(dx+c)\right)\left(A+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*co
s(d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2(c + dx))\cos^m(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c + d\,x)^m\,\left(C\cos(c + d\,x)^2 + A\right)}{(b\,\cos(c + d\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(2/3), x)
```

3.181 $\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=149

$$\frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2\right.}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-A
*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(-9*m^2-3*m+2)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{3\left(\frac{A}{1-3m}-\frac{C}{3m+2}\right)\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3C\sin(c+dx)\cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) +
(3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 +
3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)\left(A+C\cos^2(c+dx)\right)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+m\right)\right)\sqrt[3]{}\right.}{b\left(\frac{2}{3}+m\right)}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(1-3m)-A(2+3m))\cos}{bd(1-3}$$

Mathematica [A]
time = 0.29, size = 142, normalized size = 0.95

$$-\frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(5+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)+C(-1+3m)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-1+3m)(5+3m)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (
-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + C*(-1 + 3*m)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-1 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m(dx+c))\left(A+C(\cos^2(dx+c))\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*
cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + C\cos^2{(c + dx)})\cos^m{(c + dx)}}{(b\cos{(c + dx)})^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(4/3), x)
```

3.182 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)$

Optimal. Leaf size=144

$$\frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)}-\frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos}{ad(1+m+n}$$

```
[Out] C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+A
*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1
/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(sin(d
*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {20, 3093, 2722}

$$\frac{C\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n}{ad(m+n+2)}-\frac{(A(m+n+2)+C(m+n+1))\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{ad(m+n+1)(m+n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m +
n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c +
d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a\cos(c+dx))^m(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \left((a\cos(c+dx))^{-n}(b\cos(c+dx))^n\right)\int (a\cos($$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+}{ad(2+m+n)}$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+}{ad(2+m+n)}$$

Mathematica [A]
time = 0.27, size = 132, normalized size = 0.92

$$-\frac{(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(A(3+m+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\cos^2(c+dx)\right)+C(1+m+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3+m+n);\frac{1}{2}(5+m+n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+m+n)(3+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + m + n)*Hyperg
eometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + C*(1 + m +
 n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(3 + m + n)))
```

Maple [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c))^m\,(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos{(c+dx)})^m\,(b\cos{(c+dx)})^n\,\left(A+C\cos^2{(c+dx)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a\,\cos\left(c+d\,x\right)\right)^m\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n,x)
```

```
[Out] int((A + C*cos(c + d*x)^2)*(a*cos(c + d*x))^m*(b*cos(c + d*x))^n, x)
```

3.183 $\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=117

$$\frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)}-\frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x+
c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{b^3d(n+4)}-\frac{(A(n+4)+C(n+3))\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)(n+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A
*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{2+n}\left(A+C\cos^2(c+dx)\right)\,dx}{b^2}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} + \frac{\left(A+\frac{C(3+n)}{4+n}\right.}{}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)} - \frac{\left(A+\frac{C(3+n)}{4+n}\right.}{}$$

Mathematica [A]
time = 0.22, size = 122, normalized size = 1.04

$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(5+n)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)+C(3+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5+n}{2};\frac{7+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(3+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(5 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]^2*H
ypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(3 + n)*(5 + n)))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fr
icas")
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="gi
ac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.184 $\int \cos(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=117

$$\frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}-\frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\mathrm{si}}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x+
c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d
/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3093, 2722}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{b^2d(n+3)}-\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A
*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{1+n}\left(A+C\cos^2(c+dx)\right)\,dx}{b}$$

$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} + \frac{\left(A+\frac{C(2+n)}{3+n}\right)\int}{}$$

$$= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{\left(A+\frac{C(2+n)}{3+n}\right)(}{}$$

Mathematica [A]
time = 0.20, size = 120, normalized size = 1.03

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(A(4+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)+C(2+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(2 + n)*(4 + n)))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.185 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=117

$$\frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)}-\frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c)
)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3093, 2722}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}-\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(
2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx = \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} + \left(A+\frac{C(1+n)}{2+n}\right)\int (b\cos$$

$$= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{\left(A+\frac{C(1+n)}{2+n}\right)(b\cos(c+d}{b}$$

Mathematica [A]
time = 0.18, size = 114, normalized size = 0.97

$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\left(A(3+n)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)+C(1+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]^2*Hypergeometric2F
1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 n)*(3 + n)))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^n \left(A+C\left(\cos^2\left(dx+c\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos(c+d\,x)^2+A\right)\,(b\,\cos(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
[Out] int((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.186 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}-\frac{(A+An+Cn)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hypergeo
m([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3093, 2722}

$$\frac{C\sin(c+dx)(b\cos(c+dx))^n}{d(n+1)}-\frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c
 + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d
*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{-1+n}\left(A+C\cos^2(c+dx)\right)\,dx$$
$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{(b(A+An+Cn)}{}$$
$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} - \frac{(A+An+Cn)(b}{}$$

Mathematica [A]
time = 0.25, size = 111, normalized size = 1.11

$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(A(2+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)+Cn\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{dn(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]^2*Hypergeometric2F1[1
/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(2 +
n)))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(A+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos(c+d\,x)^2 + A\right)\,\left(b\,\cos(c+d\,x)\right)^n}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x), x)
```

3.187 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}-\frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx\right.}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-1
+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n
)/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}}{dn}-\frac{b(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(
b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int (b\cos(c+dx))^{-2+n}\left(A+C\cos^2(c+dx)\right)\,d$$

$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} - \frac{(b^2(C(1-}{}$$

$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn} - \frac{b(C(1-n)}{}$$

Mathematica [A]
time = 0.22, size = 117, normalized size = 1.04

$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(A(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)+C(-1+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-1+n)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/
2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(-1 + n)*(1 + n)))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^n\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^2, x)
```

3.188 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$

Optimal. Leaf size=125

$$-\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}+\frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\text{co}\right.}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*co
s(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/d/(n^2-3*n+2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{b^2(A(1-n)+C(2-n))\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2C\sin(c+dx)(b\cos(c+dx))^{n-2}}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] -((b^2*C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(1 - n))) + (b^2*(A*(1
- n) + C*(2 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)
/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*(2 - n)*Sqrt[Sin[c + d*x]
^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx &= b^3\int (b\cos(c+dx))^{-3+n}\left(A+C\cos^2(c+dx)\right)\,dx\\ &= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}+\Bigg(b^3\Bigg(A+\\ &= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}+\frac{b^2\Big(A+\frac{C}{}}{}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 114, normalized size = 0.91

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(An\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)+C(-2+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{d(-2+n)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-2 + n)/2,
n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, n/2
, (2 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 +
 n)*n))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^3, x)
```

3.189 $\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=127

$$-\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}+\frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*co
s(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*sin
(d*x+c)/d/(n^2-5*n+6)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3093, 2722}

$$\frac{b^3(A(2-n)+C(3-n))\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3C\sin(c+dx)(b\cos(c+dx))^{n-3}}{d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -((b^3*C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(2 - n))) + (b^3*(A*(2
- n) + C*(3 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)
/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*(3 - n)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx &= b^4\int (b\cos(c+dx))^{-4+n}\left(A+C\cos^2(c+dx)\right)\,d \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}+\Bigg(b^4\Big(A \\
&= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}+\frac{b^3\Big(A+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 122, normalized size = 0.96

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos^2(c+dx)\right)+C(-3+n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\right)\sec^3(c+dx)\sqrt{\sin^2(c+dx)}}{d(-3+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-3 +
 n)/2, (-1 + n)/2, Cos[c + d*x]^2] + C*(-3 + n)*Cos[c + d*x]^2*Hypergeometr
ic2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[
c + d*x]^2])/(d*(-3 + n)*(-1 + n)))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^n\left(A+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4,x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^4, x)
```

3.190 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=142

$$\frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)} - \frac{2(C(7+2n)+A(9+2n))\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1(\ldots}{d(7+2n)(9+2n)\sqrt{\sin\ldots}}$$

```
[Out] 2*C*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(9+2*n)-2*(C*(7+2*n)+A*(
9+2*n))*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+
1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+32*n+63)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+9)} - \frac{2\left(\frac{A}{2n+7}+\frac{C}{2n+9}\right)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(9 + 2*n)) - (2
*(A/(7 + 2*n) + C/(9 + 2*n))*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx=\left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx$$

$$=\frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)}+$$

$$=\frac{2C\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(9+2n)}-$$

Mathematica [A]
time = 0.24, size = 140, normalized size = 0.99

$$-\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(11+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)+C(7+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(11+2n);\frac{1}{4}(15+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(7+2n)(11+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(11 + 2*n)*Hyperg
eometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*C
os[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(11 + 2*n))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(C\,\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.191 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx$

Optimal. Leaf size=142

$$\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)}-\frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\right.}{d(5+2n)(7+2n)\sqrt{\sin^2}}$$

```
[Out] 2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*(
7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+24*n+35)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {20, 3093, 2722}

$$\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+7)}-\frac{2\left(\frac{A}{2n+5}+\frac{C}{2n+7}\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2
*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(A+C\cos^2(c+dx)\right) dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right) \int \cos^{\frac{3}{2}+n}(c+ \\
&= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} + \\
&= \frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)} -
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 140, normalized size = 0.99

$$
-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(9+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)+C(5+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(9+2n);\frac{1}{4}(13+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(5+2n)(9+2n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hyperge
ometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos
[c + d*x]^2*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(9 + 2*n))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$
\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right) dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*
x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.192 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,(A+C\cos^2(c+dx)$

Optimal. Leaf size=142

$$\frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)}-\frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1(}{d(3+2n)(5+2n)\sqrt{\sin}}$$

```
[Out] 2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*(
5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+16*n+15)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+5)}-\frac{2\left(\frac{A}{2n+3}+\frac{C}{2n+5}\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2
*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
```

```
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} +$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)} -$$

Mathematica [A]
time = 0.20, size = 140, normalized size = 0.99

$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(A(7+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)+C(3+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(3+2n)(7+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos
[c + d*x]^2*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^
2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(7 + 2*n))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(A+C\left(\cos^2(dx+c)\right)\right)\left(\sqrt{\cos}\,(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n, x)
```

3.193 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}-\frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1(}{d(1+2n)(3+2n)\sqrt{\sin}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(3+2*n)-2*(C+2*C*n+A*(3+
2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(4*n^2+8*n+3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n}{d(2n+3)}-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2
*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeome
tric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1
+ 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\left(A+C\right.\\ &= \frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}+\frac{\left(\left(C\left(\frac{1}{2}+n\right)\right.\right.}{}\\ &= \frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}-\frac{2(C+2Cn}{}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 140, normalized size = 1.00

$$-\frac{2\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\csc(c+dx)\left(A(5+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)+C(1+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+2n)(5+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos
[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(5 + 2*n))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(1/2), x)
```

3.194 $\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}}+\frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos\ldots\right.}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+2
*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d\left(1-4n^2\right)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(2n+1)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2
*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 +
2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[
c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\left(A+C\cos\right.\\ &= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{\left(\left(C\left(-\frac{1}{2}+n\right)+A\left(\frac{1}{2}+n\right)\right)\right.}{}\\ &= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1-2n)+2An)(b}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 140, normalized size = 1.03

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)+C(-1+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-1+2n)(3+2n)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-1 + 2*n)*(3 + 2*n)*Sqrt[Cos[c + d*x]])
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cos\left(dx+c\right)\right)^{n}\left(A+C\left(\cos^{2}\left(dx+c\right)\right)\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n (A + C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + C*cos(c + d*x)**2)/cos(c + d*x)**(3/2), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(3/2), x)
```

3.195 $\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}+\frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\mathrm{co}\right.}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)-
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(4*n^2-8*n+3)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {20, 3093, 2722}

$$\frac{2\left(\frac{A}{3-2n}+\frac{C}{1-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Cos[c + d*x]^(3/2)) + (
2*(C/(1 - 2*n) + A/(3 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)
*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\left(A+C\right.\\ &= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}+\frac{\left(\left(C\left(-\frac{3}{2}+n\right)+A\left(-\frac{1}{2}+\right.\right.\right.}{}\\ &= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}+\frac{2(A(1-2n)+C(3-2n}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 140, normalized size = 1.00

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)+C(-3+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-3+2n)(1+2n)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]^2*Hyper
geometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(5/2), x)
```

3.196 $\int \frac{(b\cos(c+dx))^n(A+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}+\frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx}}$$

[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C*(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2-16*n+15)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2\left(\frac{A}{5-2n}+\frac{C}{3-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(5/2)) + (2*(C/(3 - 2*n) + A/(5 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\left(A+C\,\text{co}\right.$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{5}{2}+n\right)+A\left(-\frac{3}{2}+\right.\right.\right.}{}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2(A(3-2n)+C(5-2n)}{}$$

Mathematica [A]
time = 0.21, size = 140, normalized size = 0.99

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)+C(-5+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-5+2n)(-1+2n)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-5 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^n}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(7/2), x)
```

3.197 $\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)}+\frac{2(A(5-2n)+C(7-2n))(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n)\right.}{d(5-2n)(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(7/2)+2*(A*(5-2*n)+C*(7-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2-24*n+35)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)

Rubi [A]

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3093, 2722}

$$\frac{2\left(\frac{A}{7-2n}+\frac{C}{5-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(7/2)) + (
2*(C/(5 - 2*n) + A/(7 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7
 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(7/2
)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f
```

```
*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])
^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\left(A+C\right.$$
$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(\left(C\left(-\frac{7}{2}+n\right)+A\left(-\frac{5}{2}+\right.\right.\right.}{}$$
$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(5-2n)\cos^{\frac{7}{2}}(c+dx)} + \frac{2(A(5-2n)+C(7-2n}{}$$

Mathematica [A]
time = 0.20, size = 140, normalized size = 0.99

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(A(-3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n);\cos^2(c+dx)\right)+C(-7+2n)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-7+2n)(-3+2n)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + C*(-7 + 2*n)*Cos[c + d*x]^2*Hyp
ergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c
+ d*x]^2])/(d*(-7 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(7/2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(A+C\left(\cos^2(dx+c)\right)\right)}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{{\cos\left(c+d\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + C*cos(c + d*x)^2)*(b*cos(c + d*x))^n)/cos(c + d*x)^(9/2), x)
```

3.198 $\int (a+a\cos(e+fx))^m (A+C\cos^2(e+fx))\,dx$

Optimal. Leaf size=170

$$-\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f(2+3m+m^2)}+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\frac{2^{\frac{1}{2}+m}(C(1+m+m^2)+A(2$$

[Out] -C*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(m^2+3*m+2)+C*(a+a*cos(f*x+e))^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(C*(m^2+m+1)+A*(m^2+3*m+2))*(1+cos(f*x+e))^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f*x+e))*sin(f*x+e)/f/(m^2+3*m+2)

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3103, 2830, 2731, 2730}

$$\frac{2^{m+\frac{1}{2}}(A(m^2+3m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx))\right)}{f(m+1)(m+2)}-\frac{C\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m^2+3m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]

[Out] -((C*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(2 + 3*m + m^2))) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx &= \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\frac{\int (a+a\cos(e+\\ &= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f\left(2+3m+m^2\right)}+\frac{C(a+a\cos(e+}{af(}\\ &= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f\left(2+3m+m^2\right)}+\frac{C(a+a\cos(e+}{af(}\\ &= -\frac{C(a+a\cos(e+fx))^m\sin(e+fx)}{f\left(2+3m+m^2\right)}+\frac{C(a+a\cos(e+}{af(}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.65, size = 238, normalized size = 1.40

$$\frac{i4^{-1-m}e^{-2i(e+fx)}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m\left(C(-2+m)m\,{}_2F_1\left(-2-m,-2m;-1-m;-e^{i(e+fx)}\right)+e^{2i(e+fx)}(2+m)\left(Ce^{2i(e+fx)}m\,{}_2F_1\left(2-m,-2m;3-m;-e^{i(e+fx)}\right)+2(2A+C)(-2+m)\,{}_2F_1\left(-2m,-m;1-m;-e^{i(e+fx)}\right)\right)\right)}{f(-2+m)m(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Cos
[e + f*x]))^m*(C*(-2 + m)*m*Hypergeometric2F1[-2 - m, -2*m, -1 - m, -E^(I*(
e + f*x))] + E^((2*I)*(e + f*x))*(2 + m)*(C*E^((2*I)*(e + f*x))*m*Hypergeom
etric2F1[2 - m, -2*m, 3 - m, -E^(I*(e + f*x))] + 2*(2*A + C)*(-2 + m)*Hyper
geometric2F1[-2*m, -m, 1 - m, -E^(I*(e + f*x))])))/(E^((2*I)*(e + f*x))*(1
+ E^(I*(e + f*x)))^(2*m)*f*(-2 + m)*m*(2 + m)*Cos[(e + f*x)/2]^(2*m))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (a+a\cos(fx+e))^m \left(A+C\left(\cos^2(fx+e)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos(e + fx) + 1))^m \left(A + C\cos^2(e + fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*(cos(e + f*x) + 1))**m*(A + C*cos(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(e+f\,x\right)^2+A\right)\,\left(a+a\,\cos\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
[Out] int((A + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

3.199 $\int (a+a\cos(c+dx))^{2/3} \left(A+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=135

$$-\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad}+\frac{(40A+19C)(a+a\cos(c+$$

```
[Out] -9/40*C*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^(5/3)*si
n(d*x+c)/a/d+1/20*(40*A+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeom([-1/6, 1/2],
[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(7/6)
```

Rubi [A]
time = 0.12, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3103, 2830, 2731, 2730}

$$\frac{(40A+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{10\ 2^{5/6}d(\cos(c+dx)+1)^{7/6}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}-\frac{9C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-9*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a + a*Cos[c +
 d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 19*C)*(a + a*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(
10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad}+\frac{3\int(a+a\cos(c\cdots}{\cdots}$$

$$= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(\cdots}{\cdots}$$

$$= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(\cdots}{\cdots}$$

$$= -\frac{9C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(\cdots}{\cdots}$$

Mathematica [A]
time = 0.90, size = 175, normalized size = 1.30

$$\frac{(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(6\,2^{5/6}(40A+28C+14C\cos(c+dx)+5C\cos(2(c+dx)))\sqrt[6]{1-\cos\left(dx-2\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)}\sin(c+dx)-4(40A+19C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\cos^2\left(\frac{dx}{2}-\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)\right)\sin\left(dx-2\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)\right)}{320\,2^{5/6}d\sqrt[6]{1-\cos\left(dx-2\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(6*2^(5/6)*(40*A + 28*C +
14*C*Cos[c + d*x] + 5*C*Cos[2*(c + d*x)])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]
])^(1/6)*Sin[c + d*x] - 4*(40*A + 19*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Co
s[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(320*2^(5/
6)*d*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (a+a\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(2/3), x)
```

3.200 $\int \sqrt[3]{a+a\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=135

$$-\frac{9C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+\frac{(28A+13C)\sqrt[3]{a+a\cos(c+d}}{14\sqrt[6]{2}}$$

```
[Out] -9/28*C*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^(4/3)*si
n(d*x+c)/a/d+1/28*(28*A+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[
3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3103, 2830, 2731, 2730}

$$\frac{(28A+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{14\sqrt[6]{2}\,d(\cos(c+dx)+1)^{5/6}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}-\frac{9C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{28d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-9*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a + a*Cos[c +
 d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 13*C)*(a + a*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(1
4*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{a+a\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx &= \frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+\frac{3\int\sqrt[3]{a+a\cos(\\ &= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c\\ &= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c\\ &= -\frac{9C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.93, size = 240, normalized size = 1.78

$$\frac{3\sqrt[3]{a(1+\cos(c+dx))}\left(-4(28A+13C)\cot\left(\frac{c}{2}\right)+4C\cos(dx)\sin(c)+\frac{(28A+13C)\csc\left(\frac{c}{4}\right)\left(2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{idx}(\cos(c)+i\sin(c))\right)+e^{idx}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{idx}(\cos(c)+i\sin(c))\right)\right)\sec\left(\frac{c}{4}\right)\sqrt[3]{1+e^{idx}\cos(c)+ie^{idx}\sin(c)}}{\left(1+e^{idx}\right)\cos\left(\frac{c}{2}\right)+i\left(-1+e^{idx}\right)\sin\left(\frac{c}{2}\right)}+8C\cos(2dx)\sin(2c)+4C\cos(c)\sin(dx)+8C\cos(2c)\sin(2dx)\right)}{112d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*(a*(1 + Cos[c + d*x]))^(1/3)*(-4*(28*A + 13*C)*Cot[c/2] + 4*C*Cos[d*x]*S
in[c] + ((28*A + 13*C)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I
*d*x)*(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(
E^(I*d*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*
x)*Sin[c])^(1/3))/((1 + E^(I*d*x))*Cos[c/2] + I*(-1 + E^(I*d*x))*Sin[c/2])
+ 8*C*Cos[2*d*x]*Sin[2*c] + 4*C*Cos[c]*Sin[d*x] + 8*C*Cos[2*c]*Sin[2*d*x]))
/(112*d)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (a+a\cos(dx+c))^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+C\cos^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + C*cos(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + a*cos(c + d*x))^(1/3), x)
```

3.201 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}+\frac{(10A+7C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)\right)}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\ \sqrt[3]{a+a\cos}}$$

```
[Out] -9/10*C*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^(2/3)*si
n(d*x+c)/a/d+1/10*(10*A+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*cos(d*x+c))
*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3103, 2830, 2731, 2730}

$$\frac{(10A+7C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\ \sqrt[3]{a\cos(c+dx)+a}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}-\frac{9C\sin(c+dx)}{10d\sqrt[3]{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-9*C*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a + a*Cos[c +
 d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A + 7*C)*Hypergeometric2F1[1/2, 5
/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1 + Cos[c + d*x]
)^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx &= \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{3\int \frac{\frac{1}{3}a(5A+2C)-aC\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx}{5a} \\
&= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{1}{10}(10A \\
&= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{((10A -}{} \\
&= -\frac{9C\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{(10A +}{5\ 2}
\end{aligned}$$

Mathematica [A]
time = 0.48, size = 144, normalized size = 1.07

$$-\frac{3\ 2^{5/6}C\sqrt[6]{1-\cos\left(dx-2\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)}\,(\sin(c+dx)-\sin(2(c+dx)))+2(10A+7C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\cos^2\left(\frac{dx}{2}-\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)\right)\sin\left(dx-2\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)}{20d\sqrt[3]{a(1+\cos(c+dx))}\sqrt[6]{\sin^2\left(\frac{dx}{2}-\text{ArcTan}\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] -1/20*(3*2^(5/6)*C*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*(Sin[c + d*x]
- Sin[2*(c + d*x)]) + 2*(10*A + 7*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(
d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]])/(d*(a*(1 + Cos
[c + d*x]))^(1/3)*(Sin[(d*x)/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{A+C\left(\cos^2\left(dx+c\right)\right)}{\left(a+a\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^2{(c + dx)}}{\sqrt[3]{a(\cos{(c + dx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

3.202 $\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=138

$$\frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad}-\frac{(4A+7C)\sqrt[3]{a+a\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(\ldots\right.}{2\sqrt[6]{2}\,ad(1+\cos(c+d\ldots}$$

```
[Out] 3*(A+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)*si
n(d*x+c)/a/d-1/4*(4*A+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[3/2
],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$,
Rules used = {3103, 2829, 2731, 2730}

$$-\frac{(4A+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}\,ad(\cos(c+dx)+1)^{5/6}}+\frac{3(A+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c
 + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A + 7*C)*(a + a*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2
*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad} + \frac{3\int \frac{\frac{1}{3}a(4A+C)-aC\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx}{4a}$$

$$= \frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad} - \frac{(4A+7C)}{}$$

$$= \frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad} - \frac{\left((4A+7C\right.}{}$$

$$= \frac{3(A+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad} - \frac{(4A+7C)}{}$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{A+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
[Out] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{A+C(\cos^2(dx+c))}{(a+a\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^2{(c + dx)}}{(a(\cos{(c + dx)} + 1))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+A}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

3.203 $\int (a+b\cos(c+dx))^{2/3} \left(A + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=277

$$\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3a(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos}{4\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/8*a*(a+b)*C*AppellF1(1/2,-5/3
,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*
sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2
)+1/8*(3*a^2*C+b^2*(8*A+5*C))*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a
+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d
*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3103, 2835, 2744, 144, 143}

$$\frac{(3a^2C+b^2(8A+5C))\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}-\frac{3aC(a+b)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*a*(a + b)*C*Appe
llF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a +
 b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((3*a^2*C + b^2*(8*A + 5
*C))*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*
x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{2/3}\left(A+C\cos^2(c+dx)\right)\,dx=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{3\int(a+b\cos(c+\cdots}{\cdots}$$

$$=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{(3aC)\int(a+b\,\mathrm{co}\cdots}{8\cdots}$$

$$=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{(3aC\sin(c+dx)\cdots}{8b^2d\sqrt{\cdots}}$$

$$=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{(3a(-a-b)C(a\cdots}{\cdots}$$

$$=\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3a(a+b)CF_1\left(\frac{1}{2}\cdots\right.}{\cdots}$$

Mathematica [A]
time = 2.78, size = 279, normalized size = 1.01

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(60a(a^2-b^2)CF_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+4(40Ab^2-6a^2C+25b^2C)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-20b^2C(2a+5b\cos(c+dx))\sin^2(c+dx)\right)}{800b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(60*a*(a^2 - b^2)*C*AppellF1[2/
3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b
)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(
a - b))] + 4*(40*A*b^2 - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (
a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 +
Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c
 + d*x]) - 20*b^2*C*(2*a + 5*b*Cos[c + d*x])*Sin[c + d*x]^2))/(800*b^3*d)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{2}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)
```

[Out] int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)

[Out] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)

3.204 $\int \sqrt[3]{a+b\cos(c+dx)}\,(A+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=277

$$\frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}-\frac{3\sqrt{2}\,a(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+}}{7b^2d\sqrt{1+\cos(c+dx)}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d-3/7*a*(a+b)*C*AppellF1(1/2,-4/3
,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*
sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2
)+1/7*(3*a^2*C+b^2*(7*A+4*C))*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a
+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a
+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3103, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\,(3a^2C+b^2(7A+4C))\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3\sqrt{2}\,aC(a+b)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*Sqrt[2]*a*(a + b
)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x
]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(3*a^2*C + b^2*
(7*A + 4*C))*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Co
s[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqr
t[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+b\cos(c+dx)}\,\left(A+C\cos^2(c+dx)\right)\,dx = \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{3\int \sqrt[3]{a+b\cos(c-\cdots}{\cdots}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{(3aC)\int(a+b\cos\cdots}{7b^2\cdots}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{(3aC\sin(c+dx))\cdots}{7b^2d\sqrt{\cdots}}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{\left(3a(-a-b)C\sqrt[3]{a\cdots}\right.}{7b^2\cdots}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{3\sqrt{2}\,a(a+b)CF_1\cdots}{\cdots}$$

Mathematica [A]
time = 2.75, size = 276, normalized size = 1.00

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(12a(a^2-b^2)CF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+(28Ab^2-3a^2C+16b^2C)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-4b^2C(a+4b\cos(c+dx))\sin^2(c+dx)\right)}{112b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(12*a*(a^2 - b^2)*C*AppellF1[1/
3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b
)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(
a - b))] + (28*A*b^2 - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a
+ b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Co
s[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c +
 d*x]) - 4*b^2*C*(a + 4*b*Cos[c + d*x])*Sin[c + d*x]^2))/(112*b^3*d)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{1}{3}}\left(A+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
[Out] int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C\cos^2\left(c + dx\right)\right)\sqrt[3]{a + b\cos\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
[Out] Integral((A + C*cos(c + d*x)**2)*(a + b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(c+d\,x\right)^2+A\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

3.205 $\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=274

$$\frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}-\frac{3\sqrt{2}\,aCF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/5*a*C*AppellF1(1/2,-2/3,1/2,3
/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*
x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)+1/5*
(3*a^2*C+b^2*(5*A+2*C))*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2
-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/(a
+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3103, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\,(3a^2C+b^2(5A+2C))\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{a+b\cos(c+dx)}}-\frac{3\sqrt{2}\,aC\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*C*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a +
 b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x
]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2*C + b^2*(5*A + 2
*C))*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x
]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sq
rt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{3\int \frac{\frac{1}{3}b(5A+2C)-aC\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{5b}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{(3aC)\int(a+b\cos(c+dx))^{2/3}\,dx}{5b^2} + \frac{1}{5}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{(3aC\sin(c+dx))\text{Subst}\left(\int \frac{(a+bx)^{2}}{\sqrt{1-x}\,\sqrt{}}\right.}{5b^2d\sqrt{1-\cos(c+dx)}\,\sqrt{1}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\left(3aC(a+b\cos(c+dx))^{2/3}\sin(c+dx\right.}{5b^2d\sqrt{1-\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{3\sqrt{2}\,aCF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+\right.}{5b^2d\sqrt{1+\text{co}}}$$

Mathematica [A]
time = 1.82, size = 256, normalized size = 0.93

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2+3a^2C+2b^2C)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}-6aCF_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-10b^2C\sin^2(c+dx)\right)}{50b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 + 3*a^2*C + 2*b^2*C
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
 + d*x]))/(-a + b)] - 6*a*C*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x
])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 10*b^2
*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{A+C\left(\cos^2\left(dx+c\right)\right)}{\left(a+b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
[Out] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^2{\left(c + dx \right)}}{\sqrt[3]{a + b\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
[Out] Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos(c+d\,x)^2+A}{(a+b\,\cos(c+d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

3.206 $\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=272

$$\frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd}-\frac{3aCF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}}{2\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d-3/4*a*C*AppellF1(1/2,-1/3,1/2,3
/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*
x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)+1/4*
(3*a^2*C+b^2*(4*A+C))*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1
/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)/b^2/d/(a+b*cos(d*x
+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3103, 2835, 2744, 144, 143}

$$\frac{(3a^2C+b^2(4A+C))\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,(a+b\cos(c+dx))^{2/3}}-\frac{3aC\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*a*C*AppellF1[1/2
, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a
+ b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x
]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((3*a^2*C + b^2*(4*A + C))*Appel
lF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b
)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt
[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{3\int \frac{\frac{1}{3}b(4A+C)-aC\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{(3aC)\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{4b^2} + \frac{1}{4}\Big(4$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{(3aC\sin(c+dx))\text{Subst}\Big(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\,\sqrt{1}}}{4b^2d\sqrt{1-\cos(c+dx)}\,\sqrt{1-}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{\Big(3aC\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)\Big)}{4b^2d\sqrt{1-\cos(c+dx)}\,\sqrt{}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{3aC F_1\Big(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),}{2\sqrt{2}\,b^2d\sqrt{1+\cos(c}}$$

Mathematica [A]
time = 2.03, size = 256, normalized size = 0.94

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\,\csc(c+dx)\left(4(4Ab^2+(3a^2+b^2)C)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}+C\left(-3aF_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-4b^2\sin^2(c+dx)\right)\right)}{16b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 + (3*a^2 + b^2)*C)*
AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d
*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c +
 d*x]))/(-a + b)] + C*(-3*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x
])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
 + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*
Sin[c + d*x]^2)))/(16*b^3*d)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{A+C(\cos^2(dx+c))}{(a+b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + C\cos^2{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos{(c+d\,x)}^2+A}{{(a+b\,\cos{(c+d\,x)})}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
[Out] int((A + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

3.207 $\int (a+b\cos(e+fx))^m \left(A - A\cos^2(e+fx)\right) dx$

Optimal. Leaf size=211

$$-\frac{4\sqrt{2}\, A F_1\left(\frac{1}{2};-\frac{3}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\sin(e+}{f\sqrt{1+\cos(e+fx)}}$$

```
[Out] -4*A*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a
+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f
*x+e))^(1/2)+4*A*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*co
s(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))
^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3097, 2834, 144, 143, 2863}

$$\frac{4\sqrt{2}\, A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};-\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}-\frac{4\sqrt{2}\, A\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};-\frac{3}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
```

```
[Out] (-4*Sqrt[2]*A*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Co
s[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[
e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (4*Sqrt[2]*A*AppellF1[1/2, -1
/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*C
os[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x]
)/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3097

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, Int[(a + b*Sin[e + f*x])^m*(1 + S
in[e + f*x]), x], x] + Dist[C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]
)^2, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A + C, 0] &&  !Intege
rQ[2*m]
```

Rubi steps

$$\int (a+b\cos(e+fx))^m \left(A-A\cos^2(e+fx)\right) dx = -\left(A\int (1+\cos(e+fx))^2(a+b\cos(e+fx))^m\, dx\right) + $$

$$= \frac{(A\sin(e+fx))\text{Subst}\left(\int \frac{(1+x)^{3/2}(a+bx)^m}{\sqrt{1-x}}\, dx, x, \cos(e+f\right.}{f\sqrt{1-\cos(e+fx)}\sqrt{1+\cos(e+fx)}}$$

$$= \frac{\left(A(a+b\cos(e+fx))^m\left(-\frac{a+b\cos(e+fx)}{-a-b}\right)^{-m}\sin(e+fx\right.}{f\sqrt{1-\cos(e+fx)}}$$

$$= -\frac{4\sqrt{2}AF_1\left(\frac{1}{2}; -\frac{3}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e-}{a+b}\right.}{f\sqrt{1+\text{c}}}$$

Mathematica [A]
time = 0.39, size = 119, normalized size = 0.56

$$\frac{4AF_1\left(\frac{3}{2}; -\frac{1}{2}, -m; \frac{5}{2}; \sin^2\left(\frac{1}{2}(e+fx)\right), \frac{2b\sin^2\left(\frac{1}{2}(e+fx)\right)}{a+b}\right)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)}(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\sin(e+fx)\tan^2\left(\frac{1}{2}(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2),x]
```

```
[Out] (4*A*AppellF1[3/2, -1/2, -m, 5/2, Sin[(e + f*x)/2]^2, (2*b*Sin[(e + f*x)/2]
^2)/(a + b)]*Sqrt[Cos[(e + f*x)/2]^2]*(a + b*Cos[e + f*x])^m*Sin[e + f*x]*T
an[(e + f*x)/2]^2)/(3*f*((a + b*Cos[e + f*x])/(a + b))^m)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (a+b\cos(fx+e))^m \left(A-A\left(\cos^2(fx+e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

```
[Out] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="maxima")
[Out] -integrate((A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="fricas")
[Out] integral(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(A-A*cos(f*x+e)**2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="giac")
[Out] integrate(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A-A\cos\left(e+f\,x\right)^2\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
[Out] int((A - A*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.208 $\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right)\,dx$

Optimal. Leaf size=285

$$\frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}-\frac{\sqrt{2}\,a(a+b)CF_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(2+m)\sqrt{1+\cos(e}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-a*(a+b)*C*AppellF1(1/2,-1-m,1
/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*
x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)+
(a^2*C+b^2*(C*(1+m)+A*(2+m)))*AppellF1(1/2,-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b
),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((
a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3103, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\sin(e+fx)\left(a^2C+b^2(A(m+2)+C(m+1))\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}-\frac{\sqrt{2}\,aC(a+b)\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a
 + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e
 + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt
[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^
2*(C*(1 + m) + A*(2 + m)))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2
, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2
*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3103

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m
*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a,
 b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^m \left(A+C\cos^2(e+fx)\right) dx = \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \frac{\int (a+b\cos(e+ \cdots}{}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{(aC)\int (a+b\,\mathrm{co} \cdots}{b^2(2 \cdots}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \frac{(aC\sin(e+fx \cdots}{b^2f(2+ \cdots}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\left(a(-a-b)C(\cdots\right.}{}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\sqrt{2}\,a(a+b)C \cdots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10805 vs. $2(285) = 570$.
time = 26.52, size = 10805, normalized size = 37.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{m} \left(A+C\left(\cos^{2}\left(fx+e\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(e+f\,x\right)^2+A\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

```
[Out] int((A + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.209 $\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)$

Optimal. Leaf size=141

$$-\frac{B(a\cos(e+fx))^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{a^2f(2+m)\sqrt{\sin^2(e+fx)}}-\frac{C(a\cos(e+fx))^{3+m}\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\right.}{a^3f(3+m)\sqrt{\sin^2(e}}$$

```
[Out] -B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*si
n(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1/2)-C*(a*cos(f*x+e))^(3+m)*hypergeom(
[1/2, 3/2+1/2*m],[5/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^3/f/(3+m)/(sin(f*x+
e)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3089, 2827, 2722}

$$-\frac{C\sin(e+fx)(a\cos(e+fx))^{m+3}\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(e+fx)\right)}{a^3f(m+3)\sqrt{\sin^2(e+fx)}}-\frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{a^2f(m+2)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -((B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])) - (C*(a
*Cos[e + f*x])^(3 + m)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e +
 f*x]^2]*Sin[e + f*x])/(a^3*f*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx &= \frac{\int (a\cos(e+fx))^{1+m}(B+C\cos(e+fx))\,dx}{a} \\
&= \frac{B\int (a\cos(e+fx))^{1+m}\,dx}{a} + \frac{C\int (a\cos(e+fx))}{a^2} \\
&= -\frac{B(a\cos(e+fx))^{2+m}\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+\right.}{a^2 f(2+m)\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.29, size = 118, normalized size = 0.84

$$
-\frac{\cos(e+fx)(a\cos(e+fx))^m\cot(e+fx)\left(B(3+m)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)+C(2+m)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(e+fx)\right)\right)\sqrt{\sin^2(e+fx)}}{f(2+m)(3+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Cot[e + f*x]*(B*(3 + m)*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] + C*(2 + m)*Cos[e + f*x]*Hyper
geometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^
2])/(f*(2 + m)*(3 + m)))
```

Maple [**F**]
time = 0.25, size = 0, normalized size = 0.00

$$
\int \left(a\cos\left(fx+e\right)\right)^{m}\left(B\cos\left(fx+e\right)+C\left(\cos^{2}\left(fx+e\right)\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m (B+C\cos(e+fx))\cos(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos(e+f\,x))^m\,\left(C\cos(e+f\,x)^2+B\,\cos(e+f\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.210 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,(B\cos(c+dx)+C\cos$

Optimal. Leaf size=167

$$-\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)\sqrt{\sin^2(c+dx)}}-\frac{3C\cos^{3+m}(c$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/6
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x
+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/3+1/2*m],cos(d
*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+10);\frac{1}{6}(3m+16);\cos^2(c+dx)\right)}{d(3m+10)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7
 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeom
etric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\,\int \cos^{\frac{1}{3}+m}(c+dx)}{\sqrt[3]{\cos(}}$$

$$= \frac{\sqrt[3]{b\cos(c+dx)}\,\int \cos^{\frac{4}{3}+m}(c+dx)(}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{\left(B\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+}{\sqrt[3]{\cos(c+dx)}}$$

$$= -\frac{3B\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx}}{}$$

Mathematica [A]
time = 0.41, size = 140, normalized size = 0.84

$$-\frac{3\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\csc(c+dx)\left(C(7+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2};\frac{8}{3}+\frac{m}{2};\cos^2(c+dx)\right)+B(10+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(7+3m)(10+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(7 + 3*m)*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] +
B*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*
x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x
 + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\,(B+C\cos(c+dx))\cos(c+dx)\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(B + C*cos(c + d*x))*cos(c + d*x)*cos(c +
d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*
x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{1/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.211 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=167

$$-\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8+3m);\frac{1}{6}(14+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(8+3m)\sqrt{\sin^2(c+dx)}}-\frac{3C\cos^{3+m}(}{}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7/3+
1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x+
c)^(3+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 11/6+1/2*m],[17/6+1/2*m],cos(
d*x+c)^2)*sin(d*x+c)/d/(11+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+8);\frac{1}{6}(3m+14);\cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}}-\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+11);\frac{1}{6}(3m+17);\cos^2(c+dx)\right)}{d(3m+11)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (8
 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeom
etric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(11 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+d\ldots}{\cos\ldots}$$

$$= \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{5}{3}+m}(c+d\ldots}{\cos^{\frac{2}{3}}(c+d\ldots}$$

$$= \frac{\left(B(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{5}{3}+m}(c\ldots}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= -\frac{3B\cos^{2+m}(c+dx)(b\cos(c+dx\ldots}{\ldots}$$

Mathematica [A]
time = 0.40, size = 140, normalized size = 0.84

$$-\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\left(B(11+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8+3m);\frac{7}{3}+\frac{m}{2};\cos^2(c+dx)\right)+C(8+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(11+3m);\frac{1}{6}(17+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(8+3m)(11+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(B*(11 + 3*m)*
Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2] + C*(8 + 3*m
)*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d
*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(8 + 3*m)*(11 + 3*m))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{2}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*
x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*x
 + c)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*
x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos \left(c+d\,x\right)^m\,{\left(b\,\cos \left(c+d\,x\right)\right)}^{2/3}\,\left(C\,{\cos \left(c+d\,x\right)}^2+B\,\cos \left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.212 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=169

$$-\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{6}(10+3m);\frac{1}{6}(16+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(10+3m)\sqrt{\sin^2(c+dx)}}-\frac{3bC\cos^{4+}}{}$$

```
[Out] -3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1/2*m],[8/
3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)-3*b*C*cos
(d*x+c)^(4+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 13/6+1/2*m],[19/6+1/2*m]
,cos(d*x+c)^2)*sin(d*x+c)/d/(13+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+10);\frac{1}{6}(3m+16);\cos^2(c+dx)\right)}{d(3m+10)\sqrt{\sin^2(c+dx)}}-\frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+4}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+13);\frac{1}{6}(3m+19);\cos^2(c+dx)\right)}{d(3m+13)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2,
(10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqr
t[Sin[c + d*x]^2]) - (3*b*C*Cos[c + d*x]^(4 + m)*(b*Cos[c + d*x])^(1/3)*Hyp
ergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(d*(13 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+}{\sqrt[3]{c}}$$

$$= \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{7}{3}+m}(c+}{\sqrt[3]{\cos(c+}}$$

$$= \frac{\left(bB\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{7}{3}+m}(c}{\sqrt[3]{\cos(c+dx)}}$$

$$= -\frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+}}{}$$

Mathematica [A]
time = 0.57, size = 140, normalized size = 0.83

$$-\frac{3\cos^{2+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\left(B(13+3m)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2};\frac{8}{3}+\frac{m}{2};\cos^2(c+dx)\right)+C(10+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(13+3m);\frac{1}{6}(19+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(10+3m)(13+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(13 + 3*m)*
Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + C*(10 + 3*m)
*Cos[c + d*x]*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(10 + 3*m)*(13 + 3*m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{4}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*
x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3)*c
os(d*x + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*
x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.213 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=167

$$-\frac{3B\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3C\cos^{3+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8\ \ldots\right.}{d(8+3m)\sqrt[3]{\ldots}}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)
*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x
+c)^(3+m)*hypergeom([1/2, 4/3+1/2*m],[7/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(8+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+8);\frac{1}{6}(3m+14);\cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
1/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (8 + 3*m)
/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*(b*Cos[c + d*x
])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\left(B\cos(c+dx)+\right.}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{\frac{2}{3}+m}(c+dx)(B+C\cos(c+dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}} + \frac{\left(C\sqrt[3]{c\right.}{}$$

$$= -\frac{3B\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m)\right.}{d(5+3m)\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2}}$$

Mathematica [A]
time = 0.47, size = 140, normalized size = 0.84

$$-\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(8+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)+C(5+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8+3m);\frac{7}{3}+\frac{m}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(5+3m)(8+3m)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(8 + 3*m)*Hypergeometric2F1[1/2, (
5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 + 3*m)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]
^2])/(d*(5 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^m(dx+c)\right)\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)\cos^m(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
1/3), x)
```

3.214 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{3B\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{1}{6}(10+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}-\frac{3C\cos^{3+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7\right.}{d(7+3m)(b}$$

```
[Out] -3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*x+
c)^(3+m)*hypergeom([1/2, 7/6+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d
/(7+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3C\sin(c+dx)\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
2/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Si
n[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (7 + 3*m)
/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(b*Cos[c + d*x
])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(B\cos(c+dx)+C\right.}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{\frac{1}{3}+m}(c+dx)(B+C\cos(c+dx))}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{\left(B\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}} + \frac{\left(C\cos^{\frac{2}{3}}\right.}{}$$

$$= -\frac{3B\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{1}{6}(10+3m)\right.}{d(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin}}$$

Mathematica [A]
time = 0.45, size = 140, normalized size = 0.84

$$-\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(7+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\cos^2(c+dx)\right)+C(4+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(4+3m)(7+3m)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(7 + 3*m)*Hypergeometric2F1[1/2, (
4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + C*(4 + 3*m)*Cos[c + d*x]*Hypergeom
etric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]
^2])/(d*(4 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^m(dx+c)\right)\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B + C\cos{(c + dx)})\cos{(c + dx)}\cos^{m}{(c + dx)}}{(b\cos{(c + dx)})^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\left(b\,\cos(c+d\,x)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
2/3), x)
```

3.215 $\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=173

$$-\frac{3B\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3C\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5\right.}{bd(5+3m)\sqrt[3]{}}$$

```
[Out] -3*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*
sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*C*cos(d*
x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)
/b/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3C\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{bd(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(
4/3),x]
```

```
[Out] (-3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2]) - (3*C*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m
)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(5 + 3*m)*(b*Cos[c +
d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^m(c+dx)\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\sqrt[3]{\cos(c+dx)}\int\cos^{-\frac{4}{3}+m}(c+dx)\left(B\cos(c+dx)+\right.}{b\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\sqrt[3]{\cos(c+dx)}\int\cos^{-\frac{1}{3}+m}(c+dx)(B+C\cos(c+}{b\sqrt[3]{b\cos(c+dx)}}\\
&= \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int\cos^{-\frac{1}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}}+\frac{\left(C\sqrt[3]{\right.}}{}\\
&= -\frac{3B\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\right.}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2}}
\end{aligned}$$

Mathematica [A]
time = 0.39, size = 140, normalized size = 0.81

$$-\frac{3\cos^{2+m}(c+dx)\csc(c+dx)\left(B(5+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)+C(2+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+3m)(5+3m)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d
*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(5 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^m(dx+c)\right)\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*co
s(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)\cos^{m}(c + dx)}{(b\cos(c + dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3
),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))
**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c
))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
4/3),x)
```

```
[Out] int((cos(c + d*x)^m*(B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))^(
4/3), x)
```

3.216 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos\right.$

Optimal. Leaf size=167

$$-\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2+m+n);\frac{1}{2}(4+m+n);\cos^2(c+dx)\right)\sin(c+dx)}{a^2d(2+m+n)\sqrt{\sin^2(c+dx)}}-\frac{C(a\cos}{}$$

```
[Out] -B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/2*m+1/2*n],[2+
1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin(d*x+c)^2)^(1/2)-C*
(a*cos(d*x+c))^(3+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/2+1/2*m+1/2*n],[5/2
+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^3/d/(3+m+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{C\sin(c+dx)(a\cos(c+dx))^{m+3}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+3);\frac{1}{2}(m+n+5);\cos^2(c+dx)\right)}{a^3d(m+n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(a\cos(c+dx))^{m+2}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+2);\frac{1}{2}(m+n+4);\cos^2(c+dx)\right)}{a^2d(m+n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] -((B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2
+ m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)
*Sqrt[Sin[c + d*x]^2])) - (C*(a*Cos[c + d*x])^(3 + m)*(b*Cos[c + d*x])^n*Hy
pergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(a^3*d*(3 + m + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left((a\cos(c+dx))^{-n}(b\cos(c+dx)\right.$$

$$= \frac{\left((a\cos(c+dx))^{-n}(b\cos(c+dx)\right.}{}$$

$$= \frac{\left(B(a\cos(c+dx))^{-n}(b\cos(c+d\right.}{a}$$

$$= -\frac{B(a\cos(c+dx))^{2+m}(b\cos(c+}{}$$

Mathematica [A]
time = 0.37, size = 136, normalized size = 0.81

$$-\frac{\cos(c+dx)(a\cos(c+dx))^m(b\cos(c+dx))^n\cot(c+dx)\left(B(3+m+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2+m+n);\frac{1}{2}(4+m+n);\cos^2(c+dx)\right)+C(2+m+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3+m+n);\frac{1}{2}(5+m+n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+m+n)(3+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 +
m + n)*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]
 + C*(2 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m
+ n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m + n)*(3 + m + n)))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(dx+c\right)\right)^{m}\left(b\cos\left(dx+c\right)\right)^{n}\left(B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
 + c))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
+ c))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n (B+C\cos(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c
 + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x
 + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cos(c+d\,x))^m\,(b\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(\right.$

Optimal. Leaf size=141

$$-\frac{B(b\cos(c+dx))^{4+n}\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^4d(4+n)\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^{5+n}\,{}_2F_1\left(\frac{1}{2},\frac{5+n}{2};\frac{7+n}{2};\cos^2(\right.}{b^5d(5+n)\sqrt{\sin^2(c+dx}}$$

```
[Out] -B*(b*cos(d*x+c))^(4+n)*hypergeom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(5+n)*hypergeom(
[1/2, 5/2+1/2*n],[7/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^5/d/(5+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3089, 2827, 2722}

$$-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+5}\,{}_2F_1\left(\frac{1}{2},\frac{n+5}{2};\frac{n+7}{2};\cos^2(c+dx)\right)}{b^5d(n+5)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+6}{2};\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x
]
```

```
[Out] -((B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(5 + n)*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^5*d*(5 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{2+n}\left(B\cos(c+dx)\right.}{b^2}$$

$$= \frac{\int (b\cos(c+dx))^{3+n}(B+C\cos(c+}{b^3}$$

$$= \frac{B\int (b\cos(c+dx))^{3+n}\,dx}{b^3} + \frac{C\int (b\,c}{}$$

$$= -\frac{B(b\cos(c+dx))^{4+n}\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6}{}\right.}{b^4 d(4+n)\sqrt{\sin}}$$

Mathematica [A]
time = 0.49, size = 120, normalized size = 0.85

$$-\frac{\cos^3(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(B(5+n)\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c+dx)\right)+C(4+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5+n}{2};\frac{7+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] -((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(5 + n)*Hypergeometric
2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] + C*(4 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(4 + n)*(5 + n)))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^n\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x
)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),
x)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=141

$$-\frac{B(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^{4+n}\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c\right.}{b^4d(4+n)\sqrt{\sin^2(c+dx}}$$

```
[Out] -B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(4+n)*hyperg
eom([1/2, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,
Rules used = {16, 3089, 2827, 2722}

$$-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+6}{2};\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{1+n}\,(B\cos(c+dx)+}{b}$$
$$= \frac{\int (b\cos(c+dx))^{2+n}(B+C\cos(c+}{b^2}$$
$$= \frac{B\int (b\cos(c+dx))^{2+n}\,dx}{b^2} + \frac{C\int (b\cos}{}$$
$$= -\frac{B(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+}{2}\right.}{b^3 d(3+n)\sqrt{\sin^2}}$$

Mathematica [A]
time = 0.30, size = 120, normalized size = 0.85

$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n \cot(c+dx)\left(B(4+n)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)+C(3+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(4 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(3 + n)*(4 + n)))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)\,(b\cos(c+dx))^n\left(C\cos(c+dx)^2+B\cos(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.219 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=141

$$-\frac{B(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(3+n)*hypergeom(
[1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3089, 2827, 2722}

$$-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b
*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
```

```
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right) dx = \frac{\int (b\cos(c+dx))^{1+n}(B+C\cos(c+dx))\,dx}{b}$$
$$= \frac{B\int (b\cos(c+dx))^{1+n}\,dx}{b} + \frac{C\int (b\cos(c+dx))^{2-}}{b^2}$$
$$= -\frac{B(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+c\right.}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.25, size = 118, normalized size = 0.84

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\left(B(3+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)+C(2+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(2 + n)*(3 + n)))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos (c+dx))^n (B+C\cos (c+dx))\cos (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\cos (c+d\,x))^n\,\left(C\cos (c+d\,x)^2+B\,\cos (c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.220 $\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx$

Optimal. Leaf size=141

$$-\frac{B(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(2+n)*hypergeo
m([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {16, 3089, 2827, 2722}

$$-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x],x]
```

```
[Out] -((B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (C*(b*C
os[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d
*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{-1+n}\left(B\cos(c+dx)\right.$$
$$= \int (b\cos(c+dx))^n(B+C\cos(c+dx))$$
$$= B\int (b\cos(c+dx))^n\,dx + \frac{C\int (b\cos(c}{}$$
$$= -\frac{B(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\right.}{bd(1+n)\sqrt{\sin^2(c}}$$

Mathematica [A]
time = 0.18, size = 112, normalized size = 0.79

$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\left(B(2+n)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)+C(1+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(2 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n
)*(2 + n)))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n (B+C\cos(c+dx))\cos(c+dx)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))*cos(c + d*x)*sec(c + d*x)
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),x
)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x),
x)
```

3.221 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=132

$$-\frac{B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+\right.}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+
c)/d/n/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*
n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3089, 2827, 2722}

$$-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x
]
```

```
[Out] -((B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (C*(b*Cos[c + d*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int (b\cos(c+dx))^{-2+n}\left(B\cos(c+dx\right.$$

$$= b\int (b\cos(c+dx))^{-1+n}(B+C\cos(c+$$

$$= (bB)\int (b\cos(c+dx))^{-1+n}\,dx + C\int$$

$$= -\frac{B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos\right.}{dn\sqrt{\sin^2(c+dx}}$$

Mathematica [A]
time = 0.21, size = 109, normalized size = 0.83

$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\left(B(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)+Cn\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{dn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(B*(1 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n)
))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n\left(C\cos(c+dx)^2+B\cos(c+dx)\right)}{\cos(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^2
,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^2
, x)
```

3.222 $\int (b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec$ $dx)\,dx$

Optimal. Leaf size=131

$$\frac{bB(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{C(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos\right.}{dn\sqrt{\sin^2(c+}}$$

```
[Out] b*B*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c
)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)-C*(b*cos(d*x+c))^n*hypergeom([
1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {16, 3089, 2827, 2722}

$$\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{C\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x
]
```

```
[Out] (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (C*(b*Co
s[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int (b\cos(c+dx))^{-3+n}\left(B\cos(c+dx\right.$$
$$= b^2\int (b\cos(c+dx))^{-2+n}(B+C\cos(c$$
$$= \left(b^2B\right)\int (b\cos(c+dx))^{-2+n}\,dx + (bC$$
$$= \frac{bB(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+\right.}{d(1-n)\sqrt{\sin}}$$

Mathematica [A]
time = 0.19, size = 109, normalized size = 0.83

$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\left(Bn\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)+C(-1+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-1+n)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(B*n*Hypergeometric2F1[1/2, (-1
 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]*Hypergeometri
c2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n
)*n))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n \left(C\cos(c+dx)^2 + B\cos(c+dx)\right)}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^3
,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^3
, x)
```

3.223 $\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\mathrm{se}$ $dx)\,dx$

Optimal. Leaf size=139

$$\frac{b^2B(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bC(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1\right.}{d(1-n)\sqrt{\mathrm{s}}}$$

```
[Out] b^2*B*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*C*(b*cos(d*x+c))^(-1+n)*hypergeo
m([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c
)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3089, 2827, 2722}

$$\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x
]
```

```
[Out] (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*C*(b*Cos[
c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int (b\cos(c+dx))^{-4+n}\left(B\cos(c+dx\right.$$

$$= b^3\int (b\cos(c+dx))^{-3+n}(B+C\cos(c$$

$$= \left(b^3B\right)\int (b\cos(c+dx))^{-3+n}\,dx + \left(b^2C\right.$$

$$= \frac{b^2B(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+\right.}{d(2-n)\sqrt{\sin^2}}$$

Mathematica [A]
time = 0.18, size = 118, normalized size = 0.85

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\left(B(-1+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)+C(-2+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{d(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^
2])/(d*(-2 + n)*(-1 + n)))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{n}\left(B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\left(\sec^{4}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c
)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x +
c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n\left(C\cos(c+dx)^2+B\cos(c+dx)\right)}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^4
,x)

[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^4
, x)
```

3.224 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2\right.$

Optimal. Leaf size=163

$$-\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(9+2n);\frac{1}{4}(13+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(9+2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\cos^{\frac{11}{2}}(c+d\ldots}{}$$

```
[Out] -2*B*cos(d*x+c)^(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^
(11/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 11/4+1/2*n],[15/4+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(11+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+9);\frac{1}{4}(2n+13);\cos^2(c+dx)\right)}{d(2n+9)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+dx)\cos^{\frac{11}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+11);\frac{1}{4}(2n+15);\cos^2(c+dx)\right)}{d(2n+11)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (9 + 2*n
)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*C*Cos[c + d*x]^(11/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/
2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(11 + 2*n)*
Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \mathrm{co}\\ &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \mathrm{co}\\ &= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int\\ &= -\frac{2B\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F}{d(9}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 138, normalized size = 0.85

$$-\frac{2\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(11+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(9+2n);\frac{1}{4}(13+2n);\cos^2(c+dx)\right)+C(9+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(11+2n);\frac{1}{4}(15+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(9+2n)(11+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(11 + 2*n)*Hyperg
eometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2] + C*(9 + 2*n)*C
os[c + d*x]*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(9 + 2*n)*(11 + 2*n))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{5}{2}}(dx+c)\right)(b\cos(dx+c))^n\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{5/2}\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.225 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2\right.$

Optimal. Leaf size=163

$$-\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\cos^{\frac{9}{2}}(c+dx}{}$$

```
[Out] -2*B*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2
*n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^
(9/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 9/4+1/2*n],[13/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(9+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+9);\frac{1}{4}(2n+13);\cos^2(c+dx)\right)}{d(2n+9)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n
)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*C*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2
, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \mathrm{co}$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \mathrm{co}$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int$$

$$= -\frac{2B\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F}{d(7}$$

Mathematica [A]
time = 0.43, size = 138, normalized size = 0.85

$$-\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(9+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)+C(7+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(9+2n);\frac{1}{4}(13+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(7+2n)(9+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(9 + 2*n)*Hyperge
ometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*Co
s[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2
),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x +
c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.226 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,(B\cos(c+dx)+C$

Optimal. Leaf size=163

$$-\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\cos^{\frac{7}{2}}(c+dx)}{}$$

```
[Out] -2*B*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^(
7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (-2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n
)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \\ &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \\ &= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right) \\ &= -\frac{2B\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 138, normalized size = 0.85

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(7+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)+C(5+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(5+2n)(7+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(7 + 2*n)*Hyperge
ometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]
)*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sqrt{\cos}\,(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d
*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*
x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
, algorithm="giac")
```

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2),x)

[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2), x)

3.227 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\cos^{\frac{5}{2}}(c+dx)}{}$$

```
[Out] -2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*C*cos(d*x+c)^(
5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d
*x]],x]
```

```
[Out] (-2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n
)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+d$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+$$

$$= -\frac{2B\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n\right.}{d(3+2n)\sqrt{\sin^2(}}$$

Mathematica [A]
time = 0.26, size = 138, normalized size = 0.85

$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\left(B(5+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)+C(3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(3+2n)(5+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(5 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d
*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2
),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d
*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)\right)}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
1/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
1/2), x)
```

3.228 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2B\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}-\frac{2C\cos^{\frac{3}{2}}(c+dx}{}$$

```
[Out] -2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(d*x+c
))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
3/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n
)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx \\ &= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx \\ &= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+ \\ &= -\frac{2B\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+\right.}{d(1+2n)\sqrt{\sin^2(}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\csc(c+dx)\left(B(3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)+C(1+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(1+2n)(3+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(3 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n (B + C\cos(c+dx))}{\sqrt{\cos(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n\left(C\cos(c+dx)^2+B\cos(c+dx)\right)}{\cos(c+dx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
3/2), x)
```

3.229 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+d}{}$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*C*(b*cos(d*x+c
))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
5/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) - (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+$$

$$= \frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\right.}{d(1-2n)\sqrt{\cos(c+dx)}\,\sqrt{\sin^2(}}$$

Mathematica [A]
time = 0.27, size = 133, normalized size = 0.82

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)+C(-1+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d\left(-1+4n^2\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\,(B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n\,(B+C\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2
),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(B + C*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
5/2), x)
```

3.230 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}+\frac{2C(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-\right.}{d(1-2n)\sqrt{}}$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*x+c
))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-
2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}+\frac{2C\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\,\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
7/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 +
 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+$$

$$= \frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\right.}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c}}$$

Mathematica [A]
time = 0.23, size = 138, normalized size = 0.85

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(-1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)+C(-3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-3+2n)(-1+2n)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + 2*n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\left(B\cos(dx+c)+C(\cos^2(dx+c))\right)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n \left(C\cos(c+dx)^2+B\cos(c+dx)\right)}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
7/2), x)
```

3.231 $\int \frac{(b\cos(c+dx))^n\left(B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}+\frac{2C(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}\right.}{d(3-2n)}$$

```
[Out] 2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*C*(b*cos(d*x+
c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3
-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {20, 3089, 2827, 2722}

$$\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}+\frac{2C\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(
9/2),x]
```

```
[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1
+ 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3089

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] +
(C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x
])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\,(B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx$$

$$= \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx$$

$$= \left(B\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+$$

$$= \frac{2B(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n\right.}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(}}$$

Mathematica [A]
time = 0.24, size = 138, normalized size = 0.85

$$-\frac{2(b\cos(c+dx))^n\csc(c+dx)\left(B(-3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)+C(-5+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\right)\sqrt{\sin^2(c+dx)}}{d(-5+2n)(-3+2n)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n\,(B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(9/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2
),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x +
c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^n\,\left(C\cos(c+dx)^2+B\cos(c+dx)\right)}{\cos(c+dx)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*x)^(
9/2), x)
```

3.232 $\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx\right.$

Optimal. Leaf size=173

$$-\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)}+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\frac{2^{\frac{1}{2}+m}(Bm(2+}{}$$

```
[Out] -(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e))
^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1))*(1+cos(f*x+e)
)^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*cos(f*x+
e))*sin(f*x+e)/f/(m^2+3*m+2)
```

Rubi [A]
time = 0.15, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3102, 2830, 2731, 2730}

$$\frac{2^{m+\frac{1}{2}}(Bm(m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx))\right)}{f(m+1)(m+2)}-\frac{(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m))
) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 +
 m)*(B*m*(2 + m) + C*(1 + m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Co
s[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Si
n[e + f*x])/(f*(1 + m)*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx &= \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\frac{\int(a\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e}{f(1+m)(2+m)}\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e}{f(1+m)(2+m)}\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e}{f(1+m)(2+m)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 44.93, size = 356, normalized size = 2.06

$$\frac{i4^{-1-m}e^{-2i(e+fx)}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m\left(Cm(2-m-2m^2+m^3)\,{}_2F_1\left(-2-m,-2m;-1-m;-e^{i(e+fx)}\right)+e^{i(e+fx)}(2+m)\left(2Bm(2-3m+m^2)\,{}_2F_1\left(-1-m,-2m;-m;-e^{i(e+fx)}\right)+e^{i(e+fx)}(1+m)\left(2Be^{i(e+fx)}(-2+m)m\,{}_2F_1\left(1-m,-2m;2-m;-e^{i(e+fx)}\right)+C(-1+m)\left(e^{2i(e+fx)}m\,{}_2F_1\left(2-m,-2m;3-m;-e^{i(e+fx)}\right)+2(-2+m)\,{}_2F_1\left(-2m,-m;1-m;-e^{i(e+fx)}\right)\right)\right)\right)\right)}{f(-2+m)(-1+m)m(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Cos
[e + f*x]))^m*(C*m*(2 - m - 2*m^2 + m^3)*Hypergeometric2F1[-2 - m, -2*m, -1
 - m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(2 + m)*(2*B*m*(2 - 3*m + m^2)*Hy
pergeometric2F1[-1 - m, -2*m, -m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(1 +
m)*(2*B*E^(I*(e + f*x))*(-2 + m)*m*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E
^(I*(e + f*x))] + C*(-1 + m)*(E^((2*I)*(e + f*x))*m*Hypergeometric2F1[2 - m
, -2*m, 3 - m, -E^(I*(e + f*x))] + 2*(-2 + m)*Hypergeometric2F1[-2*m, -m, 1
 - m, -E^(I*(e + f*x))])))))/(E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))^(2*
m)*f*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*Cos[(e + f*x)/2]^(2*m))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (a+a\cos (fx+e))^{m} \left(B\cos (fx+e)+C\left(\cos^{2}(fx+e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
[Out] int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos (e+fx)+1))^{m} (B+C\cos (e+fx))\cos (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
[Out] Integral((a*(cos(e + f*x) + 1))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)\right)\,\left(a+a\,\cos\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m,x)
```

```
[Out] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + a*cos(e + f*x))^m, x)
```

3.233 $\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx\right.$

Optimal. Leaf size=295

$$\frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}-\frac{\sqrt{2}\,(a+b)(aC-bB(2+m))F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)\right.}{b^2f(2+m)\sqrt{1-}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-(a+b)*(a*C-b*B*(2+m))*AppellF
1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+
e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+(a^2*C+b^2*C*(1+m)-a*b*B*(2+m))*AppellF1(1/2,-m,1/2,3/2,b*(1-co
s(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/b
^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\sin(e+fx)\left(a^2C-abB(m+2)+b^2C(m+1)\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}-\frac{\sqrt{2}(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}}+\frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
 b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1
 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*
Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(
a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^m \left(B\cos(e+fx)+C\cos^2(e+fx)\right) dx = \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \cdots$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \cdots$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \cdots$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} + \cdots$$
$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 13441 vs. $2(295) = 590$.
time = 26.66, size = 13441, normalized size = 45.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
[Out] Result too large to show
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{m} \left(B\cos\left(fx+e\right)+C\left(\cos^{2}\left(fx+e\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
[Out] int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(e+f\,x\right)^2+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m,x)
```

```
[Out] int((B*cos(e + f*x) + C*cos(e + f*x)^2)*(a + b*cos(e + f*x))^m, x)
```

3.234 $\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx\right.$

Optimal. Leaf size=284

$$\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{(a+b)(8bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\,\left(\frac{a+b\cos(c+dx}{a+b}\right.}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(a+b)*(8*B*b-3*C*a)*AppellF
1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*
x+c))^(1/2)-1/8*(8*B*a*b-3*C*a^2-5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-co
s(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d
/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$-\frac{(-3a^2C+8abB-5b^2C)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - ((8*a*b*B - 3*
a^2*C - 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[
2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^{2/3}\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{3\int \cdots}{\cdots} \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(8b \cdots}{\cdots} \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} - \frac{((8 \cdots}{\cdots} \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{((- \cdots}{\cdots} \\ &= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} + \frac{(a \cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 3.04, size = 290, normalized size = 1.02

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-a^2+b^2)(8bB-3aC)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+(16abB-6a^2C+25b^2C)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}(a+b\cos(c+dx))-5b^2(8bB+2aC+5bC\cos(c+dx))\sin^2(c+dx)\right)}{200b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-a^2 + b^2)*(8*b*B - 3*a*C)
*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (16*a*b*B - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1
/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*
(a + b*Cos[c + d*x]) - 5*b^2*(8*b*B + 2*a*C + 5*b*C*Cos[c + d*x])*Sin[c + d
*x]^2))/(200*b^3*d)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{2}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3),x)
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(2/3), x)
```

3.235 $\int \sqrt[3]{a+b\cos(c+dx)}\,(B\cos(c+dx)+C\cos^2(c+d$

Optimal. Leaf size=284

$$\frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}+\frac{\sqrt{2}\,(a+b)(7bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx}{a+b}\right.}{7b^2d\sqrt{1+\cos(c+dx)}\,\sqrt[3]{\frac{a+b\cos(c+}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*(a+b)*(7*B*b-3*C*a)*AppellF
1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*
x+c))^(1/2)-1/7*(7*B*a*b-3*C*a^2-4*C*b^2)*AppellF1(1/2,-1/3,1/2,3/2,b*(1-co
s(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/
2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$-\frac{\sqrt{2}\,(-3a^2C+7abB-4b^2C)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}\,(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7
*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - C
os[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(7*a*
b*B - 3*a^2*C - 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2
, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/
(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+b\cos(c+dx)}\,\left(B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{3\int \sqrt[3]{}}{}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{(7bB}{}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \frac{((7bB}{}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\left((-a\right.}{}$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \frac{\sqrt{2}\,(a}{}$$

Mathematica [A]
time = 2.98, size = 289, normalized size = 1.02

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\,\csc(c+dx)\left(4(-a^2+b^2)\,(7bB-3aC)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+(7abB-3a^2C+16b^2C)\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}\,(a+b\cos(c+dx))-4b^2(7bB+aC+4bC\cos(c+dx))\sin^2(c+dx)\right)}{112b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/
2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-(
(b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*(
a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*x])*Sin[c + d*x]
^2))/(112*b^3*d)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{1}{3}}\left(B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (B + C\cos(c + dx))\sqrt[3]{a + b\cos(c + dx)}\cos(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)\right)\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3),x)
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)*(a + b*cos(c + d*x))^(1/3), x)
```

3.236 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=281

$$\frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{\sqrt{2}\,(5bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a}{5b^2d\sqrt{1+\cos(c+dx)}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*(5*B*b-3*C*a)*AppellF1(1/2,
-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2
/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^
(1/2)-1/5*(5*B*a*b-3*C*a^2-2*C*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1
/2)/b^2/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$-\frac{\sqrt{2}\,(-3a^2C+5abB-2b^2C)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\,\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}\,(5bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*(5*a*b*B - 3*
a^2*C - 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])
/(5*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{3\int \frac{\frac{2bC}{3}+\frac{1}{3}(5bB-3aC)\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}} \cdots}{5b}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{(5bB-3aC)\int(a+b\cos(c \cdots}{5b^2}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)\sin(c+dx)) \cdots}{5b^2d\sqrt{1-\cdots}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)(a+b\cos(c \cdots}{5b^2d\sqrt{1-\cdots}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\sqrt{2}\,(5bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},\cdots\right.}{\cdots}$$

Mathematica [A]
time = 2.20, size = 263, normalized size = 0.94

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(-5abB+3a^2C+2b^2C)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+2(5bB-3aC)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-10b^2C\sin^2(c+dx)\right)}{50b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-5*a*b*B + 3*a^2*C + 2*b^2*
C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c
+ d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Co
s[c + d*x]))/(a - b))] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (a
+ b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Co
s[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c +
 d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+C(\cos^2(dx+c))}{(a+b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B + C\cos(c + dx))\cos(c + dx)}{\sqrt[3]{a + b\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

3.237 $\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=281

$$\frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd}+\frac{(4bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}}{2\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2,
-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1
/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^
(1/2)-1/4*(4*B*a*b-3*C*a^2-C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))
/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)/b^2/d/
(a+b*cos(d*x+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3102, 2835, 2744, 144, 143}

$$-\frac{(-3a^2C+4abB-b^2C)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,(a+b\cos(c+dx))^{2/3}}+\frac{(4bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*Ap
pellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a
 + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - ((4*a*b*B - 3*a^2*C -
b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2
]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{3\int \frac{\frac{bC}{3}+\frac{1}{3}(4bB-3aC)\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx}{4b}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{(4bB-3aC)\int \sqrt[3]{a+b\cos(c}{4b^2}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{((4bB-3aC)\sin(c+dx))\mathrm{Su}}{4b^2d\sqrt{1-\cos}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{\Big((4bB-3aC)\sqrt[3]{a+b\cos(c+}}{4b^2d\sqrt{1-}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{(4bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}\right.}{2\sqrt{2}\,b}$$

Mathematica [**A**]
time = 2.18, size = 261, normalized size = 0.93

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\,\csc(c+dx)\left(4(-4abB+3a^2C+b^2C)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+(4bB-3aC)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-4b^2C\sin^2(c+dx)\right)}{16b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-4*a*b*B + 3*a^2*C + b^2*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*
Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c
+ d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x
]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+C(\cos^2(dx+c))}{(a+b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B + C\cos{(c + dx)})\cos{(c + dx)}}{(a + b\cos{(c + dx)})^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)/(a + b*cos(c + d*x))**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
[Out] int((B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

3.238 $\int (a\cos(e+fx))^m (A + B\cos(e+fx) + C\cos^2(e+f$

Optimal. Leaf size=187

$$\frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}-\frac{(C(1+m)+A(2+m))(a\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+f\right.}{af(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] C*(a*cos(f*x+e))^(1+m)*sin(f*x+e)/a/f/(2+m)-(C*(1+m)+A*(2+m))*(a*cos(f*x+e)
)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a/f
/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-B*(a*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+
1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/a^2/f/(2+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3102, 2827, 2722}

$$-\frac{B\sin(e+fx)(a\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{a^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{(A(m+2)+C(m+1))\sin(e+fx)(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)(m+2)\sqrt{\sin^2(e+fx)}}+\frac{C\sin(e+fx)(a\cos(e+fx))^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) - ((C*(1 + m) + A*(
2 + m))*(a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2])
 - (B*(a*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{\int (a\cos}{}$$

$$= \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} + \frac{B\int (a\,c}{}$$

$$= \frac{C(a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)} - \frac{\left(A+\right.}{}$$

Mathematica [A]
time = 0.31, size = 142, normalized size = 0.76

$$-\frac{\cos(e+fx)(a\cos(e+fx))^m\sin(e+fx)\left(\left(C(1+m)+A(2+m)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)+(1+m)\left(B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(e+fx)\right)-C\sqrt{\sin^2(e+fx)}\right)\right)}{f(1+m)(2+m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Sin[e + f*x]*((C*(1 + m) + A*(2 + m))*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2] + (1 + m)*(B*Cos
[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] - C*
Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m \left(A+B\cos(fx+e)+C\left(\cos^2(fx+e)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
[Out] int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\cos\left(e+f\,x\right)\right)}^m\,\left(C\,{\cos\left(e+f\,x\right)}^2+B\,\cos\left(e+f\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.239 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=209

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\,\sin}{21d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^(5/
2)*sin(d*x+c)/b^2/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^3/d+10/21*b*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[C
os[c + d*x]]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d
*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*B*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c
 + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,(A+B\cos(c}{b^2}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin}{45bd}$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\big(\tfrac{1}{2}}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\big(\tfrac{1}{2}}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 1.12, size = 125, normalized size = 0.60

$$\frac{\sqrt{b\cos(c+dx)}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(c+dx)\right)}{630d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*Ell
ipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x] +
 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(6
30*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.42, size = 382, normalized size = 1.83

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1120*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126
*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 191, normalized size = 0.91

$$\frac{-75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\sqrt{2}(-9iA-7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}(9iA+7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35C\cos(dx+c)^3+45B\cos(dx+c)^2+7(9A+7C)\cos(dx+c)+75B\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
```

```
+ c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.240 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=180

$$\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+}}{21d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/b^2/d+2/21*b*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2b(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*S
qrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2
1*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*C*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c\right.}{b}$$
$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$
$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$
$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin}{21d}$$
$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin}{21d}$$
$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.84, size = 111, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d
*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.41, size = 351, normalized size = 1.95

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 177, normalized size = 0.98

$$-\frac{5\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-63i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+63i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(15\,C\cos(dx+c)^2+21\,B\cos(dx+c)+35\,A+25\,C\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(
d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.241 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+d$

Optimal. Leaf size=145

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\,\sin(c}{3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/3*b*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5*(5*A+
3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Co
s[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2\int\sqrt{b\cdots}}{}$$
$$=\frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{B\int(b\cos\cdots}{}$$
$$=\frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2C(b\cos\cdots}{}$$
$$=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{5d\sqrt{\cos(c+dx)}}$$
$$=\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.38, size = 94, normalized size = 0.65

$$\frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c +
 d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.40, size = 317, normalized size = 2.19

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(24*C*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/
2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 163, normalized size = 1.12

$$\frac{-5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(5i\,A+3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3\,C\cos(dx+c)+5\,B)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.242 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\frac{2b(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.24, size = 83, normalized size = 0.74

$$\frac{b\left(6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x],x]
```

```
[Out] (b*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [**A**]
time = 0.39, size = 283, normalized size = 2.53

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\; b\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{2\,(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 149, normalized size = 1.33

$$\frac{\sqrt{2}\,(-3i\,A-i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,A+i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,\sqrt{b\cos(dx+c)}\,C\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos{(c+dx)}}\left(A+B\cos{(c+dx)}+C\cos^2{(c+dx)}\right)\sec{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(
c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.243 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))\,dx$

Optimal. Leaf size=109

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d
/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^2,x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c +
 d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)+C\,\mathrm{co}}{(b\cos(c+dx))^{3/}}$$

$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b}{\sqrt{b}}}{}$$

$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + (bB)\int \frac{}{\sqrt{}}$$

$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(bB\sqrt{\cos}\right.}{}$$

$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E(}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.31, size = 78, normalized size = 0.72

$$\frac{2b\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A]
time = 0.40, size = 260, normalized size = 2.39

method	result
default	$\frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\ \left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\right.}\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 180, normalized size = 1.65

$$\frac{-i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}B\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\sqrt{2}(-iA+iC)\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\sqrt{2}(iA-iC)\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}A\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.244 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+d$ $dx)\,dx$

Optimal. Leaf size=140

$$-\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\cdots}{d\cdots}$$

```
[Out] 2/3*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b*B*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)+2/3*b*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^3,x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2
*b*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos^{2}}{(b\cos(c+dx))^{5/2}}$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2}{3}\int \frac{\frac{3b^2B}{2}}{}$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + (b^2B)\int$$

$$= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2bB\sin(}{d\sqrt{b\cos(}}$$

$$= \frac{2b(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{3d\sqrt{b\cos(c+dx)}}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]

time = 0.42, size = 90, normalized size = 0.64

$$\frac{2b\left(-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3B\cos(c+dx))\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^3,x]
```

```
[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]
))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(176) = 352$.

time = 0.71, size = 505, normalized size = 3.61

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 199, normalized size = 1.42

$$\frac{\sqrt{2}\,(-iA-3iC)\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(iA+3iC)\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3i\,\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3i\,\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,(3\,B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.245 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+$ $dx)\,dx$

Optimal. Leaf size=181

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^2*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^4,x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[C
os[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2))
+ (2*b^2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*(3*A + 5*C)*Si
n[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx &= b^4\int \frac{A+B\cos(c+dx)+C\operatorname{co}}{(b\cos(c+dx))^{7/}} \\ &= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}(2b)\int \\ &= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\left(b^3B\right)\int \\ &= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2B\,\mathrm{s}}{3d(b\cos} \\ &= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2B\,\mathrm{s}}{3d(b\cos} \\ &= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E}{5d\sqrt{\cos(c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.64, size = 122, normalized size = 0.67

$$-\frac{\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\sin(c+dx)-9A\sin(2(c+dx))-15C\sin(2(c+dx))-6A\tan(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] -1/15*(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(6*(3*A + 5*C)*Cos[c + d*x]^(3/2
)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2
, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] - 6
*A*Tan[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(209) = 418$.
time = 0.93, size = 804, normalized size = 4.44

method	result	size
default	Expression too large to display	804

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/
2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3
6*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-120*
C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin
(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 220, normalized size = 1.22

$$\frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 3\sqrt{2}(3iA+5iC)\sqrt{b}\cos(dx+c)^3 \,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3\sqrt{2}(-3iA-5iC)\sqrt{b}\cos(dx+c)^3 \,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(3(3A+5C)\cos(dx+c)^2+5B\cos(dx+c)+3A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.246 $\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+d$ $dx)\,dx$

Optimal. Leaf size=210

$$-\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+$$

```
[Out] 2/7*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^3*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^2*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b*B*s
in(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6bB\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*
x]^5,x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*
Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2))
 + (2*b^3*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*(5*A + 7*C)
*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b*B*Sin[c + d*x])/(5*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)+C\cos^2}{(b\cos(c+dx))^{9/2}}$$

$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}(2b^2)\int$$

$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + (b^4B)\int$$

$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3B\sin}{5d(b\cos(}$$

$$= \frac{2Ab^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^3B\sin}{5d(b\cos(}$$

$$= \frac{2b(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{21d\sqrt{b\cos(c+dx)}}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 1.04, size = 143, normalized size = 0.68

$$\frac{2\sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\left(-63B\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+21B\sin(c+dx)+63B\cos^2(c+dx)\sin(c+dx)+\frac{25}{2}A\sin(2(c+dx))+\frac{35}{2}C\sin(2(c+dx))+15A\tan(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^5,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(-63*B*Cos[c + d*x]^(5/2)*EllipticE[
(c + d*x)/2, 2] + 5*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2
] + 21*B*Sin[c + d*x] + 63*B*Cos[c + d*x]^2*Sin[c + d*x] + (25*A*Sin[2*(c +
 d*x)])/2 + (35*C*Sin[2*(c + d*x)])/2 + 15*A*Tan[c + d*x]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(234) = 468.
time = 1.19, size = 725, normalized size = 3.45

method	result	size
default	Expression too large to display	725

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(1/5*B/b/sin
(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+s
in(1/2*d*x+1/2*c)^2*b)^(1/2)+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c)/
b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1
))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 231, normalized size = 1.10

$$-\frac{5\sqrt{2}\,(5i\,A+7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\,(-5i\,A-7i\,C)\sqrt{b}\,\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+63i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-63i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\left(63\,B\cos\left(dx+c\right)^{3}+5\,(5\,A+7\,C)\cos\left(dx+c\right)^{2}+21\,B\cos\left(dx+c\right)+15\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
```

```
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(
d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.247 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=210

$$\frac{2b(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10bB\sqrt{b\cos(c+dx}}{21d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)
*sin(d*x+c)/b/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d+10/21*b^2*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*b*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{2b(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}+\frac{10bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt
[Cos[c + d*x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
21*d*Sqrt[b*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin
[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos\right.}{b}$$
$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx}{9b^2d}$$
$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx}{9b^2d}$$
$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\,\mathrm{s}}{45d}$$
$$= \frac{10bB\sqrt{b\cos(c+dx)}\,\sin(c+}{21d}$$
$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}\,E}{15d\sqrt{\cos(c+dx}}$$
$$= \frac{2b(9A+7C)\sqrt{b\cos(c+dx)}\,E}{15d\sqrt{\cos(c+dx}}$$

Mathematica [A]

time = 0.71, size = 128, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(c+dx)\right)}{630bd\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x]
 + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/
(630*b*d*Cos[c + d*x]^(5/2))
```

Maple [A]

time = 0.39, size = 384, normalized size = 1.83

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*co
s(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 195, normalized size = 0.93

$$\frac{-75i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}\,(9A+7C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,(9A+7C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35Cb\cos(dx+c)^3+45Bb\cos(dx+c)^2+7(9A+7C)b\cos(dx+c)+75Bb\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt
(2)*(9*A + 7*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(35*C*b*cos(d*x + c)^3 + 45*B*b*cos(d*x
```

```
+ c)^2 + 7*(9*A + 7*C)*b*cos(d*x + c) + 75*B*b)*sqrt(b*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*co
s(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.248 $\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+d$

Optimal. Leaf size=181

$$\frac{6bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sqrt{b\cos(c}{21d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*x+
c)/b/d+2/21*b^2*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2
)+2/21*b*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*b*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2b^2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*
x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{2\int(b\,\text{c}$$

$$=\frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}+\frac{B\int(b\,\text{c}}{}$$

$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}$$

$$=\frac{2b(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}$$

$$=\frac{6bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b}{}$$

Mathematica [A]
time = 0.11, size = 108, normalized size = 0.60

$$\frac{(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))

Maple [**A**]
time = 0.36, size = 353, normalized size = 1.95

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNVERBOSE)

[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 183, normalized size = 1.01

$$\frac{-5i\sqrt{2}\,(7A+5C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,(7A+5C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}\,Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}\,Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(15Cb\cos(dx+c)^2+21Bb\cos(dx+c)+5(7A+5C)b\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*C)*b^(3/2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*B*b^(3/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6
3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(15*C*b*cos(d*x + c)^2 + 21*B*b*cos(d*x +
c) + 5*(7*A + 5*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.249 $\int (b\cos(c+dx))^{3/2}\,(A + B\cos(c+dx) + C\cos^2(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{2b(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5*b*(
5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2b(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[
Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*
d*Sqrt[b*Cos[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) +
 (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \sqrt{b\cos(c+dx)}\,(A+B\cos($$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{2bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}\,E(}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b(5A+3C)\sqrt{b\cos(c+dx)}\,E(}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.27, size = 95, normalized size = 0.65

$$\frac{2b\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c
 + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.35, size = 319, normalized size = 2.18

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(24*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 165, normalized size = 1.13

$$\frac{-5i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}\,Bb^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3i\sqrt{2}\,(5A+3C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\sqrt{2}\,(5A+3C)b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,(3\,Cb\cos(dx+c)+5\,Bb)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(
5*A + 3*C)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*C*b*cos(d*x + c) + 5*B*b)*sqrt(b*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.250 $\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*C*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\frac{2b^2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2bC\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.18, size = 85, normalized size = 0.73

$$\frac{b^2\left(6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (b^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*C
os[c + d*x]])
```

Maple [A]
time = 0.35, size = 285, normalized size = 2.46

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{2}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 148, normalized size = 1.28

$$\frac{-i\sqrt{2}\,(3A+C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(3A+C)b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,Bb^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Cb\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(3/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*
B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*b*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.251 $\int (b\cos(c+dx))^{3/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=114

$$-\frac{2b(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*b*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (-2*b*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c
 + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{3/2}}$$

$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + 2\int \frac{\frac{b^2B}{2} -}{}$$

$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \left(b^2B\right)\int$$

$$= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(b^2B\sqrt{\cos}\right.}{}$$

$$= -\frac{2b(A-C)\sqrt{b\cos(c+dx)}\,E(}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 0.70

$$\frac{2b^2\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,F\left(\tfrac{1}{2}(c+dx)\middle|2\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c +
d*x]])
```

Maple [A]
time = 0.44, size = 262, normalized size = 2.30

method	result
default	$\frac{2b^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\mathrm{si}\right.}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 179, normalized size = 1.57

$$\frac{-i\sqrt{2}\,Bb^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,Bb^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(A-C)b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Ab\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + I*sqrt(2)*(A - C)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x
 + c))*A*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.252 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c + dx) + C\cos^2(c + dx) dx$

Optimal. Leaf size=145

$$-\frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^2*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*b^2*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/
2)-2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
+ (2*b^2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2} \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^5}$$

$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{3}(2b)$$

$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + (b^3B)$$

$$= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^2B\,s}{d\sqrt{b\,c}}$$

$$= \frac{2b^2(A+3C)\sqrt{\cos(c+dx)}\,F}{3d\sqrt{b\cos(c+dx)}}$$

$$= -\frac{2bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.35, size = 92, normalized size = 0.63

$$\frac{2b^2\left(-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3B\cos(c+dx))\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(181) = 362.
time = 0.65, size = 506, normalized size = 3.49

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 200, normalized size = 1.38

$$\frac{-i\sqrt{2}(A+3C)b^{\frac{3}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3C)b^{\frac{3}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3Bb\cos(dx+c)+Ab)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(3/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B
*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^2*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) + 2*(3*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.253 $\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+d\right.$ $\left.dx\right)dx$

Optimal. Leaf size=186

$$-\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b^2*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*b*(3*A+5*C)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2b(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (-2*b*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt
[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3
*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/
2)) + (2*b^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2*(3*A + 5
*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{7/}}$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(2b^2\right)$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \left(b^4B\right)\int$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\,\mathrm{s}}{3d(b\cos}$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\,\mathrm{s}}{3d(b\cos}$$

$$= -\frac{2b(3A+5C)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.49, size = 122, normalized size = 0.66

$$-\frac{(b\cos(c+dx))^{3/2}\sec^3(c+dx)\left(6(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-10B\sin(c+dx)-9A\sin(2(c+dx))-15C\sin(2(c+dx))-6A\tan(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] -1/15*((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(6*(3*A + 5*C)*Cos[c + d*x]^(3
/2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)
/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] -
 6*A*Tan[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(214) = 428$.
time = 0.94, size = 805, normalized size = 4.33

method	result	size
default	Expression too large to display	805

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12
0*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)
/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

<u>Maxima</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^5, x)
```

<u>Fricas</u> [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 223, normalized size = 1.20

$$\frac{-5i\sqrt{2}\,Bb^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,Bb^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,(3A+5C)b^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,(3A+5C)b^{\frac{3}{2}}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3(3A+5C)b\cos(dx+c)^2+5Bb\cos(dx+c)+3Ab\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*C)*
b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*C)*b^(3/2)*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*(3*A + 5*C)*b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*A*b
)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.254 $\int (b\cos(c+dx))^{3/2} (A + B\cos(c + dx) + C\cos^2(c + dx) dx$

Optimal. Leaf size=215

$$-\frac{6bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/}}$$

```
[Out] 2/7*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^4*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^3*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^2*B
*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b^2*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^3(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (-6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(2
1*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*b^4*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^3*(5*A +
7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^2*B*Sin[c + d*x])/(
5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = b^6\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^9}$$

$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}(2b^3)$$

$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + (b^5B)$$

$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B}{5d(b\,\mathrm{co}}$$

$$= \frac{2Ab^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^4B}{5d(b\,\mathrm{co}}$$

$$= \frac{2b^2(5A+7C)\sqrt{\cos(c+dx)}\,F}{21d\sqrt{b\cos(c+d}}$$

$$= -\frac{6bB\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c-\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 1.83, size = 134, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{3/2}\sec^5(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(110A+70C+273B\cos(c+dx)+10(5A+7C)\cos(2(c+dx))+63B\cos(3(c+dx)))\sin(c+dx)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5*(-504*B*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2,
 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)
] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(239) = 478.
time = 1.25, size = 727, normalized size = 3.38

method	result	size
default	Expression too large to display	727

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(1/5*B/b/s
in(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/
42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*
d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 235, normalized size = 1.09

$$\frac{-5i\sqrt{2}(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7C)b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-63i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+63i\sqrt{2}Bb^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(63Bb\cos(dx+c)^3+5(5A+7C)b\cos(dx+c)^2+21Bb\cos(dx+c)+15Ab\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*C)*b^(3/2)*cos
```

```
(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*
I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*B*b^(3/2)*cos(d*x
+ c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(63*B*b*cos(d*x + c)^3 + 5*(5*A + 7*C)*b*cos(d*x + c)^2 +
 21*B*b*cos(d*x + c) + 15*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*se
c(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6, x)
```

3.255 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+d\right.$

Optimal. Leaf size=212

$$\frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{10b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2B\sqrt{b\cos(c+d}}{21d}$$

```
[Out] 2/45*b*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/
2)*sin(d*x+c)/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d+10/21*b^3*B*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/d+2/15*b^2*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2b^2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2b(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{10b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sq
rt[Cos[c + d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])
/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2
*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (2*C*(b*Cos[c + d*x])^(7/2)
*Sin[c + d*x])/(9*b*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{2\int(b\,c$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}+\frac{B\int(b\,c$$

$$= \frac{2b(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}$$

$$= \frac{10b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2b(9$$

$$= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{15d\sqrt{\cos(c+dx)}}$$

$$= \frac{2b^2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{15d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.48, size = 125, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{5/2}\left(84(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+300BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(c+dx)\right)}{630d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*E
llipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x]
 + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/
(630*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.37, size = 384, normalized size = 1.81

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 203, normalized size = 0.96

$$\frac{-75i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,(9A+7C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35Cb^2\cos(dx+c)^3+45Bb^2\cos(dx+c)^2+7(9A+7C)b^2\cos(dx+c)+75Bb^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 21*I*sqrt(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt
(2)*(9*A + 7*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(35*C*b^2*cos(d*x + c)^3 + 45*B*b^2*cos(
d*x + c)^2 + 7*(9*A + 7*C)*b^2*cos(d*x + c) + 75*B*b^2)*sqrt(b*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (b\cos(c+dx))^{5/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
[Out] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.256 $\int (b\cos(c+dx))^{5/2} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=183

$$\frac{6b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sqrt{b\cos \cdots}}{2 \cdots}$$

```
[Out] 2/5*b*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(d*
x+c)/d+2/21*b^3*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2
)+2/21*b^2*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*b^2*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2b^3(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(21*d) + (2*b*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{3/2}\left(A+B\,\mathrm{co}\right.$$
$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx}{7d}$$
$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx}{7d}$$
$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\,\mathrm{s}}{21d}$$
$$= \frac{2b^2(7A+5C)\sqrt{b\cos(c+dx)}\,\mathrm{s}}{21d}$$
$$= \frac{6b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 109, normalized size = 0.60

$$\frac{b(b\cos(c+dx))^{3/2}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (b*(b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)
*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c +
 d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.37, size = 353, normalized size = 1.93

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 189, normalized size = 1.03

$$\frac{-5i\sqrt{2}\,(7\,A+5\,C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}\,(7\,A+5\,C)b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+63i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-63i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(15\,Cb^2\cos(dx+c)^2+21\,Bb^2\cos(dx+c)+5\,(7\,A+5\,C)b^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*C)*b^(5/2)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*B*b^(5/2)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 6
3*I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(15*C*b^2*cos(d*x + c)^2 + 21*B*b^2*cos(d*
x + c) + 5*(7*A + 5*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.257 $\int (b\cos(c+dx))^{5/2} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^2(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] 2/5*b*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b^3*B*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2/5
*b^2*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2b^2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bC\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqr
t[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(
3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int\sqrt{b\cos(c+dx)}\,(A+B\,c$$

$$= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{2bC(b\cos(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{2b^2B\sqrt{b\cos(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx}}$$

$$= \frac{2b^2(5A+3C)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.20, size = 97, normalized size = 0.64

$$\frac{2b^2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*
EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin
[c + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.40, size = 319, normalized size = 2.11

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(24*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 169, normalized size = 1.12

$$\frac{-5i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,Bb^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,(5A+3C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(5A+3C)b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3\,Cb^{2}\cos(dx+c)+5\,Bb^{2})\sqrt{b\cos(dx+c)}\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(
5*A + 3*C)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) + 2*(3*C*b^2*cos(d*x + c) + 5*B*b^2)*sqrt(b*cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.258 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)dx$

Optimal. Leaf size=120

$$\frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2C\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] 2/3*b^3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*
C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\frac{2b^3(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*
Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.22, size = 79, normalized size = 0.66

$$\frac{2(b\cos(c+dx))^{5/2}\left(3BE\left(\frac{1}{2}(c+dx)\middle|2\right)+(3A+C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(3*B*EllipticE[(c + d*x)/2, 2] + (3*A + C)*Ellipt
icF[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]
^(5/2))
```

Maple [A]
time = 0.35, size = 285, normalized size = 2.38

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 150, normalized size = 1.25

$$\frac{-i\sqrt{2}\,(3A+C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(3A+C)b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,Bb^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,Bb^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Cb^2\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + I*sqrt(2)*(3*A + C)*b^(5/2)*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*
B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*b^2*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.259 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal. Leaf size=116

$$-\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (-2*b^2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqr
t[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{3/2}}$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + (2b)\int \frac{\frac{b^2B}{2}}{}$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \left(b^3B\right)\int -$$

$$= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(b^3B\sqrt{\cos}\right.}{}$$

$$= -\frac{2b^2(A-C)\sqrt{b\cos(c+dx)}\,E}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.31, size = 80, normalized size = 0.69

$$\frac{2b^3\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c +
d*x]])
```

Maple [A]
time = 0.45, size = 262, normalized size = 2.26

method	result
default	$\frac{2b^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\mathrm{si}\right.}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 181, normalized size = 1.56

$$\frac{-i\sqrt{2}\,Bb^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,Bb^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,(A-C)b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,(A-C)b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,Ab^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + I*sqrt(2)*(A - C)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x
 + c))*A*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}\,\left(C\cos(c+dx)^2+B\,\cos(c+dx)+A\right)}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.260 $\int (b\cos(c+dx))^{5/2} (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec^5(c+dx)\,dx$

Optimal. Leaf size=147

$$-\frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*b^3*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(1/2)+2/3*b^3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/
2)-2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^5,x]
```

```
[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d
*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)
) + (2*b^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^5}$$

$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}(2b^2)$$

$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+(b^4B)$$

$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\,s}{d\sqrt{b\,c}}$$

$$= \frac{2b^3(A+3C)\sqrt{\cos(c+dx)}\,F}{3d\sqrt{b\cos(c+dx)}}$$

$$= -\frac{2b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.33, size = 92, normalized size = 0.63

$$\frac{2b^3\left(-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3B\cos(c+dx))\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*
x]))/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(183) = 366.
time = 0.66, size = 508, normalized size = 3.46

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 204, normalized size = 1.39

$$\frac{-i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(A+3C)b^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3Bb^2\cos(dx+c)+Ab^2)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*C)*b^(5/2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B
*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^2*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
```

```
) + 2*(3*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^5, x)
```

3.261 $\int (b\cos(c+dx))^{5/2} (A + B\cos(c+dx) + C\cos^2(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=188

$$-\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*A*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^4*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(3/2)+2/5*b^3*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^3*B*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*b^2*(3*A+5*C)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2b^3(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2b^2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^6,x]
```

```
[Out] (-2*b^2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sq
rt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(
5/2)) + (2*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^3*(3*A +
 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^6(c+dx)\,dx = b^6\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{7/2}}$$

$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(2b^3\right)\int$$

$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \left(b^5B\right)\int$$

$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin}{3d(b\cos}$$

$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin}{3d(b\cos}$$

$$= -\frac{2b^2(3A+5C)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.44, size = 121, normalized size = 0.64

$$-\frac{2b^4\left(3(3A+5C)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)-5B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-5B\sin(c+dx)-\frac{9}{2}A\sin(2(c+dx))-\frac{15}{2}C\sin(2(c+dx))-3A\tan(c+dx)\right)}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (-2*b^4*(3*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2
*(c + d*x)])/2 - (15*C*Sin[2*(c + d*x)])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Co
s[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(216) = 432$.
time = 1.01, size = 807, normalized size = 4.29

method	result	size
default	Expression too large to display	807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-
120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/
2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 229, normalized size = 1.22

$$\frac{-5i\sqrt{2}\,Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\sqrt{2}\,Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-3i\sqrt{2}\,(3A+5C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,(3A+5C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,(3A+5C)b^{2}\cos\left(dx+c\right)^{2}+5\,Bb^{2}\cos\left(dx+c\right)+3\,Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(3*A + 5*C)*
b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(3*A + 5*C)*b^(5/2)*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(3*(3*A + 5*C)*b^2*cos(d*x + c)^2 + 5*B*b^2*cos(d*x + c) + 3
*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^6, x)
```

3.262 $\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+\right.$ $\left.dx\right)dx$

Optimal. Leaf size=217

$$-\frac{6b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^7}$$

```
[Out] 2/7*A*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^5*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b^4*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*b^3*B
*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*b^3*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^4(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^5B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^3B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^7,x]
```

```
[Out] (-6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/
(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^
(7/2)) + (2*b^5*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^4*(5*A
+ 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^3*B*Sin[c + d*x])
/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^7(c+dx)\,dx = b^7\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^{9}}$$

$$= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(2b^4)$$

$$= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+(b^6B)$$

$$= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^5B}{5d(b\,co}$$

$$= \frac{2Ab^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b^5B}{5d(b\,co}$$

$$= \frac{2b^3(5A+7C)\sqrt{\cos(c+dx)}\,F}{21d\sqrt{b\cos(c+d}}$$

$$= -\frac{6b^2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{5d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.82, size = 134, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{5/2}\sec^6(c+dx)\left(-504B\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(5A+7C)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(110A+70C+273B\cos(c+dx)+10(5A+7C)\cos(2(c+dx))+63B\cos(3(c+dx)))\sin(c+dx)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6*(-504*B*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2,
 2] + 2*(110*A + 70*C + 273*B*Cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)
] + 63*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(241) = 482$.
time = 1.15, size = 727, normalized size = 3.35

method	result	size
default	Expression too large to display	727

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(1/5*B/b/s
in(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/
42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*
d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 243, normalized size = 1.12

$$\frac{-5i\sqrt{2}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(5A+7C)b^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-63i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+63i\sqrt{2}Bb^{\frac{5}{2}}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(63Bb^2\cos(dx+c)^3+5(5A+7C)b^2\cos(dx+c)^2+21Bb^2\cos(dx+c)+15Ab^2\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*C)*b^(5/2)*cos
```

```
(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*
I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*B*b^(5/2)*cos(d*x
+ c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(63*B*b^2*cos(d*x + c)^3 + 5*(5*A + 7*C)*b^2*cos(d*x + c)
^2 + 21*B*b^2*cos(d*x + c) + 15*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**7,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^7,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^7, x)
```

3.263 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=214

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\,\sin}{21bd}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^3/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^4/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos
(d*x+c))^(1/2)/b/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d
*x+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt
[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d
*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*B*(b*
Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*S
in[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(\right.}{b^3}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{2\int (b\cos(c+}{}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{B\int (b\cos(c+}{b^4}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2B(}{}$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2(9A+7C)(}{}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} +$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.71, size = 127, normalized size = 0.59

$$\frac{168(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(2(c+dx))}{1260d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.42, size = 381, normalized size = 1.78

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A
-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b
*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 194, normalized size = 0.91

$$\frac{-75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\sqrt{2}(-9iA-7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}(9iA+7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35C\cos(dx+c)^3+45B\cos(dx+c)^2+7(9A+7C)\cos(dx+c)+75B\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{315bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
```

```
+ c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7320 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2), x)
```

3.264 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+}{21bd}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^3/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*B*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*
cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*S
qrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2
1*b*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*C*(b*Cos[
c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos\right.}{b^2}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{2\int (b\cos(c\right.}{}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{B\int (b\cos(c}{b}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2B(}{}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2B(}{}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(7A+}{}$$

Mathematica [A]
time = 0.69, size = 108, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.40, size = 350, normalized size = 1.89

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 180, normalized size = 0.97

$$-\frac{5\sqrt{2}(7iA+5iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}(-7iA-5iC)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-63i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+63i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(15C\cos(dx+c)^2+21B\cos(dx+c)+35A+25C\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/2), x)
```

3.265 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=150

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\,\sin(c}{3bd}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+2/5*(5*
A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$,
Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d
*x]],x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[
Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sq
rt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*
C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(\right.}{b}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2\int \sqrt{b\cos(c}}{}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{B\int (b\cos(c-}{b^2}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2C(b\cos(c+}{}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} +$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.32, size = 97, normalized size = 0.65

$$\frac{2\sqrt{b\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Elli
pticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c +
 d*x]))/(15*b*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.37, size = 316, normalized size = 2.11

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*
c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 166, normalized size = 1.11

$$\frac{-5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(-5iA-3iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(5iA+3iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3C\cos(dx+c)+5B)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/2), x)
```

3.266 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c}{3bd}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/b/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3102, 2827, 2721, 2720, 2719}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{1}{3}($$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int}{3\sqrt{b\cos(c+}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx}}{3d\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 0.70

$$\frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c +
 d*x]])
```

Maple [A]
time = 0.33, size = 282, normalized size = 2.41

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 152, normalized size = 1.30

$$\frac{\sqrt{2}\,(-3i\,A-i\,C)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,A+i\,C)\sqrt{b}\;\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3i\,\sqrt{2}\,B\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\,\sqrt{2}\,B\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,\sqrt{b\cos(dx+c)}\,C\sin(dx+c)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Mupad [**B**]
time = 0.39, size = 128, normalized size = 1.09

$$\frac{2\,C\,\sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{3\,b\,d}+\frac{2\,A\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}+\frac{2\,B\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos\left(c+d\,x\right)}}+\frac{2\,C\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*C*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)
*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d*x
)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*C*cos(
c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))
```

3.267 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=110

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*
cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d
*x]],x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c
 + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx - \cdots$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{\cdots}{\sqrt{\cos(\cdots}}}{\sqrt{b\cos(c+dx)}}$$

$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2I\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.33, size = 803, normalized size = 7.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-2*(-2*A + C + C*Co
s[2*c])*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(Sqrt[b*C
os[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*B*Cos[
c + d*x]^(3/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan
[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[
c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]
^2]) + (2*A*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d
*x])) - (2*C*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x]
)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*
d*x]))
```

Maple [A]
time = 0.41, size = 259, normalized size = 2.35

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\right.}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
```

```
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 183, normalized size = 1.66

$$\frac{-i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(-i\,A+i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(i\,A-i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\sqrt{b\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/sqrt(b*cos(c
 + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/2)), x)
```

3.268 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2}{d\sqrt{}}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(
1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B
*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}}{3b}$$

$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + (bB)\int \frac{1}{(b\cos(c+dx))^{3/2}}$$

$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{B\int}{}$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2}{3d(}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2(A+3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.35, size = 757, normalized size = 5.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c]
)/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(
A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[
c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x]
 + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos
[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^(5/
2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]*
(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2*
```

```
B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d
*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(175) = 350$.

time = 0.73, size = 508, normalized size = 3.65

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 202, normalized size = 1.45

$$\frac{\sqrt{2}\,(-i\,A-3i\,C)\sqrt{b}\,\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+\sqrt{2}\,(i\,A+3i\,C)\sqrt{b}\,\cos{(dx+c)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})-3i\,\sqrt{2}\,B\sqrt{b}\,\cos{(dx+c)}^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))+3i\,\sqrt{2}\,B\sqrt{b}\,\cos{(dx+c)}^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)}))+2\,(3\,B\cos{(dx+c)}+A)\sqrt{b\cos{(dx+c)}}\,\sin{(dx+c)}}{3\,bd\cos{(dx+c)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos{(c+dx)}+C\cos^2{(c+dx)})\sec^2{(c+dx)}}{\sqrt{b\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/sqrt(b*co
s(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/2)), x)
```

3.269 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{3}{}$$

```
[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*
x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2(3A+5C)\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt
[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*S
qrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2))
+ (2*b*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c
+ d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2}{5}\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)}{(b\cos(c+dx)}$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + (b^2B)\int \frac{1}{(b\cos(c+dx))^{5/}}$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(}{}$$

$$= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2(}{}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.49, size = 116, normalized size = 0.64

$$\frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+9A\sin(c+dx)+15C\sin(c+dx)+5B\tan(c+dx)+3A\sec(c+dx)\tan(c+dx)\right)}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(208) = 416.
time = 1.06, size = 807, normalized size = 4.48

method	result	size
default	Expression too large to display	807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1
/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12
0*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)
/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 223, normalized size = 1.24

$$\frac{-5i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)) + 5i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)) - 3\sqrt{2}\,(3i\,A+5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))) - 3\sqrt{2}\,(-3i\,A-5i\,C)\sqrt{b}\,\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))) + 2\left(3\,(3\,A+5\,C)\cos(dx+c)^2+5\,B\cos(dx+c)+3\,A\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,bd\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/2)), x)
```

3.270 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=209

$$-\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+$$

```
[Out] 2/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b^2*B*sin(d*x+c)/d/(b*cos(d
*x+c))^(5/2)+2/21*b*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d
*x+c)/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.18, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2b(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6B\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c +
 d*x]],x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d
*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*
Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2))
 + (2*b^2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*(5*A + 7*C)*S
in[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[b
*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^4\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$$

$$= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}(2b)\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A}{(b\cos(c}$$

$$= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + (b^3B)\int \frac{1}{(b\cos(c+dx))}$$

$$= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} +$$

$$= \frac{2Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2b^2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} +$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} +$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(5A}{}$$

Mathematica [A]
time = 0.75, size = 133, normalized size = 0.64

$$\frac{2\left(-63B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+63B\sin(c+dx)+25A\tan(c+dx)+35C\tan(c+dx)+21B\sec(c+dx)\tan(c+dx)+15A\sec^2(c+dx)\tan(c+dx)\right)}{105d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*C
os[c + d*x]],x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(233) = 466$.
time = 1.15, size = 726, normalized size = 3.47

method	result	size
default	Expression too large to display	726

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*B/b/sin(1/2
*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1
/2*d*x+1/2*c)^2*b)^(1/2)+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/6*cos(1/2*d*x+1/2*c)/
b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*
x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1
))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos
(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 234, normalized size = 1.12

$$-\frac{5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iA-7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(63B\cos(dx+c)^3+5(5A+7C)\cos(dx+c)^2+21B\cos(dx+c)+15A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105bd\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
```

```
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
 + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos
(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\cos\left(c+d\,x\right)}^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x)
)^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^4*(b*cos(c + d*x)
)^(1/2)), x)
```

3.271 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\,\sin}{21b^2d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^4/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^5/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*c
os(d*x+c))^(1/2)/b^2/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d
/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sq
rt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*b*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b
^2*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2
*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d) + (2*C*(b*Cos[c + d*x])^(
7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos^2(}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{2\int (b\cos(c+}{}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{B\int (b\cos(c+}{b^5}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2B(}{}$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2(9A+7C)(}{}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + $$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + $$

Mathematica [A]

time = 0.76, size = 130, normalized size = 0.60

$$\frac{168(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(2(c+dx))}{1260bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]

time = 0.37, size = 384, normalized size = 1.77

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1120*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)
^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126
*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 194, normalized size = 0.89

$$\frac{-75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\sqrt{2}(-9iA-7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}(9iA+7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35C\cos(dx+c)^3+45B\cos(dx+c)^2+7(9A+7C)\cos(dx+c)+75B\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{315b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
```

```
+ c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.272 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+}}{21b^2d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^4/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^
(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+6/5*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b
*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]
)/(21*b^2*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*C*(
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos\right.}{b^3}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{2\int(b\cos(c\cdot}{}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{B\int(b\cos(c}{}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2B(}{}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2B(}{}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(7A+}{}$$

Mathematica [A]
time = 0.79, size = 108, normalized size = 0.57

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.38, size = 353, normalized size = 1.88

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*C*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 180, normalized size = 0.96

$$-\frac{5\sqrt{2}\left(7i\,A+5i\,C\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-7i\,A-5i\,C\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-63i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+63i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\left(15\,C\cos\left(dx+c\right)^{2}+21\,B\cos\left(dx+c\right)+35\,A+25\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.273 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\,\sin(c}{3b^2d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+2/5
*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqr
t[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*
d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
 + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2}{b^2}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2\int \sqrt{b\cos(}}{}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{B\int (b\cos(c}{b}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2C(b\cos(c+}{}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} +$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.40, size = 94, normalized size = 0.61

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.37, size = 319, normalized size = 2.08

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(24*C*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/
2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 166, normalized size = 1.08

$$\frac{-5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(-5i\,A-3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(5i\,A+3i\,C\right)\sqrt{b}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(3\,C\cos\left(dx+c\right)+5\,B\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(3/2), x)
```

3.274 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c}{3b^2d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*C*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d
/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(3/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sq
rt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}b}{\sqrt{b\cos(c}}}{3b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{B\int\sqrt{b\cos(c+}}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{\Big((3A+C)\sqrt{\mathrm{co}}}{3}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+C)}{}$$

Mathematica [A]
time = 0.20, size = 85, normalized size = 0.71

$$\frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [A]
time = 0.35, size = 285, normalized size = 2.38

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 152, normalized size = 1.27

$$\frac{\sqrt{2}\left(-3iA-iC\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{2}\left(3iA+iC\right)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+3i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-3i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\sqrt{b\cos\left(dx+c\right)}C\sin\left(dx+c\right)}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(3/2), x)
```

3.275 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d
/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3100, 2827, 2721, 2720, 2719}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos
[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^3}$$

$$= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b} - \frac{(A-C)\int \sqrt{b \cdots}}{\cdots}$$

$$= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}} \cdots$$

$$= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+d \cdots}}{bd\sqrt{b\,\text{co}\cdots}}$$

Mathematica [**A**]
time = 0.20, size = 80, normalized size = 0.69

$$\frac{2\left(-\left((A-C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A]
time = 0.38, size = 262, normalized size = 2.26

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}\right.}{b\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 183, normalized size = 1.58

$$\frac{-i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\sqrt{2}\,(-iA+iC)\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(iA-iC)\sqrt{b}\,\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,A\sin(dx+c)}{b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
```

```
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(3/2), x)
```

3.276 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\ldots$$

```
[Out] 2/3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(
1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.180, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(3/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*S
qrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2
*B*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx}{(b\cos(c+dx))^{3/2}}}{3b^2}$$

$$= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{B}{}$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{}{3d}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2(A+3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.31, size = 761, normalized size = 5.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] ((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c
])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*
(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos
[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x
] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Co
s[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^(5
/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]
*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2
```

```
*B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/
b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(180) = 360$.
time = 0.72, size = 508, normalized size = 3.53

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 202, normalized size = 1.40

$$\frac{\sqrt{2}\left(-iA-3iC\right)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(iA+3iC\right)\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3i\,\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3i\,\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{2}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(3B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,b^{2}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*
cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(b\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d
*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(3/2)), x)
```

3.277 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+$$

```
[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c))
^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*
x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)
) + (2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c
+ d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+}{(b\cos(c+dx))^{5/2}}}{5b}$$

$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + (bB)\int \frac{1}{(b\cos(c+dx))^{5/}}$$

$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2}{}$$

$$= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2}{}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.50, size = 119, normalized size = 0.65

$$\frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+9A\sin(c+dx)+15C\sin(c+dx)+5B\tan(c+dx)+3A\sec(c+dx)\tan(c+dx)\right)}{15bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(211) = 422$.
time = 1.02, size = 807, normalized size = 4.41

method	result	size
default	Expression too large to display	807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-
120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/
2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 223, normalized size = 1.22

$$\frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}(3iA+5iC)\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}(-3iA-5iC)\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3(3A+5C)\cos(dx+c)^2+5B\cos(dx+c)+3A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{15b^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)} + C\cos^2{(c + dx)})\sec^2{(c + dx)}}{(b\cos{(c + dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c
+ d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A}{{\cos\left(c+d\,x\right)}^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(3/2)), x)
```

3.278 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+ 5$$

```
[Out] 2/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*b*B*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c
)/b/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(
b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{2(5A+7C)\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6B\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x
])^(3/2),x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*
b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*b*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*S
in[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b*d*Sqrt
[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$$

$$= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2}{7}\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C}{(b\cos(c+dx}$$

$$= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + (b^2B)\int \frac{1}{(b\cos(c+dx))^{7/}}$$

$$= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(}{2}$$

$$= \frac{2Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2bB\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2(}{2}$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2}{7d}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(5A+}{}$$

Mathematica [A]
time = 0.77, size = 136, normalized size = 0.64

$$\frac{2\left(-63B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+63B\sin(c+dx)+25A\tan(c+dx)+35C\tan(c+dx)+21B\sec(c+dx)\tan(c+dx)+15A\sec^2(c+dx)\tan(c+dx)\right)}{105bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c
 + d*x])^(3/2),x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(236) = 472$.
time = 1.19, size = 729, normalized size = 3.44

method	result	size
default	Expression too large to display	729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(2/5*B/b/sin(1
/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d
*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin
(1/2*d*x+1/2*c)^2*b)^(1/2)+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/6*cos(1/2*d*x+1/2*c
)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*
d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 234, normalized size = 1.10

$$-\frac{5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iA-7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(63B\cos(dx+c)^3+5(5A+7C)\cos(dx+c)^2+21B\cos(dx+c)+15A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105b^2d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
```

```
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d
*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(3/2)), x)
```

3.279 $\int \frac{\cos^5(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\,\mathrm{si}}{21b^3d}$$

```
[Out] 2/45*(9*A+7*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*cos(d*x+c))^(
5/2)*sin(d*x+c)/b^5/d+2/9*C*(b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^6/d+10/21*B*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b
*cos(d*x+c))^(1/2)/b^3/d+2/15*(9*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2715, 2721, 2719, 2720}

$$\frac{2(9A+7C)\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}+\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{10B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sq
rt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21
*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*b^3*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) +
(2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d) + (2*C*(b*Cos[c + d*x])
^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx)+C\cos}{b^5}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{2\int (b\cos(c-}{}$$

$$= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{B\int (b\cos(c}{b}$$

$$= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2E}{}$$

$$= \frac{10B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2(9A+7C)}{}$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} +$$

$$= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.75, size = 130, normalized size = 0.60

$$\frac{168(9A+7C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+600B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7(36A+43C)\cos(c+dx)+5(78B+18B\cos(2(c+dx))+7C\cos(3(c+dx))))\sin(2(c+dx))}{1260b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5
*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(
1260*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.43, size = 384, normalized size = 1.77

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720B+2240C)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1120
*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*
c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x
 + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 194, normalized size = 0.89

$$\frac{-75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+75i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21\sqrt{2}(-9iA-7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21\sqrt{2}(9iA+7iC)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(35C\cos(dx+c)^3+45B\cos(dx+c)^2+7(9A+7C)\cos(dx+c)+75B\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{315b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/315*(-75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 75*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 21*sqrt(2)*(-9*I*A - 7*I*C)*sqrt(b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqr
t(2)*(9*I*A + 7*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(35*C*cos(d*x + c)^3 + 45*B*cos(d*x
```

```
+ c)^2 + 7*(9*A + 7*C)*cos(d*x + c) + 75*B)*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^5\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^5*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.280 $\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(7A+5C)\sqrt{b\cos(c+d}}{21b^3d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*C*(b*cos(d*x+c))^(5/2)*sin(
d*x+c)/b^5/d+2/21*(7*A+5*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c)
)^(1/2)+2/21*(7*A+5*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6/5*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$,
Rules used = {16, 3102, 2827, 2715, 2721, 2720, 2719}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{2(7A+5C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b
^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*
x])/(21*b^3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*C
*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(\right.}{b^4}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{2\int (b\cos(c+}{}$$

$$= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{B\int (b\cos(c+}{b^5}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2B(b}{}$$

$$= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2B(b}{}$$

$$= \frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(7A+5}{}$$

Mathematica [**A**]
time = 0.64, size = 111, normalized size = 0.59

$$\frac{\sqrt{\cos(c+dx)}\left(126BE\left(\frac{1}{2}(c+dx)\middle|2\right)+10(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(70A+65C+42B\cos(c+dx)+15C\cos(2(c+dx)))\sin(c+dx)\right)}{105b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*Ellip
ticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]
+ 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [**A**]
time = 0.41, size = 353, normalized size = 1.88

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168B-360C)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*C
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*C*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1
/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 180, normalized size = 0.96

$$-\frac{5\sqrt{2}\,(7i\,A+5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-7i\,A-5i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-63i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+63i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\left(15\,C\cos(dx+c)^2+21\,B\cos(dx+c)+35\,A+25\,C\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{105\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*A + 5*I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*A - 5*I*C)*sqrt(b)*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*B*sqrt(b)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 63*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*C*cos(d*x + c)^2 + 21*B*cos(d*x +
 c) + 35*A + 25*C)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.281 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{b\cos(c+dx)}\,\sin(}{3b^3d}$$

```
[Out] 2/5*C*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+2
/5*(5*A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3102, 2827, 2721, 2719, 2715, 2720}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqr
t[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^
2*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*
d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c\right.}{b^3}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2\int \sqrt{b\cos(c}}{}$$

$$= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{B\int (b\cos(c+}{b^4}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{2C(b\cos(c+d}{}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A]
time = 0.33, size = 97, normalized size = 0.63

$$\frac{2\sqrt{\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3C\cos(c+dx))\sin(c+dx)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.40, size = 319, normalized size = 2.08

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-20B-24C)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(24*C*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+
1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 166, normalized size = 1.08

$$\frac{-5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\sqrt{2}\,(-5i\,A-3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\sqrt{2}\,(5i\,A+3i\,C)\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3\,C\cos(dx+c)+5\,B)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{15\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*A - 3*I*C)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*
(5*I*A + 3*I*C)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) + 2*(3*C*cos(d*x + c) + 5*B)*sqrt(b*cos(d*x
+ c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^3\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.282 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+}{3b^3d}$$

```
[Out] 2/3*(3*A+C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*C*si
n(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {16, 3102, 2827, 2721, 2720, 2719}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*
Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\cdots}{\sqrt{b\cos\cdots}}}{3\cdots}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{B\int \sqrt{b\cos(\cdots}}{b^3\cdots}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d} + \frac{\left((3A+C)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2(3A+\cdots}{\cdots}$$

Mathematica [A]
time = 0.20, size = 85, normalized size = 0.71

$$\frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [A]
time = 0.35, size = 285, normalized size = 2.38

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4C\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\mathrm{si}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-2*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+C*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/s
in(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 152, normalized size = 1.27

$$\frac{\sqrt{2}\,(-3i\,A-i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,A+i\,C)\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3i\,\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,\sqrt{b\cos(dx+c)}\,C\sin(dx+c)}{3\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A - I*C)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) + sqrt(2)*(3*I*A + I*C)*sqrt(b)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt
(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*C*sin(d*x + c))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(5/2), x)
```

3.283 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b
^2/d/(b*cos(d*x+c))^(1/2)-2*(A-C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 3100, 2827, 2721, 2720, 2719}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])
^(5/2),x]
```

```
[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos
[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqr
t[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b} \\
&= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2\int \frac{\frac{b^2B}{2}-\frac{1}{2}b^2(A-C)\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^4} \\
&= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2} \\
&= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{}{\sqrt{}}}{b^2\sqrt{b\cos(c+}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^3d\sqrt{\cos(c+dx)}}+
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.23, size = 807, normalized size = 6.96

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] ((Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-2*(-2*A + C + C*C
os[2*c])*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(Sqrt[b*
Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*B*Cos
[c + d*x]^(3/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos
[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c
]^2]) + (2*A*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x]
)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*
d*x])) - (2*C*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x
])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2
*d*x])))/b^2
```

Maple [A]
time = 0.39, size = 262, normalized size = 2.26

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\right.\right.}}{b^2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
```

```
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 183, normalized size = 1.58

$$\frac{-i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(-iA+iC\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\sqrt{2}\left(iA-iC\right)\sqrt{b}\,\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\,A\sin\left(dx+c\right)}{b^{3}d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(-I*A + I*C)*sqrt(b)*cos(d*x
+ c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) + sqrt(2)*(I*A - I*C)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*co
s(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(5/2), x)
```

3.284 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c
))^(1/2)+2/3*(A+3*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d
*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
+ (2*B*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2\int \frac{\frac{3b^2B}{2}+\frac{1}{2}b^2(A+3C)\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{3b^3}$$

$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b}+\frac{(A+3C)\int \frac{}{\sqrt{b}}}{3}$$

$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{B\int\sqrt{b\cos(c+}}{b^3}$$

$$= \frac{2(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2(A+3C)\sqrt{\cos(c+d}}{3b^2d\sqrt{b\cos}}$$

Mathematica [A]
time = 0.42, size = 92, normalized size = 0.63

$$\frac{2\left(-3B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+3B\cos(c+dx))\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]))
/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(183) = 366$.
time = 0.70, size = 508, normalized size = 3.46

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*d
*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^4+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 202, normalized size = 1.37

$$\frac{\sqrt{2}\,(-iA-3iC)\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(iA+3iC)\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3i\sqrt{2}\,B\sqrt{b}\,\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,(3B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{3\,b^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A - 3*I*C)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A + 3*I*C)*sqrt(b)*cos(d*x +
 c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(
2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*
cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A}{{\left(b\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(5/2), x)
```

3.285 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{}{3}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c))
^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {16, 3100, 2827, 2716, 2721, 2720, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(5/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)
) + (2*B*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[
c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}}{5b^2}$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2(}{5}$$

$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2(}{5}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.46, size = 119, normalized size = 0.64

$$\frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+9A\sin(c+dx)+15C\sin(c+dx)+5B\tan(c+dx)+3A\sec(c+dx)\tan(c+dx)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*C
os[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(213) = 426$.
time = 0.97, size = 807, normalized size = 4.36

method	result	size
default	Expression too large to display	807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x,metho
d=_RETURNVERBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-
120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/
2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 1.21

$$\frac{-5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}B\sqrt{b}\cos(dx+c)^3 \text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 3\sqrt{2}(3iA+5iC)\sqrt{b}\cos(dx+c)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3\sqrt{2}(-3iA-5iC)\sqrt{b}\cos(dx+c)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2(3(3A+5C)\cos(dx+c)^2+5B\cos(dx+c)+3A)\sqrt{b\cos(dx+c)}\sin(dx+c)}{15b^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(5/2)), x)
```

3.286 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+$$

```
[Out] 2/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+2/5*B*sin(d*x+c)/d/(b*cos(d*x+c))
^(5/2)+2/21*(5*A+7*C)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+6/5*B*sin(d*x+c)/
b^2/d/(b*cos(d*x+c))^(1/2)+2/21*(5*A+7*C)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d
/(b*cos(d*x+c))^(1/2)-6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3100, 2827, 2716, 2721, 2719, 2720}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}-\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{6B\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(5/2),x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*
b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7
/2)) + (2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin
[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b^2*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{9/2}}\,dx$$

$$= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2\int \frac{\frac{7b^2B}{2}+\frac{1}{2}b^2(5A+7C)\cos(c+}{(b\cos(c+dx))^{7/2}}}{7b}$$

$$= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + (bB)\int \frac{1}{(b\cos(c+dx))^{7/}}$$

$$= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2}{}$$

$$= \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{2B\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2}{}$$

$$= \frac{2(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{}{7}$$

$$= -\frac{6B\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2(5A}{}$$

Mathematica [A]
time = 0.52, size = 136, normalized size = 0.64

$$\frac{2\left(-63B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(5A+7C)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+63B\sin(c+dx)+25A\tan(c+dx)+35C\tan(c+dx)+21B\sec(c+dx)\tan(c+dx)+15A\sec^2(c+dx)\tan(c+dx)\right)}{105b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(5/2),x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c +
 d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d
*x]^2*Tan[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(236) = 472$.
time = 1.15, size = 729, normalized size = 3.44

method	result	size
default	Expression too large to display	729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(2/5*B/b/sin
(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2
*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+s
in(1/2*d*x+1/2*c)^2*b)^(1/2)+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/
42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*C*(-1/6*cos(1/2*d*x+1/2
*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/
2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 234, normalized size = 1.10

$$-\frac{5\sqrt{2}(5iA+7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iA-7iC)\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}B\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(63B\cos(dx+c)^3+5(5A+7C)\cos(dx+c)^2+21B\cos(dx+c)+15A\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{105b^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A + 7*I*C)*sqrt(b)*cos(d*x + c)^4*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A - 7*I*C)*sqrt(b
```

```
)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 63*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*B*sqrt(b)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(63*B*cos(d*x + c)^3 + 5*(5*A + 7*C)*cos(d*x + c)^2
+ 21*B*cos(d*x + c) + 15*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d
*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5
/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(5/2)), x)
```

3.287 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*x
+c))^(3/2)+2/5*(3*A+5*C)*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)/b^3/d/(b*cos(d*x+c))^(1/2)-2/5*(3*A+5*C)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(
b*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3100, 2827, 2716, 2721, 2720, 2719}

$$-\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/
2)) + (2*B*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*
Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2\int \frac{\frac{5b^2B}{2}+\frac{1}{2}b^2(3A+5C)\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx}{5b^3}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{b} + \frac{(3A+5C)\int \frac{}{(}}{5}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)}{5b^3d\sqrt{b\cos}}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{2(3A+5C)}{5b^3d\sqrt{b\cos}}$$

$$= -\frac{2(3A+5C)\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c-}}{3b^3d}$$

Mathematica [A]
time = 0.13, size = 119, normalized size = 0.63

$$\frac{2\left(-3(3A+5C)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+9A\sin(c+dx)+15C\sin(c+dx)+5B\tan(c+dx)+3A\sec(c+dx)\tan(c+dx)\right)}{15b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*C
os[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(216) = 432$.
time = 1.01, size = 807, normalized size = 4.29

method	result	size
default	Expression too large to display	807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x
+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^4+120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*C*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)
^4+36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-
120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*C*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+30*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-15*C*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/
2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 223, normalized size = 1.19

$$\frac{-5i\,\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5i\,\sqrt{2}\,B\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\,\sqrt{2}\,\left(3i\,A+5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\,\sqrt{2}\,\left(-3i\,A-5i\,C\right)\sqrt{b}\,\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\,\left(3\,\left(3\,A+5\,C\right)\cos\left(dx+c\right)^{2}+5\,B\cos\left(dx+c\right)+3\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,b^{4}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*A + 5*I*C
)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*A - 5*I*C)*sqrt(b)*cos(d*
x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c))) + 2*(3*(3*A + 5*C)*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(7/2), x)
```

3.288 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+$

Optimal. Leaf size=223

$$\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(5A+4C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3B\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+c}{8d}$$

```
[Out] 1/4*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*C*cos(d*x+c)^(
7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x
+c)^(1/2)+1/5*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-
1/15*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*B*s
in(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.09, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$-\frac{(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((5*A + 4*C)*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(5*d) - ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)
/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)\,(}{\sqrt{c}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

$$= \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

$$= \frac{(5A+4C)\sqrt{b\cos(c+dx)}\,\sin(c}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(5A+4}{}$$

Mathematica [A]

time = 0.31, size = 109, normalized size = 0.49

$$\frac{\sqrt{b\cos(c+dx)}\,(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))+40A\sin(3(c+dx))+50C\sin(3(c+dx))+15B\sin(4(c+dx))+6C\sin(5(c+dx)))}{480d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x] +
120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] + 15
*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 3.02, size = 134, normalized size = 0.60

method	result
default	$\frac{\sqrt{b\cos(dx+c)}\,(24C(\cos^4(dx+c))\sin(dx+c)+30B(\cos^3(dx+c))\sin(dx+c)+40A(\cos^2(dx+c))\sin(dx+c)+32C\sin(dx+c)(c}{120d\sqrt{\cos(dx+c)}}$
risch	$\frac{3\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}Bx}{4(e^{2i(dx+c)}+1)}-\frac{i\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80(e^{2i(dx+c)}+1)d}-\frac{i\sqrt{b\cos(d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120/d*(b*cos(d*x+c))^(1/2)*(24*C*cos(d*x+c)^4*sin(d*x+c)+30*B*cos(d*x+c)^
3*sin(d*x+c)+40*A*cos(d*x+c)^2*sin(d*x+c)+32*C*sin(d*x+c)*cos(d*x+c)^2+45*B
*cos(d*x+c)*sin(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*C*sin(d*x+c))/cos(d*
x+c)^(1/2)
```

Maxima [A]
time = 0.67, size = 159, normalized size = 0.71

$$\frac{15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)B\sqrt{b}+2\,C\sqrt{b}\left(3\,\sin(5\,dx+5\,c)+25\,\sin\left(\frac{3}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)+150\,\sin\left(\frac{1}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)\right)+40\,A\sqrt{b}\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25
*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) + 150*sin(1/5*arctan2
(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 40*A*sqrt(b)*(sin(3*d*x + 3*c) + 9
*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

Fricas [A]
time = 0.42, size = 292, normalized size = 1.31

$$\left[\frac{45\,B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(24\,C\cos\left(dx+c\right)^{4}+30\,B\cos\left(dx+c\right)^{3}+8\,\left(5\,A+4\,C\right)\cos\left(dx+c\right)^{2}+45\,B\cos\left(dx+c\right)+80\,A+64\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)},\ \frac{45\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(24\,C\cos\left(dx+c\right)^{4}+30\,B\cos\left(dx+c\right)^{3}+8\,\left(5\,A+4\,C\right)\cos\left(dx+c\right)^{2}+45\,B\cos\left(dx+c\right)+80\,A+64\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*cos(d*x + c
)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c
) + 80*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)), 1/120*(45*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*cos(d*x + c)^4 + 30*B*co
s(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80*A + 64
*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0Simplification
```

Mupad [B]
time = 3.64, size = 141, normalized size = 0.63

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\,\sin\left(4\,c+4\,d\,x\right)+135\,B\,\sin\left(3\,c+3\,d\,x\right)+15\,B\,\sin\left(5\,c+5\,d\,x\right)+350\,C\,\sin\left(2\,c+2\,d\,x\right)+56\,C\,\sin\left(4\,c+4\,d\,x\right)+6\,C\,\sin\left(6\,c+6\,d\,x\right)+360\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{480\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*sin(
2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*sin(5*
c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6*c +
 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.289 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=184

$$\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(}{8d}$$

```
[Out] 1/4*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*(4*A+3*C)*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)-1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/
8*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c +
 d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x
]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)}{\sqrt{}}$$

$$=\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

$$=\frac{C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

$$=\frac{(4A+3C)\sqrt{\cos(c+dx)}\sqrt{b\,c}}{8d}$$

$$=\frac{(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+$$

Mathematica [A]
time = 0.29, size = 92, normalized size = 0.50

$$\frac{\sqrt{b\cos(c+dx)}\,(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c +
 d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c +
 d*x)]))/(96*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.35, size = 114, normalized size = 0.62

method	result
default	$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(6C\left(\cos^{3}(dx+c)\right)\sin(dx+c)+8B\left(\cos^{2}(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)\right.}{24d\sqrt{\cos\left(dx+c\right)}}$
risch	$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{i(dx+c)}(8A+6C)x}{8\,\mathrm{e}^{2i(dx+c)}+8}-\frac{i\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{i\sqrt{b\cos}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*s
in(d*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.67, size = 116, normalized size = 0.63

$$\frac{24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))A\sqrt{b}+3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right))C\sqrt{b}+8\,B\sqrt{b}\,(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right))}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*
sqrt(b) + 8*B*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))))/d
```

Fricas [A]
time = 0.42, size = 276, normalized size = 1.50

$$\left[\frac{3\,(4\,A+3\,C)\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,(6\,C\cos\left(dx+c\right)^{3}+8\,B\cos\left(dx+c\right)^{2}+3\,(4\,A+3\,C)\cos\left(dx+c\right)+16\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)},\ \frac{3\,(4\,A+3\,C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+(6\,C\cos\left(dx+c\right)^{3}+8\,B\cos\left(dx+c\right)^{2}+3\,(4\,A+3\,C)\cos\left(dx+c\right)+16\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*cos(
d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(3*(
4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*
(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0Simplification
```

Mupad [**B**]
time = 2.81, size = 137, normalized size = 0.74

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(4\,c+4\,d\,x\right)+27\,C\,\sin\left(3\,c+3\,d\,x\right)+3\,C\,\sin\left(5\,c+5\,d\,x\right)+96\,A\,d\,x\,\cos\left(c+d\,x\right)+72\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{96\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.290 $\int \sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)$

Optimal. Leaf size=143

$$\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{(3A+2C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/3*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*B*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)+1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\frac{(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int \cos(c+d}{}$$

$$= \frac{C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+d}}{3d}$$

$$= \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{(3A+}{}$$

Mathematica [A]
time = 0.21, size = 75, normalized size = 0.52

$$\frac{\sqrt{b\cos(c+dx)}\,(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*C
os[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*S
in[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.26, size = 83, normalized size = 0.58

method	result
default	$\frac{\sqrt{b\cos(dx+c)}\,(2C\sin(dx+c)(\cos^2(dx+c))+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c))}{6d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}Bx}{e^{2i(dx+c)}+1} - \frac{i\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}(dx+c)\right)e^{4i(dx+c)}C}{12\left(e^{2i(dx+c)}+1\right)d} - \frac{i\sqrt{b\cos(dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(1/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(
d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.66, size = 80, normalized size = 0.56

$$\frac{3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))B\sqrt{b}\,+C\sqrt{b}\,\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)+12\,A\sqrt{b}\,\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + C*sqrt(b)*(sin(3*d*x +
 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 12*A*sqrt
(b)*sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 236, normalized size = 1.65

$$\left[\frac{3\,B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)},\ \frac{3\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*cos(d*x + c)^2
 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)
^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A]
time = 41.50, size = 241, normalized size = 1.69

$$\begin{cases} 0 & \text{for}\ c = -dx + \frac{\pi}{2} \vee c = -dx + \frac{3\pi}{2} \\ x\sqrt{b\cos\left(c\right)}\,\left(A + B\cos\left(c\right) + C\cos^{2}\left(c\right)\right)\sqrt{\cos\left(c\right)} & \text{for}\ d = 0 \\ \frac{A\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)}{d\sqrt{\cos\left(c+dx\right)}} + \frac{Bx\sqrt{b\cos\left(c+dx\right)}\,\sin^{2}\left(c+dx\right)}{2\sqrt{\cos\left(c+dx\right)}} + \frac{Bx\sqrt{b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}{2} + \frac{B\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{2d} + \frac{2C\sqrt{b\cos\left(c+dx\right)}\,\sin^{3}\left(c+dx\right)}{3d\sqrt{\cos\left(c+dx\right)}} + \frac{C\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)\cos^{\frac{3}{2}}\left(c+dx\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Piecewise((0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (x*sqrt(b*cos(c))
*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c)), Eq(d, 0)), (A*sqrt(b*cos(c + d*
x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*sin(c +
d*x)**2/(2*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/
2)/2 + B*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d) + 2*C*s
```

```
qrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*d*sqrt(cos(c + d*x))) + C*sqrt(b*cos
(c + d*x))*sin(c + d*x)*cos(c + d*x)**(3/2)/d, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0(12*sqrt(sageVA
```

Mupad [B]
time = 1.40, size = 104, normalized size = 0.73

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)+12\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.291 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=123

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*C*x*(b*cos(d*x+c))^(1/2)/cos(
d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*C*sin
(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$,
Rules used = {17, 2717, 2715, 8}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos
[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/
(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C}{\sqrt{\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)}{\sqrt{\cos(c+}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\,\sin}{d\sqrt{\cos(c+dx}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 61, normalized size = 0.50

$$\frac{\sqrt{b\cos(c+dx)}\,(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sq
rt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(
c + d*x)]))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.24, size = 63, normalized size = 0.51

method	result	size
default	$\frac{\sqrt{b\cos(dx+c)}\,(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{\cos(dx+c)}}$	63
risch	$\frac{\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{B\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x
+c)+C*(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.64, size = 64, normalized size = 0.52

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))C\sqrt{b}+8\,A\sqrt{b}\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,B\sqrt{b}\,\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*
x + c)/(cos(d*x + c) + 1)) + 4*B*sqrt(b)*sin(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 212, normalized size = 1.72

$$\left[\frac{(2\,A+C)\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,(C\cos\left(dx+c\right)+2\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)}, \frac{(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+(C\cos\left(dx+c\right)+2\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos
(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*cos(d*x + c
) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A]
time = 19.57, size = 184, normalized size = 1.50

$$\begin{cases} \frac{Ax\sqrt{b\cos\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)}}+\frac{B\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)}{d\sqrt{\cos\left(c+dx\right)}}+\frac{Cx\sqrt{b\cos\left(c+dx\right)}\,\sin^{2}\left(c+dx\right)}{2\sqrt{\cos\left(c+dx\right)}}+\frac{Cx\sqrt{b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}{2}+\frac{C\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{2d} & \text{for}\: d\neq 0 \\ \frac{x\sqrt{b\cos\left(c\right)}\,(A+B\cos\left(c\right)+C\cos^{2}\left(c\right))}{\sqrt{\cos\left(c\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x)) + B*sqrt(b*cos(c + d
*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*sin(c +
 d*x)**2/(2*sqrt(cos(c + d*x))) + C*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3
/2)/2 + C*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), Ne(d,
 0)), (x*sqrt(b*cos(c))*(A + B*cos(c) + C*cos(c)**2)/sqrt(cos(c)), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt
(cos(d*x + c)), x)
```

Mupad [**B**]
time = 0.56, size = 54, normalized size = 0.44

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x +
2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

3.292 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)+C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3102, 2814, 3855}

$$\frac{A\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(3/2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C\,\cdots}{\sqrt{\cos(c+dx)}}$$
$$= \frac{C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\sqrt{b\cos(c\cdots}}{\cdots}$$
$$= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b\cos(c+dx)}\,\sin\cdots}{d\sqrt{\cos(c+dx\cdots}}$$
$$= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx\cdots}{d\sqrt{\cos(c\cdots}}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 1.00

$$\frac{\sqrt{b\cos(c+dx)}\,\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqr
t[Cos[c + d*x]])
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.68

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$

risch	$\frac{Bx\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}-\frac{i\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{i\sqrt{b\cos\left(dx+c\right)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*co
s(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.63, size = 104, normalized size = 1.12

$$\frac{A\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\sin\left(dx+c\right)+1\right)\right)+4B\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+2C\sqrt{b}\sin\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*sqrt(b)*sin(d*x + c))/d
```

Fricas [A]
time = 0.47, size = 304, normalized size = 3.27

$$\left[-\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)-B\sqrt{-b}\cos\left(dx+c\right)\log\left(2b\cos\left(dx+c\right)^{2}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)}, \frac{2B\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c
)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*
cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\cos\left(c+dx\right)}\left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos(
c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2), x)
```

3.293 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+C*x*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(1/2)+B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3100, 2814, 3855}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(5/2),x]
```

```
[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c
+ d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
```

```
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx)+C\,\cdots}{\sqrt{\cos(c+dx)}\cdots}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b\cos(c-\cdots}}{\cdots}$$
$$= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\,\sin\cdots}{d\cos^{\frac{3}{2}}(c+dx\cdots}$$
$$= \frac{Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\tanh^{-1}(\sin(c+dx\cdots}{d\sqrt{\cos(c\cdots}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.65

$$\frac{\sqrt{b\cos(c+dx)}\,\left(Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c +
 d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.20, size = 72, normalized size = 0.77

method	result

default	$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)\right)}{d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{Cx\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}+\frac{2i\sqrt{b\cos\left(dx+c\right)}\,A}{\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\sqrt{b\cos\left(dx+c\right)}\,B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}-\frac{\sqrt{b\cos\left(dx\right.}}{\sqrt{\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(1/2)*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.64, size = 144, normalized size = 1.55

$$\frac{B\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)+4\,C\sqrt{b}\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{4\,A\sqrt{b}\,\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*C*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.45, size = 312, normalized size = 3.35

$$\left[-\frac{2\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^2-C\sqrt{-b}\,\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)^2},\ \frac{2\,C\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^2+B\sqrt{b}\,\cos\left(dx+c\right)^2\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(
b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-(b*c
os(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x +
```

```
c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2), x)
```

3.294 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\frac{(A+2C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(7/2),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*
x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx)+C\,\cdots}{\sqrt{\cos(c+dx)}}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\sqrt{b\cos(c\cdots}}{\cdots}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos\cdots\right.}}{\cdots}$$
$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+d\cdots}}{2d\sqrt{\cos(c+dx)}}$$
$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+d\cdots}}{2d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 0.62

$$\frac{\sqrt{b\cos(c+dx)}\,\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A
+ 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.22, size = 151, normalized size = 1.36

method	result
default	$-\frac{\left(A\left(\cos^{2}(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A\left(\cos^{2}(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)+4C\left(\cos^{2}(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos}{\sin(}\right.\right.}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\left(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{\sqrt{b\cos(dx+c)}\,(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((
-1+cos(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(95) = 190$.
time = 0.69, size = 780, normalized size = 7.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1)
 - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
```

```
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2
*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*
x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 8*
B*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.42, size = 233, normalized size = 2.10

$$\left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2), x)
```

3.295 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+(2$$

```
[Out] 1/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*(2*A+3*C)*sin(d*x+c)*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(9/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (B*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + ((2*A + 3*C)*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos$$

$$\sqrt{\cos(c+dx)}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+}}{}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\Big(B\sqrt{b\cos(}}{}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c}}{2d\cos}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{}}{}$$

Mathematica [A]
time = 0.47, size = 87, normalized size = 0.57

$$\frac{\sqrt{b\cos(c+dx)}\,\left(3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)\right)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*
C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Co
s[c + d*x]^(5/2))
```

Maple [A]
time = 0.24, size = 157, normalized size = 1.03

method	result
default	$\frac{\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^3(dx+c)\right)+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^3(dx+c)\right)+4A\left(\cos^2(dx+c)\right)\sin(dx+c)+}{6d\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\left(3B\,\mathrm{e}^{5i(dx+c)}-6C\,\mathrm{e}^{4i(dx+c)}-12A\,\mathrm{e}^{2i(dx+c)}-12C\,\mathrm{e}^{2i(dx+c)}-3B\,\mathrm{e}^{i(dx+c)}-4A-6C\right)}{3\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{\sqrt{b\cos(dx}}{2\sqrt{\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. 2(128) = 256.
time = 0.73, size = 1009, normalized size = 6.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
```

```
+ 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c
)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(
2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 24*C*sqrt(b)*sin(2*d*x + 2*c)/(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.43, size = 265, normalized size = 1.74

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{4}},\ -\frac{3\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{4}-\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*
B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*
x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c)
 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^4)]
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.296 $\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=193

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{(3A+4C)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] 1/4*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*(3*A+4*C)*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(3/2)+1/3*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+
c)^(7/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(11/2),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int (A+B\cos(c+dx)+C\,}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\sqrt{b\cos(c-}{}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\Big(B\sqrt{b\cos}}{}$$

$$= \frac{A\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)}{}$$

$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+}}{8d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.37, size = 110, normalized size = 0.57

$$\frac{\sqrt{b\cos(c+dx)}\,\left(3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)\right)\right)}{24d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Co
s[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 +
 Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8
*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.21, size = 248, normalized size = 1.28

method	result
default	$\frac{\left(-9A\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)-12C\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(}{}\right.\right.}{}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\left(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+c)}\right.}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*cos(d*x+c)^2*sin
(d*x+c)+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+
c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(165) = 330.
time = 0.78, size = 2611, normalized size = 13.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
```

```
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*
x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*
d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*
c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos
(2*d*x + 2*c) + 1) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*c
os(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) +
```

```
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(sin(4*d*x + 4*c) + 2*
sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x ...
```

Fricas [A]
time = 0.46, size = 299, normalized size = 1.55

$$\left[\frac{3\,(3\,A+4\,C)\sqrt{b}\,\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos\left(dx+c\right)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}\right)+2\,\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{5}},\ -\frac{3\,(3\,A+4\,C)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x
 + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16*B
*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.297 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A + B\cos(c+dx) + C\right.$

Optimal. Leaf size=229

$$\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(5A+4C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3bB\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+}{8d}$$

```
[Out] 1/4*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*b*C*cos(d*x+
c)^(7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*B*x*(b*cos(d*x+c))^(1/2)/c
os(d*x+c)^(1/2)+1/5*b*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c
)^(1/2)-1/15*b*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)+3/8*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$-\frac{b(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{b(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3bB\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}+\frac{bC\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(5*A + 4*C)*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c
 + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)
*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^(7/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^3(c$$

$$= \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+d}}{5d}$$

$$= \frac{bC\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+d}}{5d}$$

$$= \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+d}}{4d}$$

$$= \frac{b(5A+4C)\sqrt{b\cos(c+dx)}\ \mathrm{s}}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(5A}{}$$

Mathematica [A]

time = 0.35, size = 109, normalized size = 0.48

$$\frac{(b\cos(c+dx))^{3/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))+40A\sin(3(c+dx))+50C\sin(3(c+dx))+15B\sin(4(c+dx))+6C\sin(5(c+dx)))}{480d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.20, size = 134, normalized size = 0.59

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(24C\left(\cos^4(dx+c)\right)\sin(dx+c)+30B\left(\cos^3(dx+c)\right)\sin(dx+c)+40A\left(\cos^2(dx+c)\right)\sin(dx+c)+32C\sin(dx+c)\left(\cos^2(dx+c\right.\right.}{120d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{3b\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}Bx}{4\left(e^{2i(dx+c)}+1\right)}-\frac{ib\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}C}{80\left(e^{2i(dx+c)}+1\right)d}-\frac{ib\sqrt{b\cos(d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120/d*(b*cos(d*x+c))^(3/2)*(24*C*cos(d*x+c)^4*sin(d*x+c)+30*B*cos(d*x+c)^
3*sin(d*x+c)+40*A*cos(d*x+c)^2*sin(d*x+c)+32*C*sin(d*x+c)*cos(d*x+c)^2+45*B
*cos(d*x+c)*sin(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*C*sin(d*x+c))/cos(d*
x+c)^(3/2)
```

Maxima [A]
time = 0.71, size = 169, normalized size = 0.74

$$\frac{40\left(b\sin(3dx+3c)+9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)A\sqrt{b}+15\left(12(dx+c)b+b\sin(4dx+4c)+8b\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)\right)B\sqrt{b}+2\left(3b\sin(5dx+5c)+25b\sin\left(\frac{3}{5}\arctan(\sin(5dx+5c),\cos(5dx+5c))\right)+150b\sin\left(\frac{1}{5}\arctan(\sin(5dx+5c),\cos(5dx+5c))\right)\right)C\sqrt{b}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(40*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*(3*b*sin(
5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))) +
150*b*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.43, size = 309, normalized size = 1.35

$$\left[\frac{45\,B\sqrt{-b}\,b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(24\,Cb\cos\left(dx+c\right)^{4}+30\,Bb\cos\left(dx+c\right)^{3}+8\,(5\,A+4\,C)b\cos\left(dx+c\right)^{2}+45\,Bb\cos\left(dx+c\right)+16\,(5\,A+4\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)}, \frac{45\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(24\,Cb\cos\left(dx+c\right)^{4}+30\,Bb\cos\left(dx+c\right)^{3}+8\,(5\,A+4\,C)b\cos\left(dx+c\right)^{2}+45\,Bb\cos\left(dx+c\right)+16\,(5\,A+4\,C)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b*cos(d*x
 + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 45*B*b*c
os(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*b*cos(d*x
 + c)^4 + 30*B*b*cos(d*x + c)^3 + 8*(5*A + 4*C)*b*cos(d*x + c)^2 + 45*B*b*c
os(d*x + c) + 16*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x
+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c
)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0Simplification
```

Mupad [B]
time = 3.15, size = 142, normalized size = 0.62

$$\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\,\sin\left(4\,c+4\,d\,x\right)+135\,B\,\sin\left(3\,c+3\,d\,x\right)+15\,B\,\sin\left(5\,c+5\,d\,x\right)+350\,C\,\sin\left(2\,c+2\,d\,x\right)+56\,C\,\sin\left(4\,c+4\,d\,x\right)+6\,C\,\sin\left(6\,c+6\,d\,x\right)+360\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{480\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*si
n(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*sin(
5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6*c
 + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.298 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\,(A+B\cos(c+dx$

Optimal. Leaf size=189

$$\frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}$$

```
[Out] 1/4*b*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b*(4*A+3*C)*
x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)-1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^
(1/2)+1/8*b*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{bx(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}-\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b
*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[C
os[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(
5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*B*Sqrt[b*Cos[c + d*x]]*S
in[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\cos^2(c\ldots}{\ldots}$$

$$= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+\ldots}}{4d}\ldots$$

$$= \frac{bC\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+\ldots}}{4d}\ldots$$

$$= \frac{b(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{\ldots}}{8d\ldots}\ldots$$

$$= \frac{b(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}\ldots$$

Mathematica [A]
time = 0.24, size = 92, normalized size = 0.49

$$\frac{(b\cos(c+dx))^{3/2}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
 + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
 + d*x)]))/(96*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.34, size = 114, normalized size = 0.60

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\left(\cos^2(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A\right.}{24d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{b\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{i(dx+c)}(8A+6C)x}{8\,\mathrm{e}^{2i(dx+c)}+8}-\frac{ib\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{5i(dx+c)}C}{32\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{ib\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(3/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*s
in(d*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.73, size = 126, normalized size = 0.67

$$\frac{24\,(2\,(dx+c)b+b\sin(2\,dx+2\,c))A\sqrt{b}+8\left(b\sin(3\,dx+3\,c)+9\,b\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)B\sqrt{b}+3\left(12\,(dx+c)b+b\sin(4\,dx+4\,c)+8\,b\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)C\sqrt{b}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b*sin(3*d*x +
3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b)
+ 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.41, size = 285, normalized size = 1.51

$$\left[\frac{3\,(4\,A+3\,C)\sqrt{-b}\,b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)+2\left(6\,Cb\cos(dx+c)^3+8\,Bb\cos(dx+c)^2+3\,(4\,A+3\,C)b\cos(dx+c)+16\,Bb\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{48\,d\cos(dx+c)},\ \frac{3\,(4\,A+3\,C)b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+\left(6\,Cb\cos(dx+c)^3+8\,Bb\cos(dx+c)^2+3\,(4\,A+3\,C)b\cos(dx+c)+16\,Bb\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{24\,d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqr
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*b*
cos(d*x + c)^3 + 8*B*b*cos(d*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B
*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)),
 1/24*(3*(4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b*cos(d*x + c)^3 + 8*B*b*cos(d
*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B*b)*sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0Simplification
```

Mupad [**B**]
time = 1.77, size = 138, normalized size = 0.73

$$\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(4\,c+4\,d\,x\right)+27\,C\,\sin\left(3\,c+3\,d\,x\right)+3\,C\,\sin\left(5\,c+5\,d\,x\right)+96\,A\,d\,x\,\cos\left(c+d\,x\right)+72\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{96\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(
c + d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*
d*x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x)
 + 72*C*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))
```

3.299 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b(3A+2C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+}{2d}$$

[Out] 1/3*b*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$,
Rules used = {17, 3102, 2813}

$$\frac{b(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}+\frac{bC\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*(3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b*C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,(A+B\,\mathrm{c}}{\sqrt{\cos(c+dx}}$$

$$= \frac{bC\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b(3A+2C)\sqrt{b\cos(}}{3d\sqrt{\cos}}$$

Mathematica [A]
time = 0.08, size = 76, normalized size = 0.52

$$\frac{b\sqrt{b\cos(c+dx)}\,(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B
*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.26, size = 83, normalized size = 0.56

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(2C\sin(dx+c)\left(\cos^2(dx+c)\right)+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c)\right)}{6d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{bBx\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(dx+c)}}+\frac{b\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{b\,\mathrm{co}}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(3/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(
d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.66, size = 86, normalized size = 0.59

$$\frac{12\,Ab^{\frac{3}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b+b\sin\left(2\,dx+2\,c\right)\right)B\sqrt{b}+\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)C\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + 3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*
sqrt(b) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 249, normalized size = 1.69

$$\left[\frac{3\,B\sqrt{-b}\,b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(2\,Cb\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+c\right)+2\left(3\,A+2\,C\right)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)},\ \frac{3\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(2\,Cb\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+c\right)+2\left(3\,A+2\,C\right)b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x +
c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(3/2)*arctan(sqrt(b*co
s(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*
b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sq
rt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.81, size = 71, normalized size = 0.48

$$\frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+6\,B\,d\,x\right)}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*sin(2
*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

3.300 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*C*x*(b*cos(d*x+c))^(1/2)/
cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2
*b*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x
]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqr
t[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x
])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right.}{\sqrt{\cos(c}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+c}}$$

$$= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.13, size = 61, normalized size = 0.48

$$\frac{(b\cos(c+dx))^{3/2}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2
*(c + d*x)]))/(4*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.50

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\cos(dx+c)^{\frac{3}{2}}}$	63
risch	$\frac{b\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{bB\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{b\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x
+c)+C*(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.65, size = 67, normalized size = 0.53

$$\frac{8\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,Bb^{\frac{3}{2}}\sin\left(dx+c\right)+\left(2\left(dx+c\right)b+b\sin\left(2\,dx+2\,c\right)\right)C\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(3/2)*sin(
d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Fricas [A]
time = 0.43, size = 217, normalized size = 1.71

$$\left[\frac{\left(2\,A+C\right)\sqrt{-b}\,b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(Cb\cos\left(dx+c\right)+2\,Bb\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)}, \frac{\left(2\,A+C\right)b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(Cb\cos\left(dx+c\right)+2\,Bb\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*A + C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b*cos(d*x
 + c) + 2*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)), 1/2*((2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)
/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b*cos(d*x + c) + 2*B*b)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.17, size = 55, normalized size = 0.43

$$\frac{b\sqrt{b\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*x
+ 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

3.301 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=96

$$\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b*C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3102, 2814, 3855}

$$\frac{Ab\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bC\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int\left(A+B\cos(c+dx)+\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{bC\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{b\cos\right.}{}$$

$$= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+}}$$

$$= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab\tanh^{-1}(\sin(c+}{d\sqrt{\cos(}}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.97

$$\frac{(b\cos(c+dx))^{3/2}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*C
os[c + d*x]^(3/2))
```

Maple [A]
time = 0.20, size = 63, normalized size = 0.66

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)(b\cos(dx+c))^{\frac{3}{2}}}{d\cos(dx+c)^{\frac{3}{2}}}$

risch	$\frac{bBx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}} - \frac{ib\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d} + \frac{ib\sqrt{b\cos(dx+c)}\,C\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\,d} - \frac{b\sqrt{b\cos(dx}{\sqrt{\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*co
s(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.66, size = 107, normalized size = 1.11

$$\frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 2\,Cb^{\frac{3}{2}}\sin(dx+c) + \left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right) - b\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)A\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(3/2)*sin(
d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

Fricas [A]
time = 0.46, size = 308, normalized size = 3.21

$$\left[-\frac{2\,A\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c) - B\sqrt{-b}\,b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 - 2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c) - b\right) - 2\,\sqrt{b\cos(dx+c)}\,Cb\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)}, \frac{2\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c) + Ab^{\frac{3}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3 - 2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c) - 2\,b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2\,\sqrt{b\cos(dx+c)}\,Cb\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d
*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)), 1/2*(2*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(
b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d
*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c)
- 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2), x)
```

3.302 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=96

$$\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*C*x*(b*cos(d*x+c))
^(1/2)/cos(d*x+c)^(1/2)+b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3100, 2814, 3855}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(7/2),x]
```

```
[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*
Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx)+}{\sqrt{\cos(c+dx}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos}\right.}{}$$

$$= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{bCx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\tanh^{-1}(\sin(c+}{d\sqrt{\cos(}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{3/2}\left(Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c
 + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.18, size = 72, normalized size = 0.75

method	result

default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\left(-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)\right)}{d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{bCx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{b\sqrt{b\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{b\cos(}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.66, size = 147, normalized size = 1.53

$$\frac{4\,Cb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)B\sqrt{b}+\frac{4\,Ab^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d
*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/2)*sin(2*d*x + 2*c)/(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.46, size = 316, normalized size = 3.29

$$\left[-\frac{2\,B\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2-C\sqrt{-b}\,b\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\sqrt{b\cos(dx+c)}\,Ab\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)^2}, \frac{2\,Cb^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)^2+Bb^{\frac{3}{2}}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos(dx+c)}\,Ab\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*c
os(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - b) - 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^2), 1/2*(2*C*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(3/2)*cos(d*x + c)^2*log
(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin
(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2), x)
```

3.303 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{b(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b*(A+2*C)*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\frac{b(A+2C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c +
d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) +
 (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+ \ldots}{\sqrt{\cos(c+dx \ldots}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos \ldots\right.}{\ldots}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\,c \ldots\right.}{\ldots}$$

$$= \frac{b(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+ \ldots}}{2d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+ \ldots}}{2d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 69, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{3/2}\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))

Maple [A]
time = 0.20, size = 151, normalized size = 1.32

method	result
default	$-\frac{\left(A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)+4C(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{-1+}{\sin}\right.\right.}{2d\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B)}{\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^2}-\frac{b\sqrt{b\cos(dx+c)}\,(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(98) = 196.
time = 0.72, size = 813, normalized size = 7.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) + 8*B*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))

```
)) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*
c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(
2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)
*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*
c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.40, size = 240, normalized size = 2.11

$$\left[\frac{(A+2C)b^{\frac{3}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2Bb\cos(dx+c)+Ab)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2Bb\cos(dx+c)+Ab)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arcta
n(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*
x + c)^3 - (2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.304 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+$$

```
[Out] 1/3*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*b*(2*A+3*C)*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{b(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*(2*A + 3*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)}{\sqrt{\cos(c+a}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b\sqrt{b\,\mathrm{co}}\right.}{}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(bB\sqrt{b}\right.}{}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{bB\sqrt{b\,\mathrm{c}}}{2}$$

$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.06, size = 88, normalized size = 0.56

$$\frac{b\sqrt{b\cos(c+dx)}\left(3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)\right)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*
Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.22, size = 157, normalized size = 1.01

method	result
default	$\frac{\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+4A(\cos^2(dx+c))\sin(dx+c)+6C}{6d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}+\frac{b\sqrt{b\cos(dx}{2\sqrt{\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. 2(132) = 264.
time = 0.74, size = 1044, normalized size = 6.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9
*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x
```

```
 + 6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*co
s(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c
)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 +
9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(b*sin
(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos
(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c
) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x +
4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)
*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c)
 + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.43, size = 272, normalized size = 1.74

$$\left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\left(2\,A+3\,C\right)b\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+c\right)+2\,Ab\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{4}},\ -\frac{3\,B\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{4}-\left(2\left(2\,A+3\,C\right)b\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+c\right)+2\,Ab\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*s
qrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1
/6*(3*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqr
t(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*
cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.305 $\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=198

$$\frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+}}{8d\cos^{\frac{5}{2}}(c+}$$

```
[Out] 1/4*A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b*(3*A+4*C)*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b*B*sin(d*x+c)*(b*cos(d*
x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/
cos(d*x+c)^(7/2)+1/8*b*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 3853, 3855}

$$\frac{b(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(13/2),x]
```

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c
+ d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2))
 + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2
)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*B*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)+}{\sqrt{\cos(c+dx}}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos}\right.}{}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\,c}\right.}{}$$

$$= \frac{Ab\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b(3A+4C}{}$$

$$= \frac{b(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+}}{8d\sqrt{\cos(c+dx)}}$$

Mathematica [A]

time = 0.27, size = 111, normalized size = 0.56

$$\frac{b\sqrt{b\cos(c+dx)}\,(3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\,(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)))}{24d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 +
 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.21, size = 248, normalized size = 1.25

method	result
default	$\frac{\left(-9A(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))-12C(\cos^4(dx+c))\ln\left(-\frac{-1+c}{}\right.\right.}{}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx}}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*cos(d*x+c)^2*sin
(d*x+c)+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+
c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(170) = 340$.
time = 0.80, size = 2732, normalized size = 13.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4
*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
```

```
*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
+ 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)
^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)
^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d
*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x
+ 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c)
+ 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^
2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^
2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b
*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(
2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*
x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x +
 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) +
 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*
b*cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c)
+ 4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1) + 64*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*
```

```
x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3
*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2
*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*
sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4
*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
 + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d...
```

Fricas [A]
time = 0.48, size = 308, normalized size = 1.56

$$\left[\frac{3\,(3\,A+4\,C)b^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\left(16\,Bb\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)b\cos\left(dx+c\right)^{2}+8\,Bb\cos\left(dx+c\right)+6\,Ab\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{5}},-\frac{3\,(3\,A+4\,C)\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,Bb\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)b\cos\left(dx+c\right)^{2}+8\,Bb\cos\left(dx+c\right)+6\,Ab\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)
^2 + 8*B*b*cos(d*x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^5 - (16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)^2 + 8*B*b*cos(d*
x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(13/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/co
s(d*x + c)^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{3/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2), x)
```

3.306 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\,(A+B\cos(c+dx$

Optimal. Leaf size=241

$$\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{3b^2B\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin}{8d}$$

```
[Out] 1/4*b^2*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/5*b^2*C*cos(
d*x+c)^(7/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*B*x*(b*cos(d*x+c))^(
1/2)/cos(d*x+c)^(1/2)+1/5*b^2*(5*A+4*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)-1/15*b^2*(5*A+4*C)*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(
d*x+c)^(1/2)+3/8*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2713, 2715, 8}

$$-\frac{b^2(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{b^2(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}+\frac{b^2C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(5*A + 4*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt
[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*C*Cos[c + d*x]^(7/
2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b^2*(5*A + 4*C)*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^3(\ldots}{\ldots}$$

$$=\frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+\ldots}}{5d}\ldots$$

$$=\frac{b^2C\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+\ldots}}{5d}\ldots$$

$$=\frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+\ldots}}{4d}\ldots$$

$$=\frac{b^2(5A+4C)\sqrt{b\cos(c+dx)}\ldots}{5d\sqrt{\cos(c+dx)}\ldots}\ldots$$

$$=\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2\ldots}{\ldots}$$

Mathematica [A]

time = 0.30, size = 109, normalized size = 0.45

$$\frac{(b\cos(c+dx))^{5/2}(180Bc+180Bdx+60(6A+5C)\sin(c+dx)+120B\sin(2(c+dx))+40A\sin(3(c+dx))+50C\sin(3(c+dx))+15B\sin(4(c+dx))+6C\sin(5(c+dx)))}{480d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.20, size = 134, normalized size = 0.56

method	result
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(24C\left(\cos^4(dx+c)\right)\sin(dx+c)+30B\left(\cos^3(dx+c)\right)\sin(dx+c)+40A\left(\cos^2(dx+c)\right)\sin(dx+c)+32C\sin(dx+c)\left(\cos^2(dx+\right.\right.}{120d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{3b^2\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{i(dx+c)}Bx}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{ib^2\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{6i(dx+c)}C}{80\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-ib^2\sqrt{b\,\mathrm{co}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/120/d*(b*cos(d*x+c))^(5/2)*(24*C*cos(d*x+c)^4*sin(d*x+c)+30*B*cos(d*x+c)^
3*sin(d*x+c)+40*A*cos(d*x+c)^2*sin(d*x+c)+32*C*sin(d*x+c)*cos(d*x+c)^2+45*B
*cos(d*x+c)*sin(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*C*sin(d*x+c))/cos(d*
x+c)^(5/2)
```

Maxima [A]
time = 0.71, size = 185, normalized size = 0.77

$$\frac{40\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)A\sqrt{b}+15\left(12\left(dx+c\right)b^2+b^2\sin\left(4\,dx+4\,c\right)+8\,b^2\sin\left(\frac{1}{2}\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)B\sqrt{b}+2\left(3\,b^2\sin\left(5\,dx+5\,c\right)+25\,b^2\sin\left(\frac{3}{5}\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,b^2\sin\left(\frac{1}{5}\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)\right)C\sqrt{b}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/480*(40*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c)
+ 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2
*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d
*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))
*C*sqrt(b))/d
```

Fricas [A]
time = 0.44, size = 331, normalized size = 1.37

$$\left[\frac{45\,B\sqrt{-b}\,b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(24\,Cb^{2}\cos\left(dx+c\right)^{4}+30\,Bb^{2}\cos\left(dx+c\right)^{3}+8\,\left(5\,A+4\,C\right)b^{2}\cos\left(dx+c\right)^{2}+45\,Bb^{2}\cos\left(dx+c\right)+16\,\left(5\,A+4\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)},\ \frac{45\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(24\,Cb^{2}\cos\left(dx+c\right)^{4}+30\,Bb^{2}\cos\left(dx+c\right)^{3}+8\,\left(5\,A+4\,C\right)b^{2}\cos\left(dx+c\right)^{2}+45\,Bb^{2}\cos\left(dx+c\right)+16\,\left(5\,A+4\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{120\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b^2*cos
(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x + c)^2 +
45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*b^(5/2)*arctan(sqrt(b*
cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24
*C*b^2*cos(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x
 + c)^2 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0Simplification
```

Mupad [B]
time = 3.04, size = 144, normalized size = 0.60

$$\frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(120\,B\,\sin\left(c+d\,x\right)+400\,A\,\sin\left(2\,c+2\,d\,x\right)+40\,A\,\sin\left(4\,c+4\,d\,x\right)+135\,B\,\sin\left(3\,c+3\,d\,x\right)+15\,B\,\sin\left(5\,c+5\,d\,x\right)+350\,C\,\sin\left(2\,c+2\,d\,x\right)+56\,C\,\sin\left(4\,c+4\,d\,x\right)+6\,C\,\sin\left(6\,c+6\,d\,x\right)+360\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{480\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c
 + d*x)^2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(120*B*sin(c + d*x) + 400*A*
sin(2*c + 2*d*x) + 40*A*sin(4*c + 4*d*x) + 135*B*sin(3*c + 3*d*x) + 15*B*si
n(5*c + 5*d*x) + 350*C*sin(2*c + 2*d*x) + 56*C*sin(4*c + 4*d*x) + 6*C*sin(6
*c + 6*d*x) + 360*B*d*x*cos(c + d*x)))/(480*d*(cos(2*c + 2*d*x) + 1))
```

3.307 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=199

$$\frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}$$

```
[Out] 1/4*b^2*C*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/8*b^2*(4*A+3
*C)*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)+1/8*b^2*(4*A+3*C)*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^
(1/2)/d
```

Rubi [A]

time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{b^2x(4A+3C)\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}-\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[C
os[c + d*x]],x]
```

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*
Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c
+ d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b^2*B*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx=\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos^2(c+dx)\,(A+$$

$$\sqrt{\cos(c+$$

$$=\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+}{4d}$$

$$=\frac{b^2C\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+}{4d}$$

$$=\frac{b^2(4A+3C)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx}}{8d}$$

$$=\frac{b^2(4A+3C)x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\,}}{}$$

Mathematica [A]

time = 0.35, size = 92, normalized size = 0.46

$$\frac{(b\cos(c+dx))^{5/2}(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
 + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
 + d*x)]))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.34, size = 114, normalized size = 0.57

method	result
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\left(\cos^2(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(d}{24d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(8A+6C)x}{16\sqrt{\cos(dx+c)}}+\frac{3b^2B\sin(dx+c)\sqrt{b\cos(dx+c)}}{4d\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(4dx+4c)}{32\sqrt{\cos(dx+c)}\,d}+b^2\sqrt{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(5/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*s
in(d*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.68, size = 140, normalized size = 0.70

$$\frac{24\left(2(dx+c)b^2+b^2\sin(2dx+2c)\right)A\sqrt{b}+8\left(b^2\sin(3dx+3c)+9b^2\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)B\sqrt{b}+3\left(12(dx+c)b^2+b^2\sin(4dx+4c)+8b^2\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)\right)C\sqrt{b}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b^2*sin(3*
d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*
sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 303, normalized size = 1.52

$$\left[\frac{3(4A+3C)\sqrt{-b}\,b^2\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\left(6Cb^2\cos(dx+c)^3+8Bb^2\cos(dx+c)^2+3(4A+3C)b^2\cos(dx+c)+16Bb^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{48d\cos(dx+c)}, \frac{3(4A+3C)b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+\left(6Cb^2\cos(dx+c)^3+8Bb^2\cos(dx+c)^2+3(4A+3C)b^2\cos(dx+c)+16Bb^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{24d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*s
qrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*
b^2*cos(d*x + c)^3 + 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c
) + 16*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)), 1/24*(3*(4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b^2*cos(d*x + c)^3 +
 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c) + 16*B*b^2)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sq
rt(cos(d*x + c)), x)
```

Mupad [**B**]
time = 1.07, size = 94, normalized size = 0.47

$$\frac{b^2\sqrt{b\cos\left(c+d\,x\right)}\,\left(72\,B\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(3\,c+3\,d\,x\right)+24\,C\,\sin\left(2\,c+2\,d\,x\right)+3\,C\,\sin\left(4\,c+4\,d\,x\right)+48\,A\,d\,x+36\,C\,d\,x\right)}{96\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(72*B*sin(c + d*x) + 24*A*sin(2*c + 2*d*x) + 8*
B*sin(3*c + 3*d*x) + 24*C*sin(2*c + 2*d*x) + 3*C*sin(4*c + 4*d*x) + 48*A*d*
x + 36*C*d*x))/(96*d*cos(c + d*x)^(1/2))
```

3.308 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=155

$$\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2(3A+2C)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+}{2d}$$

```
[Out] 1/3*b^2*C*cos(d*x+c)^(3/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+1/2*b^2*B*x*(b
*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/3*b^2*(3*A+2*C)*sin(d*x+c)*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(
d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.04, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\frac{b^2(3A+2C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}+\frac{b^2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(3/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*(3*A + 2*C)*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos
[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b^2*C*Cos[c + d*x]^(
3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\cos(c+dx)\,(A+B}{\sqrt{\cos(c+d}}$$

$$= \frac{b^2C\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+d}{3d}$$

$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2(3A+2C)\sqrt{b\,c}}{3d\sqrt{c}}$$

Mathematica [A]
time = 0.27, size = 75, normalized size = 0.48

$$\frac{(b\cos(c+dx))^{5/2}(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B
*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.23, size = 83, normalized size = 0.54

method	result
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(2C\sin(dx+c)\left(\cos^2(dx+c)\right)+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c)\right)}{6d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{b^2Bx\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{b\cos(dx+c)}\,(4A+3C)\sin(dx+c)}{4\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(3dx+3c)}{12\sqrt{\cos(dx+c)}\,d}+\frac{b^2\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(5/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(
d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.67, size = 94, normalized size = 0.61

$$\frac{12\,Ab^{\frac{5}{2}}\sin\left(dx+c\right)+3\left(2\left(dx+c\right)b^{2}+b^{2}\sin\left(2\,dx+2\,c\right)\right)B\sqrt{b}+\left(b^{2}\sin\left(3\,dx+3\,c\right)+9\,b^{2}\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)C\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + 3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c)
)*B*sqrt(b) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))))*C*sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 263, normalized size = 1.70

$$\left[\frac{3\,B\sqrt{-b}\,b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(2\,Cb^{2}\cos\left(dx+c\right)^{2}+3\,Bb^{2}\cos\left(dx+c\right)+2\left(3\,A+2\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)},\ \frac{3\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{b}\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(2\,Cb^{2}\cos\left(dx+c\right)^{2}+3\,Bb^{2}\cos\left(dx+c\right)+2\left(3\,A+2\,C\right)b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*
x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(5/2)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + (2*C*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt
(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.72, size = 73, normalized size = 0.47

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(12\,A\,\sin\left(c+d\,x\right)+9\,C\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+6\,B\,d\,x\right)}{12\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(3/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(12*A*sin(c + d*x) + 9*C*sin(c + d*x) + 3*B*sin
(2*c + 2*d*x) + C*sin(3*c + 3*d*x) + 6*B*d*x))/(12*d*cos(c + d*x)^(1/2))
```

3.309 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\ldots}{2d}$$

```
[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*C*x*(b*cos(d*x+c))^(1
/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/
2)+1/2*b^2*C*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c +
 d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+}{\sqrt{co}}$$

$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+d}{d\sqrt{\cos(c}}$$

$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b\cos(c+}{2\sqrt{\cos(c+dx}}$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 0.45

$$\frac{(b\cos(c+dx))^{5/2}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2
*(c + d*x)]))/(4*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.20, size = 63, normalized size = 0.47

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\cos(dx+c)^{\frac{5}{2}}}$	63
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,(4A+2C)x}{4\sqrt{\cos(dx+c)}}+\frac{b^2B\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}+\frac{b^2\sqrt{b\cos(dx+c)}\,C\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x
+c)+C*(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.64, size = 71, normalized size = 0.53

$$\frac{8\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+4\,Bb^{\frac{5}{2}}\sin\left(dx+c\right)+\left(2\left(dx+c\right)b^{2}+b^{2}\sin\left(2\,dx+2\,c\right)\right)C\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(5/2)*sin(
d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d
```

Fricas [A]
time = 0.40, size = 227, normalized size = 1.68

$$\left[\frac{\left(2\,A+C\right)\sqrt{-b}\,b^{2}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\left(Cb^{2}\cos\left(dx+c\right)+2\,Bb^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)}, \frac{\left(2\,A+C\right)b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(Cb^{2}\cos\left(dx+c\right)+2\,Bb^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*A + C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b
*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b^2*cos
(d*x + c) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c)), 1/2*((2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b^2*cos(d*x + c) + 2
*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.23, size = 57, normalized size = 0.42

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(4\,B\,\sin\left(c+d\,x\right)+C\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,d\,x+2\,C\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(5/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(4*B*sin(c + d*x) + C*sin(2*c + 2*d*x) + 4*A*d*
x + 2*C*d*x))/(4*d*cos(c + d*x)^(1/2))
```

3.310 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b^2*arctanh(sin(d*x+c))*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+b^2*C*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 3102, 2814, 3855}

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(7/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx\ldots}{\sqrt{\cos(c+d\ldots}}$$

$$= \frac{b^2C\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b^2\sqrt{b\ldots}\right.}{\ldots}$$

$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2C\sqrt{b\cos(c+d\ldots}}{d\sqrt{\cos(c\ldots}}$$

$$= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\tanh^{-1}(\sin(c\ldots}{d\sqrt{c\ldots}}$$

Mathematica [A]
time = 0.17, size = 93, normalized size = 0.91

$$\frac{(b\cos(c+dx))^{5/2}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(7/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*C
os[c + d*x]^(5/2))
```

Maple [A]
time = 0.19, size = 63, normalized size = 0.62

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)(b\cos(dx+c))^{\frac{5}{2}}}{d\cos(dx+c)^{\frac{5}{2}}}$

risch	$\frac{b^2 B x \sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}} - \frac{i b^2 \sqrt{b\cos(dx+c)}\, C\, e^{i(dx+c)}}{2\sqrt{\cos(dx+c)}\, d} + \frac{i b^2 \sqrt{b\cos(dx+c)}\, C\, e^{-i(dx+c)}}{2\sqrt{\cos(dx+c)}\, d} - \frac{b^2 \sqrt{b\cos(dx}{\sqrt{co}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*(b*co
s(d*x+c))^(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.63, size = 111, normalized size = 1.09

$$\frac{4\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 2\,Cb^{\frac{5}{2}}\sin(dx+c) + \left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right) - b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)A\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(5/2)*sin(
d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/
d
```

Fricas [A]
time = 0.44, size = 316, normalized size = 3.10

$$\left[-\frac{2\,A\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c) - B\sqrt{-b}\,b^2\cos(dx+c)\log\left(2\,b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) - 2\sqrt{b\cos(dx+c)}\,Cb^2\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,d\cos(dx+c)}, \frac{2\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c) + Ab^{\frac{5}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^3 - 2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2\,b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b\cos(dx+c)}\,Cb^2\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*c
os(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x
+ c) - b) - 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)), 1/2*(2*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(5/2)*cos(d*x + c)*log(-(b
*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sq
rt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(7/2), x)
```

3.311 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*C*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/
d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3100, 2814, 3855}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(9/2),x]
```

```
[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c +
d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx \ldots}{\sqrt{\cos(c+\ldots}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b \ldots}\right.}{\ldots}$$

$$= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+d\ldots}}{d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{b^2Cx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B\tanh^{-1}(\sin(c\ldots}{d\sqrt{c\ldots}}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{5/2}\left(Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(9/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c
 + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.18, size = 72, normalized size = 0.71

method	result

default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\left(-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)\right)}{d\cos(dx+c)^{\frac{7}{2}}}$
risch	$\frac{b^2Cx\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}+\frac{2ib^2\sqrt{b\cos(dx+c)}\,A}{\sqrt{\cos(dx+c)}\,d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{b^2\sqrt{b\cos(dx+c)}\,B\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{b\cos}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(7/2)
```

Maxima [A]
time = 0.62, size = 151, normalized size = 1.48

$$\frac{4\,Cb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\frac{4\,Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)B\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*b^(5/2)*sin(
2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b))/d
```

Fricas [A]
time = 0.48, size = 324, normalized size = 3.18

$$\left[-\frac{2\,B\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2-C\sqrt{-b}\,b^2\cos(dx+c)^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\sqrt{b\cos(dx+c)}\,Ab^2\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)^2},\ \frac{2\,Cb^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)^2+Bb^{\frac{5}{2}}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos(dx+c)}\,\sqrt{b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos(dx+c)}\,Ab^2\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,d\cos(dx+c)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2
*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(
d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2), 1/2*(2*C*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(5/2)*cos(d*x + c)
^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c
))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c)
)*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(9/2), x)
```

3.312 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{b^2(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b^2*(A+2*C)*arctanh(sin(d*x+
c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 8, 3855}

$$\frac{b^2(A+2C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(11/2),x]
```

```
[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c
+ d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2
)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx}{\sqrt{\cos(c+}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\right.}{}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{\right.}{}$$

$$= \frac{b^2(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c}}{2d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c}}{2d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 69, normalized size = 0.58

$$\frac{(b\cos(c+dx))^{5/2}\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))

Maple [A]
time = 0.20, size = 151, normalized size = 1.26

method	result
default	$-\frac{\left(A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)+4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos}{\sin(}\right.\right.}{2d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{i b^2\sqrt{b\cos(dx+c)}\,(A\,e^{3i(dx+c)}-2B\,e^{2i(dx+c)}-A\,e^{i(dx+c)}-2B)}{\sqrt{\cos(dx+c)}\,d\,(e^{2i(dx+c)}+1)^2}-\frac{b^2\sqrt{b\cos(dx+c)}\,(A+2C)\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(104) = 208.
time = 0.69, size = 873, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(

```
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos
(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^
2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4
*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*
x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqr
t(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*
cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [**A**]
time = 0.46, size = 250, normalized size = 2.08

$$\left[\frac{(A+2C)b^{\frac{5}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2Bb^2\cos(dx+c)+Ab^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\left(2Bb^2\cos(dx+c)+Ab^2\right)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2
*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*
cos(d*x + c)^3 - (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(11/2), x)
```

3.313 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=164

$$\frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\sin(c+dx}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/3*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*b^2*B*sin(
d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/3*b^2*(2*A+3*C)*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*b^2*B*arctanh(sin(d*x+c))*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{b^2(2A+3C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
 + d*x]^(13/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^
2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*(2*A
 + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx)}{\sqrt{\cos(c+dx}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\,c}\right.}{}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{b}\right.}{}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B\sqrt{b\,c}}{2}$$

$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.39, size = 87, normalized size = 0.53

$$\frac{(b\cos(c+dx))^{5/2}\left(3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)\right)}{6d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(13/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A +
3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*
Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.23, size = 157, normalized size = 0.96

method	result
default	$\frac{\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+4A(\cos^2(dx+c))\sin(dx+c)+\right.}{6d\cos(dx+c)^{\frac{11}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,(3B\,e^{5i(dx+c)}-6C\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-3B\,e^{i(dx+c)}-4A-6C)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}+\frac{b^2\sqrt{b\cos(}}{2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(140) = 280.
time = 0.68, size = 1112, normalized size = 6.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) +
 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*s
```

```
in(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt
(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos
(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x
 + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) -
 3*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*
d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*
x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
 b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*s
in(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c
) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x +
 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A]
time = 0.46, size = 286, normalized size = 1.74

$$\left[\frac{3\,Bb^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\left(2\,A+3\,C\right)b^{2}\cos\left(dx+c\right)^{2}+3\,Bb^{2}\cos\left(dx+c\right)+2\,Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{4}},\ -\frac{3\,B\sqrt{-b}\,b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{4}-\left(2\left(2\,A+3\,C\right)b^{2}\cos\left(dx+c\right)^{2}+3\,Bb^{2}\cos\left(dx+c\right)+2\,Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*
b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^
4), -1/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b^2*cos(d*x + c)^
2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(13/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(13/2), x)
```

3.314 $\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=208

$$\frac{b^2(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c-}{8d\cos^{\frac{5}{2}}(c-}$$

```
[Out] 1/4*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+1/8*b^2*(3*A+4
*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+c)*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^
(1/2)/d/cos(d*x+c)^(7/2)+1/8*b^2*(3*A+4*C)*arctanh(sin(d*x+c))*(b*cos(d*x+c
))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3100, 2827, 3852, 3853, 3855}

$$\frac{b^2(3A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2(3A+4C)\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c
+ d*x]^(15/2),x]
```

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[
c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9
/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
 + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx}{\sqrt{\cos(c+}}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b}\right.}{}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{}\right.}{}$$

$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{b^2(3A}{}$$

$$= \frac{b^2(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c}}{8d\sqrt{\cos(c+dx)}}$$

Mathematica [A]

time = 0.38, size = 110, normalized size = 0.53

$$\frac{(b\cos(c+dx))^{5/2}\left(3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)\right)\right)}{24d\cos^{\frac{13}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Cos[c + d*x]^(15/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4
 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 +
 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(13/2))
```

Maple [A]
time = 0.20, size = 248, normalized size = 1.19

method	result
default	$-\frac{\left(9A\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)+12C\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos\ldots}{}\right.\right.}{}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,\left(9A\,e^{7i(dx+c)}+12C\,e^{7i(dx+c)}+33A\,e^{5i(dx+c)}+12C\,e^{5i(dx+c)}-48B\,e^{4i(dx+c)}-33A\,e^{3i(dx+c)}-12C\,e^{3i(dx+\ldots}\right.}{12\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),
x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4-16*B*cos(d*x+c)^3*sin(d*x+c)-9*A*cos(d*x+c)^2*sin
(d*x+c)-12*C*sin(d*x+c)*cos(d*x+c)^2-8*B*cos(d*x+c)*sin(d*x+c)-6*A*sin(d*x+
c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(180) = 360$.
time = 0.77, size = 2972, normalized size = 14.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d
*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2
```

```
*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*s
in(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6
*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*
c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*
d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*
sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2
+ 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x
+ 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4
*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*
x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(
4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^
2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2
 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x
 + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x
+ 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x
+ 2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^
2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^
2*cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2
*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x
 + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8
*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*
d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x
+ 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*
c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2
```

```
 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*
d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 64*(3*b^2*cos(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*
c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4
*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1) + 12*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b
^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^
2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4
*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*
x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x...
```

Fricas [A]
time = 0.43, size = 326, normalized size = 1.57

$$\left[\frac{3\,(3\,A+4\,C)b^{\frac{5}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(16\,Bb^{2}\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)b^{2}\cos\left(dx+c\right)^{2}+8\,Bb^{2}\cos\left(dx+c\right)+6\,Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{5}},\ -\frac{3\,(3\,A+4\,C)\sqrt{-b}\,b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,Bb^{2}\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)b^{2}\cos\left(dx+c\right)^{2}+8\,Bb^{2}\cos\left(dx+c\right)+6\,Ab^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x
+ c)^2 + 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b^2*a
rctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*co
s(d*x + c)^5 - (16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x + c)^2
+ 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(15/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\cos(c+d\,x))^{5/2}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(15/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^(15/2), x)
```

3.315 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=184

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+}{4d\sqrt{b\,c}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*C*cos(
d*x+c)^(7/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+c)^(
1/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1
/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin
[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^
3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int \cos^2(c+dx)\left(A+B\cos(c+\right.}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\,\int}{}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx\right.}{\sqrt{b\,\mathrm{co}}}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C\cos^{\frac{7}{2}}(}{4d\sqrt{}}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c}}$$

Mathematica [A]
time = 0.25, size = 92, normalized size = 0.50

$$\frac{\sqrt{\cos(c+dx)}\,(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.36, size = 114, normalized size = 0.62

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\left(6C\left(\cos^{3}(dx+c)\right)\sin(dx+c)+8B\left(\cos^{2}(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A\right.}{24d\sqrt{b\cos\left(dx+c\right)}}$
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(8A+6C)x}{16\sqrt{b\cos\left(dx+c\right)}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{4d\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(4dx+4c)}{32\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\sin(3}{12\sqrt{b\cos\left(dx+c\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*cos(d*x+c)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*sin(d
*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+16*
B*sin(d*x+c)+9*C*(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.71, size = 116, normalized size = 0.63

$$\frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{\sqrt{b}}+\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{\sqrt{b}}+\frac{8\,B(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right))}{\sqrt{b}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
sqrt(b) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/sqrt(b))/d
```

Fricas [A]
time = 0.44, size = 282, normalized size = 1.53

$$\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\left(6\,C\cos\left(dx+c\right)^{3}+8\,B\cos\left(dx+c\right)^{2}+3\,(4\,A+3\,C)\cos\left(dx+c\right)+16\,B\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,bd\cos\left(dx+c\right)},\ \frac{3\,(4\,A+3\,C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(6\,C\cos\left(dx+c\right)^{3}+8\,B\cos\left(dx+c\right)^{2}+3\,(4\,A+3\,C)\cos\left(dx+c\right)+16\,B\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,bd\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), 1/24*(
3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos
(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 +
 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b
*cos(d*x + c)), x)
```

Mupad [**B**]
time = 2.77, size = 140, normalized size = 0.76

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(4\,c+4\,d\,x\right)+27\,C\,\sin\left(3\,c+3\,d\,x\right)+3\,C\,\sin\left(5\,c+5\,d\,x\right)+96\,A\,d\,x\,\cos\left(c+d\,x\right)+72\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{96\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b*d*(cos(2*c + 2*d*x) + 1))
```

3.316 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=143

$$\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)^(
5/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+
c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+B\cos(c+d\right.}{\sqrt{b\cos(c+dx)}}\\ &= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int}{}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+d}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 75, normalized size = 0.52

$$\frac{\sqrt{\cos(c+dx)}\,(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin
[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.32, size = 83, normalized size = 0.58

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\left(2C\sin(dx+c)\left(\cos^2(dx+c)\right)+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c)\right)}{6d\sqrt{b\cos\left(dx+c\right)}}$
risch	$\frac{Bx\left(\sqrt{\cos}\,(dx+c)\right)}{2\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+3C)\sin(dx+c)}{4\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(3dx+3c)}{12\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\,\mathrm{si}}{4\sqrt{b\cos\left(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+
c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.68, size = 80, normalized size = 0.56

$$\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{\sqrt{b}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{\sqrt{b}}+\frac{12\,A\sin(dx+c)}{\sqrt{b}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b) + 12*A*sin(
d*x + c)/sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 242, normalized size = 1.69

$$\left[-\frac{3\,B\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\left(2\,C\cos(dx+c)^2+3\,B\cos(dx+c)+6\,A+4\,C\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{12\,bd\cos(dx+c)},\ \frac{3\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+\left(2\,C\cos(dx+c)^2+3\,B\cos(dx+c)+6\,A+4\,C\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{6\,bd\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c
))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x +
 c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b
*cos(d*x + c)), x)
```

Mupad [B]
time = 1.41, size = 107, normalized size = 0.75

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(3\,B\,\sin(c+d\,x)+12\,A\,\sin(2\,c+2\,d\,x)+3\,B\,\sin(3\,c+3\,d\,x)+10\,C\,\sin(2\,c+2\,d\,x)+C\,\sin(4\,c+4\,d\,x)+12\,B\,d\,x\,\cos(c+d\,x))}{12\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.317 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=123

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*sin
(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos
[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2
*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[
c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx)+C\cos}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \mathrm{co}}{\sqrt{b\cos(c+d}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\,\sin(c+}{d\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B}{}$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 0.50

$$\frac{\sqrt{\cos(c+dx)}\,(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.25, size = 63, normalized size = 0.51

method	result	size
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d\sqrt{b\cos(dx+c)}}$	63
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+2C)x}{4\sqrt{b\cos(dx+c)}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{d\sqrt{b\cos(dx+c)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+
C*(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.67, size = 64, normalized size = 0.52

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{\sqrt{b}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{4\,B\sin(dx+c)}{\sqrt{b}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/sqrt(b) + 4*B*sin(d*x + c)/sqrt(b))/d
```

Fricas [A]
time = 0.42, size = 218, normalized size = 1.77

$$\left[-\frac{(2\,A+C)\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,(C\cos(dx+c)+2\,B)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4\,bd\cos(dx+c)},\ \frac{(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+(C\cos(dx+c)+2\,B)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,bd\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [A]
time = 23.32, size = 184, normalized size = 1.50

$$\begin{cases}\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Cx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{Cx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d\neq 0\\ \frac{x(A+B\cos(c)+C\cos^2(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + B*sin(c + d*x)*sqr
t(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))) + C*x*sin(c + d*x)**2*sqrt(cos(c +
 d*x))/(2*sqrt(b*cos(c + d*x))) + C*x*cos(c + d*x)**(5/2)/(2*sqrt(b*cos(c +
 d*x))) + C*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(c + d*x))), Ne
(d, 0)), (x*(A + B*cos(c) + C*cos(c)**2)*sqrt(cos(c))/sqrt(b*cos(c)), True)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b
*cos(d*x + c)), x)
```

Mupad [B]
time = 1.01, size = 93, normalized size = 0.76

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+4\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\left(c+d\,x\right)+4\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b*d*(cos(2*c + 2*d*x) + 1))
```

3.318 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^
(1/2)/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3102, 2814, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c\cdot}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c}{\sqrt{b\cos(c+}}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos}\right.}{}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 1.00

$$\frac{\sqrt{\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqrt[
b*Cos[c + d*x]])
```

Maple [A]
time = 0.23, size = 63, normalized size = 0.68

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)\left(\sqrt{\cos}\,(dx+c)\right)}{d\sqrt{b\cos\left(dx+c\right)}}$

risch	$\frac{Bx\left(\sqrt{\cos}\,(dx+c)\right)}{\sqrt{b\cos\left(dx+c\right)}}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{C\sin(2dx}{2d\sqrt{\cos\left(dx+c\right)}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(d
*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.68, size = 104, normalized size = 1.12

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{2\,C\sin(dx+c)}{\sqrt{b}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x + c)/sqrt(b))/d
```

Fricas [A]
time = 0.45, size = 309, normalized size = 3.32

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)+B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)},\ \frac{2\,B\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+A\sqrt{b}\,\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
+ c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x +
 c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*
b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c
))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}\,\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*sqr
t(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{\sqrt{\cos(c + d\,x)}\,\sqrt{b\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.319 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3100, 2814, 3855}

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int (B+C\,c}{\sqrt{b\cos(}}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\big(B\,\sqrt{}}{}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.65

$$\frac{Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.22, size = 72, normalized size = 0.77

method	result
default	$\frac{-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)}{d\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}}$

risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{\sqrt{b\cos\left(dx+c\right)}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c\right.}{\sqrt{b\cos}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+C*cos(d*x+c)*(d*x+
c)+A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.67, size = 149, normalized size = 1.60

$$\frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,C\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{4\,A\sqrt{b}\,\sin(2\,dx+2\,c)}{b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*C*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(b*co
s(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b))/d
```

Fricas [A]
time = 0.44, size = 317, normalized size = 3.41

$$\left[-\frac{2\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^2+C\sqrt{-b}\,\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)^2},\ \frac{2\,C\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^2+B\sqrt{b}\,\cos\left(dx+c\right)^2\log\left(-\frac{b\cos\left(dx+c\right)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,bd\cos\left(dx+c\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(
d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqr
t(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-(b
*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(c
os(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx) + C\cos^2(c + dx)}{\sqrt{b\cos(c + dx)}\,\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(sqrt(b*cos(c + d*x))*cos
(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{\cos(c + d\,x)^{3/2}\,\sqrt{b\,\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.320 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*
x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c\\ &= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int(2B+(A}{2\sqrt{b}}\\ &= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^2(}{\sqrt{b\cos(c+dx)}}\\ &= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin}{2d\cos^{\frac{3}{2}}(c+d}\\ &= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin}{2d\cos^{\frac{3}{2}}(c+d}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 0.62

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [A]
time = 0.24, size = 150, normalized size = 1.35

method	result
risch	$-\frac{i\left(A\,e^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(A+2C)\ln\left(e^{i(dx+c)}-i\right)}{2\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)}{2\sqrt{b\cos}}$
default	$\frac{-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(}{\sin(dx}\right.}{2d\sqrt{b\cos\left(dx+c\right)}\,\cos(dx+c)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+2*B*cos(d*x+c)*sin(d*x+c)+A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(95) = 190$.
time = 0.71, size = 785, normalized size = 7.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 8*B*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(

```
2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c
)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*
x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4
*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(
2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [**A**]
time = 0.43, size = 239, normalized size = 2.15

$$\left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4bd\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2bd\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sq
rt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x +
c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.321 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=152

$$\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+$$

```
[Out] 1/3*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)/d
/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c +
d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c +
d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(3B+(2A}{3\sqrt{b}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(c}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+}}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos}}$$

Mathematica [A]
time = 0.19, size = 87, normalized size = 0.57

$$\frac{3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] +
 (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b
*Cos[c + d*x]])
```

Maple [A]
time = 0.27, size = 157, normalized size = 1.03

method	result
default	$\frac{-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+4A(\cos^2(dx+c))\sin(dx+c)+6}{6d\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)^2 d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\ln(e^{i(dx+c)}+i)}{2\sqrt{b\cos(dx+c)}\,d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. $2(128) = 256$.
time = 0.72, size = 1014, normalized size = 6.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2
*c)^2 + 2*b*cos(2*d*x + 2*c) + b) + 16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x
```

```
+ 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*s
in(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x +
 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(
3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d
*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + si
n(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*(4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*
c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*
d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x
 + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))
/d
```

Fricas [A]
time = 0.41, size = 271, normalized size = 1.78

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,bd\cos\left(dx+c\right)^{4}}, -\frac{3\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{4}-\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,bd\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1/6*(
3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(
d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x +
c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.322 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=193

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+dx}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c-}}$$

```
[Out] 1/4*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*sin(
d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/d/cos(d*x
+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c
))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[
c + d*x]]),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (
(3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (
B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*
x]^3)/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(4B+(3}{4\sqrt{}}$$

$$= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^4}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+dx}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+}}$$

$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{A\,\mathrm{s}}{4d\cos^{\frac{7}{2}}(c+}$$

Mathematica [A]

time = 0.24, size = 110, normalized size = 0.57

$$\frac{3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)\right)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[
b*Cos[c + d*x]]),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.25, size = 248, normalized size = 1.28

method	result
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(d\right.}{24\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^{3}d}$
default	$\frac{-9A\left(\cos^{4}(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^{4}(dx+c)\right)-12C\left(\cos^{4}(dx+c)\right)\ln\left(-\frac{-1+\cos(d}{\sin}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*cos(d*x+c)^2*sin
(d*x+c)+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+
c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. 2(165) = 330.
time = 0.73, size = 2611, normalized size = 13.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
```

```
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c
) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*co
s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2
*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
+ 2*c) + 1)*sqrt(b)) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3
*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*
c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B/((2*(3*cos(4*d*x + 4*c) + 3*cos
(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 +
 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c
)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2
```

```
*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*...
```

Fricas [A]
time $= 0.45$, size $= 305$, normalized size $= 1.58$

$$\left[\frac{3\,(3\,A+4\,C)\sqrt{b}\,\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,bd\cos\left(dx+c\right)^{5}},-\frac{3\,(3\,A+4\,C)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,bd\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d
*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16
*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*
sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time $= 0.00$, size $= 0$, normalized size $= 0.00$

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(9/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{9/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(9/2)*(b*cos(c +
d*x))^(1/2)), x)
```

3.323 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+ \ldots}{4bd\sqrt{b \ldots}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*C*co
s(d*x+c)^(7/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(d*x+
c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*
x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c +
d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/
2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int\cos^2(c+dx)\left(A+B\cos(c+\right.}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int}{}\\ &= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx}\right.}{b\sqrt{b\cos}}\\ &= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(}{4bd}\\ &= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 92, normalized size = 0.46

$$\frac{\cos^{\frac{3}{2}}(c+dx)(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.32, size = 114, normalized size = 0.57

method	result
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\left(6C\left(\cos^{3}(dx+c)\right)\sin(dx+c)+8B\left(\cos^{2}(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(d\right.}{24d(b\cos(dx+c))^{\frac{3}{2}}}$
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(8A+6C)x}{16b\sqrt{b\cos\left(dx+c\right)}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{4bd\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(4dx+4c)}{32b\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\sin(3}{12b\sqrt{b\cos\left(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*cos(d*x+c)^(3/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*sin(d
*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+16*
B*sin(d*x+c)+9*C*(d*x+c))/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.69, size = 116, normalized size = 0.58

$$\frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{3}{2}}}+\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{b^{\frac{3}{2}}}+\frac{8\,B(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right))}{b^{\frac{3}{2}}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
b^(3/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/b^(3/2))/d
```

Fricas [A]
time = 0.46, size = 282, normalized size = 1.42

$$\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\left(6\,C\cos(dx+c)^{3}+8\,B\cos(dx+c)^{2}+3\,(4\,A+3\,C)\cos(dx+c)+16\,B\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{48\,b^{2}d\cos(dx+c)},\ \frac{3\,(4\,A+3\,C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+\left(6\,C\cos(dx+c)^{3}+8\,B\cos(dx+c)^{2}+3\,(4\,A+3\,C)\cos(dx+c)+16\,B\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{24\,b^{2}d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), 1/24
*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2
 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [**B**]
time = 2.60, size = 140, normalized size = 0.70

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(4\,c+4\,d\,x\right)+27\,C\,\sin\left(3\,c+3\,d\,x\right)+3\,C\,\sin\left(5\,c+5\,d\,x\right)+96\,A\,d\,x\,\cos\left(c+d\,x\right)+72\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{96\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.324 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin(\ldots}{3bd\sqrt{b\cos(c+\ldots}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+c)
^(5/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/b/(b*co
s(d*x+c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c
 + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\left(A+B\cos(c+d\right.}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int}{}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.48

$$\frac{\cos^{\frac{3}{2}}(c+dx)(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin
[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.24, size = 83, normalized size = 0.54

method	result
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\left(2C\sin(dx+c)\left(\cos^2(dx+c)\right)+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c)\right)}{6d(b\cos(dx+c))^{\frac{3}{2}}}$
risch	$\frac{Bx\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{b\cos(dx+c)}}+\frac{\left(\sqrt{\cos}(dx+c)\right)(4A+3C)\sin(dx+c)}{4b\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(3dx+3c)}{12b\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\,s}{4b\sqrt{b\cos(dx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

[Out] 1/6/d*cos(d*x+c)^(3/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+
c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/(b*cos(d*x+c))^(3/2)

Maxima [A]
time = 0.71, size = 80, normalized size = 0.52

$$\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{3}{2}}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{3}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2) + 12*A*sin(
d*x + c)/b^(3/2))/d

Fricas [A]
time = 0.47, size = 242, normalized size = 1.56

$$\left[-\frac{3\,B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,b^{2}d\cos\left(dx+c\right)},\ \frac{3\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,b^{2}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^2*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x
 + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [B]
time = 1.15, size = 107, normalized size = 0.69

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)+12\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.325 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)^(
1/2)/b/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2717, 2715, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]]
)/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqr
t[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[
c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(\right.}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\cos}{b\sqrt{b\cos(c+dx}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+}{bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{}}{}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.45

$$\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.22, size = 63, normalized size = 0.47

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c))}{2d(b\cos(dx+c))^{\frac{3}{2}}}$	63
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)(4A+2C)x}{4b\sqrt{b\cos(dx+c)}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{b\cos(dx+c)}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+
C*(d*x+c))/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.67, size = 64, normalized size = 0.47

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{3}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+\frac{4\,B\sin(dx+c)}{b^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(3/2) + 4*B*sin(d*x + c)/b^(3/2))/d
```

Fricas [A]
time = 0.46, size = 218, normalized size = 1.61

$$\left[-\frac{(2\,A+C)\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,(C\cos(dx+c)+2\,B)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{4\,b^2d\cos(dx+c)},\ \frac{(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+(C\cos(dx+c)+2\,B)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{2\,b^2d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(
d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.83, size = 93, normalized size = 0.69

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(C\,\sin(c+d\,x)+4\,B\,\sin(2\,c+2\,d\,x)+C\,\sin(3\,c+3\,d\,x)+8\,A\,d\,x\,\cos(c+d\,x)+4\,C\,d\,x\,\cos(c+d\,x)\right)}{4\,b^2\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.326 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c
)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3102, 2814, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx}}{}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c-}{bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{}}{bd\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.91

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2),x]
[Out] (Cos[c + d*x]^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*(b*Co
s[c + d*x])^(3/2))
```

Maple [A]
time = 0.22, size = 63, normalized size = 0.62

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$

risch	$\frac{Bx\left(\sqrt{\cos}\,(dx+c)\right)}{b\sqrt{b\cos\left(dx+c\right)}}-\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{i(dx+c)}}{2b\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,\mathrm{e}^{-i(dx+c)}}{2b\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln(\mathrm{e}^{i(dx+c)}+i)}{b\sqrt{b\cos\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(d
*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.65, size = 104, normalized size = 1.02

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+\frac{2\,C\sin(dx+c)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x + c)/b^(3/2))/d
```

Fricas [A]
time = 0.55, size = 309, normalized size = 3.03

$$\left[-\frac{2\,A\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)+B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,b^2d\cos\left(dx+c\right)},\ \frac{2\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+A\sqrt{b}\,\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,C\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,b^2d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*
x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x
 + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) -
2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(3/2), x)
```

3.327 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/2)
/b/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3100, 2814, 3855}

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sq
rt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(B+C}{b\sqrt{b\cos}}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{(B}{}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} +$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.59

$$\frac{\sqrt{\cos(c+dx)}\left(Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.22, size = 72, normalized size = 0.71

method	result
default	$\frac{\left(-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)\right)\left(\sqrt{\cos}\,(dx+c)\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$

risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{b\sqrt{b\cos\left(dx+c\right)}}+\frac{i\mathrm{e}^{-i(dx+c)}A}{b\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b\sqrt{b\cos\left(dx+c\right)}\,d}-\frac{\left(\sqrt{\cos}\,(dx\right.}{b\sqrt{b\,\mathrm{co}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+C*cos(d*x+c)*(d*x+
c)+A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.66, size = 157, normalized size = 1.54

$$\frac{\frac{4\,A\sqrt{b}\,\sin\left(2\,dx+2\,c\right)}{b^{2}\cos\left(2\,dx+2\,c\right)^{2}+b^{2}\sin\left(2\,dx+2\,c\right)^{2}+2\,b^{2}\cos\left(2\,dx+2\,c\right)+b^{2}}+\frac{B\left(\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{4\,C\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(3/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2)
)/d
```

Fricas [A]
time = 0.44, size = 317, normalized size = 3.11

$$\left[-\frac{2\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{2}+C\sqrt{-b}\,\cos\left(dx+c\right)^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,b^{2}d\cos\left(dx+c\right)^{2}},\ \frac{2\,C\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^{2}+B\sqrt{b}\,\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos\left(dx+c\right)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}\right)+2\,\sqrt{b\cos\left(dx+c\right)}\,A\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,b^{2}d\cos\left(dx+c\right)^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-
(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos\left(c + dx\right) + C\cos^{2}\left(c + dx\right)}{\left(b\cos\left(c + dx\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c + dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/((b*cos(c + d*x))**(3/2)*
sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*s
qrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.328 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c}}$$

```
[Out] 1/2*A*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d
/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*cos(
d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c +
d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) +
 (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c\\
&= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\,\int(2B+(}{2b\sqrt{}}\\
&= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec}{b\sqrt{b\cos(c+dx}}\\
&= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin}{2bd\cos^{\frac{3}{2}}(c+}\\
&= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin}{2bd\cos^{\frac{3}{2}}(c+}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.58

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Co
s[c + d*x])^(3/2)),x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Si
n[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))

Maple [A]
time = 0.21, size = 151, normalized size = 1.26

method	result
default	$-\frac{A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)+4C(\cos^2(dx+c))\operatorname{arctanh}\left(\frac{-1+\cos(}{\sin(dx}\right.}{2d(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}$
risch	$-\frac{i(A\,e^{2i(dx+c)}-A-4B\cos(dx+c))}{2b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)d}-\frac{\left(\sqrt{\cos}(dx+c)\right)(A+2C)\ln(e^{i(dx+c)}-i)}{2b\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right.}{2b\sqrt{b\,c}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((
-1+cos(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*sin(d*x+c))/(b*cos(d
*x+c))^(3/2)/cos(d*x+c)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(104) = 208$.
time = 0.73, size = 802, normalized size = 6.68

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")

[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2

```
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*co
s(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)) + 2*C*(log
(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2
 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2))/d
```

Fricas [**A**]
time = 0.44, size = 239, normalized size = 1.99

$$\left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4b^2d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2b^2d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(
sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x
+ c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.329 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=164

$$\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+c)
/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b/d/cos
(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[
c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Si
n[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(3B+(2A}{3b\sqrt{}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3(}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c-}}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\,\mathrm{co}}}$$

Mathematica [A]
time = 0.17, size = 87, normalized size = 0.53

$$\frac{3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)}{6d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] +
 (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos
[c + d*x])^(3/2))
```

Maple [A]
time = 0.23, size = 157, normalized size = 0.96

method	result
default	$\frac{-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+4A(\cos^2(dx+c))\sin(dx+c)+6}{6d(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2 d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\ln(e^{i(dx+c)}+i)}{2b\sqrt{b\cos(dx+c)}\,d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(140) = 280.
time = 0.70, size = 1048, normalized size = 6.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x
 + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 16*((3*cos(2*d*x + 2*c) + 1)*si
```

```
n(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*
d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*
x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c)
 + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*
b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*
(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*
x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4
*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2
+ 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*
d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)))/d
```

<u>Fricas</u> **[A]**
time = 0.44, size = 271, normalized size = 1.65

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,b^2d\cos\left(dx+c\right)^4},\ -\frac{3\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^4-\left(2\left(2\,A+3\,C\right)\cos\left(dx+c\right)^2+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,b^2d\cos\left(dx+c\right)^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6
*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x
+ c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*co
s(d*x + c)^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.330 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+d}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(}$$

```
[Out] 1/4*A*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*si
n(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b/d/c
os(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*c
os(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*
cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c +
d*x])^(3/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
 + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
]) + (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Si
n[c + d*x]^3)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(4B+(3}{4b}$$

$$= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec}{b\sqrt{b\cos(c+dx}}$$

$$= \frac{A\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+d}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c}}$$

$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{A}{4bd\cos^{\frac{7}{2}}(c+}$$

Mathematica [A]

time = 0.23, size = 110, normalized size = 0.53

$$\frac{3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)\right)}{24d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Co
s[c + d*x])^(3/2)),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.22, size = 248, normalized size = 1.19

method	result
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(d}{24b\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^{3}d}$
default	$-\frac{9A\left(\cos^{4}(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^{4}(dx+c)\right)+12C\left(\cos^{4}(dx+c)\right)\ln\left(-\frac{-1+\cos(}{s}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d*(9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4-16*B*cos(d*x+c)^3*sin(d*x+c)-9*A*cos(d*x+c)^2*sin
(d*x+c)-12*C*sin(d*x+c)*cos(d*x+c)^2-8*B*cos(d*x+c)*sin(d*x+c)-6*A*sin(d*x+
c))/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(180) = 360$.
time = 0.74, size = 2660, normalized size = 12.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
```

```
 + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
 + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*A/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*
x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*
x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*
c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) +
 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)
*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*si
n(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x
 + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 64*((3*cos
(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x
+ 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x
 + 2*c))*B/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x
+ 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) +
 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*
```

```
cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(si
n(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x
 + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + ...
```

Fricas [A]
time = 0.42, size = 305, normalized size = 1.47

$$\left[\frac{3\,(3\,A+4\,C)\sqrt{b}\,\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\,\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\cos\left(dx+c\right)^2+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,b^2d\cos\left(dx+c\right)^5},-\frac{3\,(3\,A+4\,C)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^5-\left(16\,B\cos\left(dx+c\right)^3+3\,(3\,A+4\,C)\cos\left(dx+c\right)^2+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,b^2d\cos\left(dx+c\right)^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b^2*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos
(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (
16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)
^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{7/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(7/2)*(b*cos(c +
d*x))^(3/2)), x)
```

3.331 $\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=199

$$\frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/8*(4*A+3*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*C*
cos(d*x+c)^(7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/8*(4*A+3*C)*x*cos(
d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(
b*cos(d*x+c))^(1/2)-1/3*B*sin(d*x+c)^3*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {17, 3102, 2827, 2715, 8, 2713}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*
x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x
]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{9}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\left(A+B\cos(c+\right.}{b^2\sqrt{b\cos(c+dx)}}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\int}{}\\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx\right.}{b^2\sqrt{b\,c}}\\
&= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{7}{2}}(}{4b^2d}\\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c}}\end{aligned}$$

Mathematica [A]

time = 0.18, size = 95, normalized size = 0.48

$$\frac{\sqrt{\cos(c+dx)}\,(48Ac+36cC+48Adx+36Cdx+72B\sin(c+dx)+24(A+C)\sin(2(c+dx))+8B\sin(3(c+dx))+3C\sin(4(c+dx)))}{96b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.33, size = 114, normalized size = 0.57

method	result
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(6C\left(\cos^3(dx+c)\right)\sin(dx+c)+8B\left(\cos^2(dx+c)\right)\sin(dx+c)+12A\sin(dx+c)\cos(dx+c)+9C\cos(dx+c)\sin(dx+c)+12A(d\right.}{24d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)(8A+6C)x}{16b^2\sqrt{b\cos(dx+c)}}+\frac{3B\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4b^2d\sqrt{b\cos(dx+c)}}+\frac{\left(\sqrt{\cos}(dx+c)\right)C\sin(4dx+4c)}{32b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)B\sin(3}{12b^2\sqrt{b\cos(dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*cos(d*x+c)^(5/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*cos(d*x+c)^2*sin(d
*x+c)+12*A*sin(d*x+c)*cos(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+16*
B*sin(d*x+c)+9*C*(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.71, size = 116, normalized size = 0.58

$$\frac{\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{b^{\frac{5}{2}}}+\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right))C}{b^{\frac{5}{2}}}+\frac{8\,B\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/
b^(5/2) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/b^(5/2))/d
```

Fricas [A]
time = 0.43, size = 282, normalized size = 1.42

$$\left[-\frac{3\,(4\,A+3\,C)\sqrt{-b}\,\cos(dx+c)\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\,\sqrt{-b}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-b\right)-2\,\left(6\,C\cos(dx+c)^3+8\,B\cos(dx+c)^2+3\,(4\,A+3\,C)\cos(dx+c)+16\,B\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{48\,b^3d\cos(dx+c)},\ \frac{3\,(4\,A+3\,C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)+\left(6\,C\cos(dx+c)^3+8\,B\cos(dx+c)^2+3\,(4\,A+3\,C)\cos(dx+c)+16\,B\right)\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{24\,b^3d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos
(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/24
*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*c
os(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2
 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [**B**]
time = 2.34, size = 140, normalized size = 0.70

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(24\,A\,\sin\left(c+d\,x\right)+24\,C\,\sin\left(c+d\,x\right)+24\,A\,\sin\left(3\,c+3\,d\,x\right)+80\,B\,\sin\left(2\,c+2\,d\,x\right)+8\,B\,\sin\left(4\,c+4\,d\,x\right)+27\,C\,\sin\left(3\,c+3\,d\,x\right)+3\,C\,\sin\left(5\,c+5\,d\,x\right)+96\,A\,d\,x\,\cos\left(c+d\,x\right)+72\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{96\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*A*sin(c + d*x) + 24*C*sin(c
+ d*x) + 24*A*sin(3*c + 3*d*x) + 80*B*sin(2*c + 2*d*x) + 8*B*sin(4*c + 4*d*
x) + 27*C*sin(3*c + 3*d*x) + 3*C*sin(5*c + 5*d*x) + 96*A*d*x*cos(c + d*x) +
 72*C*d*x*cos(c + d*x)))/(96*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.332 $\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=155

$$\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{(3A+2C)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{5}{2}}(c+dx)\sin}{3b^2d\sqrt{b\cos(c+}}$$

```
[Out] 1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/3*C*cos(d*x+
c)^(5/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*B*x*cos(d*x+c)^(1/2)/b^2
/(b*cos(d*x+c))^(1/2)+1/3*(3*A+2*C)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*co
s(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {17, 3102, 2813}

$$\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)
*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\cos(c+dx)\left(A+B\cos(c+d\right.}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int}{}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx}}{3b^2d\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.14, size = 78, normalized size = 0.50

$$\frac{\sqrt{\cos(c+dx)}\,(6Bc+6Bdx+3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+C\sin(3(c+dx)))}{12b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin
[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.24, size = 83, normalized size = 0.54

method	result
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(2C\sin(dx+c)\left(\cos^2(dx+c)\right)+3B\cos(dx+c)\sin(dx+c)+6A\sin(dx+c)+3B(dx+c)+4C\sin(dx+c)\right)}{6d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$\frac{Bx\left(\sqrt{\cos}\,(dx+c)\right)}{2b^2\sqrt{b\cos(dx+c)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+3C)\sin(dx+c)}{4b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(3dx+3c)}{12b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B}{4b^2\sqrt{b\cos(d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*cos(d*x+c)^(5/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+
c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*C*sin(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.68, size = 80, normalized size = 0.52

$$\frac{\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}}+\frac{C\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{b^{\frac{5}{2}}}+\frac{12\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + C*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2) + 12*A*sin(
d*x + c)/b^(5/2))/d
```

Fricas [A]
time = 0.44, size = 242, normalized size = 1.56

$$\left[-\frac{3\,B\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,b^{3}d\cos\left(dx+c\right)},\ \frac{3\,B\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+\left(2\,C\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+6\,A+4\,C\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,b^{3}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^
2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x
 + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.16, size = 107, normalized size = 0.69

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(3\,B\,\sin\left(c+d\,x\right)+12\,A\,\sin\left(2\,c+2\,d\,x\right)+3\,B\,\sin\left(3\,c+3\,d\,x\right)+10\,C\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(4\,c+4\,d\,x\right)+12\,B\,d\,x\,\cos\left(c+d\,x\right)\right)}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*B*sin(c + d*x) + 12*A*sin(2*c
 + 2*d*x) + 3*B*sin(3*c + 3*d*x) + 10*C*sin(2*c + 2*d*x) + C*sin(4*c + 4*d*
x) + 12*B*d*x*cos(c + d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.333 $\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*C*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+A*x*cos(d*x+c)
^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+1/2*C*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))
^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {17, 2717, 2715, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x
]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2
*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqr
t[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \left(A+B\cos(c+dx)+C\cos^2(\right.}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos}{b^2\sqrt{b\cos(c+dx}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+}{b^2d\sqrt{b\cos(c+dx)}}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B}{}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 0.47

$$\frac{\sqrt{\cos(c+dx)}\left(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx))\right)}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c
+ d*x)]))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.22, size = 63, normalized size = 0.47

method	result	size
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)\left(C\cos(dx+c)\sin(dx+c)+2A(dx+c)+2B\sin(dx+c)+C(dx+c)\right)}{2d(b\cos(dx+c))^{\frac{5}{2}}}$	63
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(4A+2C)x}{4b^2\sqrt{b\cos\left(dx+c\right)}}+\frac{B\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{b^2d\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)C\sin(2dx+2c)}{4b^2\sqrt{b\cos\left(dx+c\right)}\,d}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*cos(d*x+c)^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+
C*(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.62, size = 64, normalized size = 0.47

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))C}{b^{\frac{5}{2}}}+\frac{8\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+\frac{4\,B\sin(dx+c)}{b^{\frac{5}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/b^(5/2) + 4*B*sin(d*x + c)/b^(5/2))/d
```

Fricas [A]
time = 0.44, size = 218, normalized size = 1.61

$$\left[-\frac{(2\,A+C)\sqrt{-b}\,\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)-2\,(C\cos\left(dx+c\right)+2\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{4\,b^{3}d\cos\left(dx+c\right)},\ \frac{(2\,A+C)\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+(C\cos\left(dx+c\right)+2\,B)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{2\,b^{3}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*co
s(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x +
c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(
d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/
(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.85, size = 93, normalized size = 0.69

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(C\,\sin\left(c+d\,x\right)+4\,B\,\sin\left(2\,c+2\,d\,x\right)+C\,\sin\left(3\,c+3\,d\,x\right)+8\,A\,d\,x\,\cos\left(c+d\,x\right)+4\,C\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(C*sin(c + d*x) + 4*B*sin(2*c +
2*d*x) + C*sin(3*c + 3*d*x) + 8*A*d*x*cos(c + d*x) + 4*C*d*x*cos(c + d*x)))
/(4*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.334 $\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x
+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+C*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b
*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$,
Rules used = {17, 3102, 2814, 3855}

$$\frac{A\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int\left(A+B\cos(c+dx)+C\cos^2(\right.}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{C\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}}{}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\,\sin(c+}{b^2d\sqrt{b\cos(c+dx)}}$$
$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{}}{b^2d\sqrt{b\cos(c+d}}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.94

$$\frac{\sqrt{\cos(c+dx)}\left(Bc+Bdx-A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+C\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(b^2*d*S
qrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.62

method	result
default	$-\frac{\left(2A\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)-C\sin(dx+c)\right)\left(\cos^{\frac{5}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$

risch	$\frac{Bx\left(\sqrt{\cos}\,(dx+c)\right)}{b^{2}\sqrt{b\cos\left(dx+c\right)}}-\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,e^{i(dx+c)}}{2b^{2}\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{i\left(\sqrt{\cos}\,(dx+c)\right)C\,e^{-i(dx+c)}}{2b^{2}\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)A\ln\left(e^{i(dx+c)}+\right.}{b^{2}\sqrt{b\cos\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-C*sin(d*x+c))*cos(d
*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.63, size = 104, normalized size = 1.02

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^{2}+\sin(dx+c)^{2}+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^{2}+\sin(dx+c)^{2}-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+\frac{2\,C\sin(dx+c)}{b^{\frac{5}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x + c)/b^(5/2))/d
```

Fricas [A]
time = 0.45, size = 309, normalized size = 3.03

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)+B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{3}d\cos\left(dx+c\right)},\ \frac{2\,B\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)+A\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\sqrt{b\cos\left(dx+c\right)}C\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{3}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x +
 c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b
) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*
x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x
 + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) -
2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2), x)
```

3.335 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+C*x*cos(d*x+c)^(1/
2)/b^2/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b
*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3100, 2814, 3855}

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(5/2),x]
```

```
[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{b^2 d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c}{}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2 d\sqrt{\cos(c+dx)}\sqrt{b\,c}}$$

$$= \frac{Cx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\tanh^{-1}(\sin(c+dx))}{b^2 d\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(Cdx\cos(c+dx)+B\tanh^{-1}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx)\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.21, size = 72, normalized size = 0.71

method	result
default	$\frac{\left(-2B\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+C\cos(dx+c)(dx+c)+A\sin(dx+c)\right)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$

risch	$\frac{Cx\left(\sqrt{\cos}\,(dx+c)\right)}{b^2\sqrt{b\cos(dx+c)}}+\frac{2i\left(\sqrt{\cos}\,(dx+c)\right)A}{b^2\sqrt{b\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\ln\left(e^{i(dx+c)}+i\right)}{b^2\sqrt{b\cos(dx+c)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)}{b^2\sqrt{b\cos(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*B*arctanh((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+C*cos(d*x+c)*(d*x+
c)+A*sin(d*x+c))*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.63, size = 157, normalized size = 1.54

$$\frac{\frac{4\,A\sqrt{b}\,\sin\left(2\,dx+2\,c\right)}{b^3\cos\left(2\,dx+2\,c\right)^2+b^3\sin\left(2\,dx+2\,c\right)^2+2\,b^3\cos\left(2\,dx+2\,c\right)+b^3}+\frac{B\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{4\,C\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{b^{\frac{5}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
*x + c) + 1))/b^(5/2) + 4*C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2)
)/d
```

Fricas [A]
time = 0.46, size = 317, normalized size = 3.11

$$\left[-\frac{2\,B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin\left(dx+c\right)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^2+C\sqrt{-b}\cos\left(dx+c\right)^2\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\,\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^3d\cos\left(dx+c\right)^2},\ \frac{2\,C\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{b}\cos\left(dx+c\right)^{\frac{3}{2}}}\right)\cos\left(dx+c\right)^2+B\sqrt{b}\cos\left(dx+c\right)^2\log\left(-\frac{b\cos\left(dx+c\right)^3-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2\,b\cos\left(dx+c\right)}{\cos\left(dx+c\right)^3}\right)+2\,\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^3d\cos\left(dx+c\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 + C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
 - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(s
qrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-
(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt
(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos
(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c +
 d*x))^(5/2), x)
```

3.336 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b
^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*(A+2*C)*arctanh(sin(d*x+c))*
cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 8, 3855}

$$\frac{(A+2C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]
]) + (B*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(2B+(}{2b^2}$$

$$= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \mathrm{se}}{b^2\sqrt{b\cos(c+d}}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\,\mathrm{s}}{2b^2d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{(A+2C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\,\mathrm{s}}{2b^2d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}\,\left((A+2C)\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(A+2B\cos(c+dx))\sin(c+dx)\right)}{2d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A +
2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.21, size = 151, normalized size = 1.26

method	result
default	$-\frac{\left(A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-A\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)+4C\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos}{\sin(}\right.\right.}{2d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$-\frac{i\left(A\,\mathrm{e}^{2i(dx+c)}-A-4B\cos(dx+c)\right)}{2b^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(A+2C)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c\right.}{2b^2\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x
+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((
-1+cos(d*x+c))/sin(d*x+c))-2*B*cos(d*x+c)*sin(d*x+c)-A*sin(d*x+c))*cos(d*x+
c)^(1/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(104) = 208$.
time = 0.70, size = 820, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
```

```
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*
cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*co
s(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*s
qrt(b)) + 2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2))/d
```

Fricas [**A**]
time = 0.42, size = 239, normalized size = 1.99

$$\left[\frac{(A+2C)\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{4b^3d\cos(dx+c)^3}, -\frac{(A+2C)\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2b^3d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*co
s(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos
(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(
sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x
+ c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*s
qrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(1/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.337 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+1/2*B*sin(d*x+
c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*(2*A+3*C)*sin(d*x+c)/b^2
/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3
*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^4(c+}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(3B+(2}{3b^2\sqrt{}}$$

$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec^3}{b^2\sqrt{b\cos(c+dx}}$$

$$= \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(}}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\,c}}$$

Mathematica [A]
time = 0.24, size = 87, normalized size = 0.53

$$\frac{\sqrt{\cos(c+dx)}\,\left(3B\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+(4A+3C+3B\cos(c+dx)+(2A+3C)\cos(2(c+dx)))\tan(c+dx)\right)}{6d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C
+ 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*C
os[c + d*x])^(5/2))
```

Maple [A]
time = 0.23, size = 157, normalized size = 0.96

method	result
default	$\frac{-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+3B\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^3(dx+c))+4A(\cos^2(dx+c))\sin(dx+c)+6}{6d(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}$
risch	$-\frac{i(3B\,e^{4i(dx+c)}-6C\,e^{3i(dx+c)}-3B+(-16A-18C)\cos(dx+c)+i(-8A-6C)\sin(dx+c))}{6b^2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)B\ln(e^{i(dx+c)}+i)}{2b^2\sqrt{b\cos(dx+c)}\,d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*cos(d*x+c)^2*sin(d
*x+c)+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))
/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(140) = 280.
time = 0.71, size = 1098, normalized size = 6.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x
 + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 16*((3*cos(2*d*x + 2*c) + 1)*si
n(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x
```

```
+ 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(
6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*
sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2
*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) +
 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c
) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*
c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + c
os(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*l
og(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c
) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*
x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*
c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c))*sqrt(b)))/d
```

Fricas [A]
time = 0.43, size = 271, normalized size = 1.65

$$\left[\frac{3\,B\sqrt{b}\,\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{12\,b^{3}d\cos\left(dx+c\right)^{4}}, -\frac{3\,B\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{4}-\left(2\,\left(2\,A+3\,C\right)\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right)+2\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{6\,b^{3}d\cos\left(dx+c\right)^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4), -1/6
*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(co
s(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x
+ c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*co
s(d*x + c)^4)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*c
os(d*x + c)^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{3/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(3/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.338 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sin(c+}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos}}$$

```
[Out] 1/4*A*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*
sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*B*sin(d*x+c)^3/b
^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+B*sin(d*x+c)/b^2/d/cos(d*x+c)^(1
/2)/(b*cos(d*x+c))^(1/2)+1/8*(3*A+4*C)*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)
/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {18, 3100, 2827, 3852, 3853, 3855}

$$\frac{(3A+4C)\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{(3A+4C)\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c +
d*x])^(5/2)),x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*
x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c
+ d*x]]) + (B*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
 + (B*Sin[c + d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx)+C\cos^2(c+dx))\sec^5(c}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int(4B+(}{4b^2}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \mathrm{sec}}{b^2\sqrt{b\cos(c+d}}$$

$$= \frac{A\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{(3A+4C)\sin(c+}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos}}$$

$$= \frac{(3A+4C)\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{A}{4b^2d\cos^{\frac{7}{2}}(c-}$$

Mathematica [A]
time = 0.24, size = 110, normalized size = 0.53

$$\frac{3(3A+4C)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+\sin(c+dx)\left(6A+3(3A+4C)\cos^2(c+dx)+24B\cos^3(c+dx)+8B\cos(c+dx)\sin^2(c+dx)\right)}{24d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Co
s[c + d*x])^(5/2)),x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3
*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c
+ d*x]^2))/(24*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.21, size = 248, normalized size = 1.19

method	result
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}+12C\,e^{6i(dx+c)}+33A\,e^{4i(dx+c)}+12C\,e^{4i(dx+c)}-48B\,e^{3i(dx+c)}-33A\,e^{2i(dx+c)}-12C\,e^{2i(dx+c)}-9A-12C-80B\cos(d}{24b^2\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^3 d}$
default	$\frac{-9A\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+9A\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)-12C\left(\cos^4(dx+c)\right)\ln\left(-\frac{-1+\cos(d}{\text{si}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/24/d*(-9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*cos(d*x+c)^2*sin
(d*x+c)+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+
c))/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2760 vs. $2(180) = 360$.
time = 0.72, size = 2760, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
```

+ 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*A/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*co
s(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*
b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c
)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b
^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x
+ 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d
*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(
2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*
d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*
c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B/((b^2*cos(6*d*
x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(
6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*

```
b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(
3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) +
b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)...
```

Fricas [A]
time = 0.44, size = 305, normalized size = 1.47

$$\left[\frac{3\,(3\,A+4\,C)\sqrt{b}\,\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\,\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{48\,b^{3}d\cos\left(dx+c\right)^{5}},\ -\frac{3\,(3\,A+4\,C)\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(16\,B\cos\left(dx+c\right)^{3}+3\,(3\,A+4\,C)\cos\left(dx+c\right)^{2}+8\,B\cos\left(dx+c\right)+6\,A\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{24\,b^{3}d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
 8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b^3*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos
(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (
16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)
^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*c
os(d*x + c)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^(5/2)*(b*cos(c +
d*x))^(5/2)), x)
```

3.339 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^2d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{88b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(}{}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^2/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c
)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*
x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{88b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyper
geometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{\int(b\cos(c+dx))^{5/3}\left(A+B\cos\right.}{b}$$
$$=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx}{11b^2d}$$
$$=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx}{11b^2d}$$
$$=\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx}{11b^2d}$$

Mathematica [A]
time = 0.28, size = 109, normalized size = 0.71

$$-\frac{3(b\cos(c+dx))^{8/3}\sin(c+dx)\left(\left(11A+8C\right){}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)+8B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)-8C\sqrt{\sin^2(c+dx)}\right)}{88b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2
, 4/3, 7/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6,
 17/6, Cos[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*b^2*d*Sqrt[Sin[c +
d*x]^2])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.340 $\int (b\cos(c+dx))^{2/3} (A+B\cos(c+dx)+C\cos^2(c+d$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5/
3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^
(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3102, 2827, 2722}

$$-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c
 + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*
x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeomet
ric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{3\int(b\,c$$

$$=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{B\int(b\,c$$

$$=\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}-\frac{3(8A+$$

Mathematica [A]
time = 0.26, size = 109, normalized size = 0.71

$$-\frac{3(b\cos(c+dx))^{5/3}\sin(c+dx)\left((8A+5C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)-5C\sqrt{\sin^2(c+dx)}\right)}{40bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2,
 5/6, 11/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3,
7/3, Cos[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*b*d*Sqrt[Sin[c + d*x]
^2])
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cos(c+dx))^{2/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.341 $\int (b\cos(c+dx))^{2/3} (A + B\cos(c + dx) + C\cos^2(c + dx)) dx$

Optimal. Leaf size=148

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d\sqrt{\sin^2(c+dx)}}-\frac{3B(\ldots}{}$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3)
*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c +
 d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(10*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F
1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.25, size = 109, normalized size = 0.74

$$-\frac{3b\left((5A+2C)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)-2C\sqrt{\sin^2(c+dx)}\right)\sin(2(c+dx))}{20d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (-3*b*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Si
n[c + d*x]^2])*Sin[2*(c + d*x)])/(20*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
d*x]^2])
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cos (dx+c))^{\frac{2}{3}} \left(A+B\cos (dx+c)+C\left(\cos^2 (dx+c)\right)\right) \sec (dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{2/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.342 $\int (b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right) dx$

Optimal. Leaf size=147

$$\frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/}}{5bd}$$

[Out] 3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/5*(2*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{4/3}}$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + \frac{3\int \frac{\frac{b^2B}{3}-\frac{1}{3}b}{\sqrt[3]{b}}}{}$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} + (bB)\int \frac{}{\sqrt[3]{}}$$

$$= \frac{3Ab\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}} - \frac{3B(b\cos(c}{}$$

Mathematica [A]
time = 0.39, size = 116, normalized size = 0.79

$$-\frac{3b\left(-10A\csc(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)+\cot(c+dx)\left(5B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2C\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)\right)\sqrt{\sin^2(c+dx)}}{10d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (-3*b*(-10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]
 + Cot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C
*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[
c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{2/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.343 $\int (b\cos(c+dx))^{2/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal. Leaf size=145

$$\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3}\ldots\right.}{8d\sqrt{\sin^2(c+dx\ldots}}$$

```
[Out] 3/4*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*b*B*hypergeom([-1/6, 1/2],[5/
6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8
*(A+4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(
d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*b*B*Hypergeometric
2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)
*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{7/3}}$$

$$= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3}{4}\int \frac{\frac{4b^2}{3}}{}$$

$$= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\left(b^2B\right)\int$$

$$= \frac{3Ab^2\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3bB\,{}_2F_1}{d\sqrt[3]{}}$$

Mathematica [A]
time = 0.26, size = 123, normalized size = 0.85

$$-\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(-A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+2\cos(c+dx)\left(-2B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)+C\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\right)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1
/3, Cos[c + d*x]^2]) + 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/
6, Cos[c + d*x]^2] + C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c
+ d*x]^2]))*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^3\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.344 $\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=152

$$\frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3b^2B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}+\frac{3b(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b^2*B*hypergeom([-2/3, 1/2]
,[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
+3/7*b*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*
cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3b(4A+7C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (3*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b^2*B*Hypergeometr
ic2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(
4/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2,
5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)+C\cos}{(b\cos(c+dx))^{10/}}$$

$$= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{1}{7}(3b)\int$$

$$= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \left(b^3B\right)\int$$

$$= \frac{3Ab^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}} + \frac{3b^2B\,{}_2F}{4d(}$$

Mathematica [A]
time = 0.22, size = 123, normalized size = 0.81

$$\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\left(4A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7\cos(c+dx)\left(B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+4C\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)\right)\sec^3(c+dx)\sqrt{\sin^2(c+dx)}}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1
/6, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, C
os[c + d*x]^2] + 4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c +
 d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*se
c(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{2/3}\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.345 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx)}{13b^2d}-\frac{3(13A+10C)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{130b^2d\sqrt{\sin^2(c+dx)}}-$$

```
[Out] 3/13*C*(b*cos(d*x+c))^(10/3)*sin(d*x+c)/b^2/d-3/130*(13*A+10*C)*(b*cos(d*x+
c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d
*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],co
s(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(13A+10C)\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{130b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{10/3}}{13b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x
]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b
*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[
c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*H
ypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^3*d*S
qrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{7/3}\left(A+B\cos(\right.}{b}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx}{13b^2d}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx}{13b^2d}$$
$$= \frac{3C(b\cos(c+dx))^{10/3}\sin(c+dx}{13b^2d}$$

Mathematica [A]
time = 0.39, size = 111, normalized size = 0.72

$$-\frac{3(b\cos(c+dx))^{10/3}\sin(c+dx)\left((13A+10C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+10\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{130b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c
 + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x]*((13*A + 10*C)*Hypergeometric2F1[1
/2, 5/3, 8/3, Cos[c + d*x]^2] + 10*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, 1
3/6, 19/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(130*b^2*d*Sqrt[Sin[
c + d*x]^2])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^4 + B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*
cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{4/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x
)^2), x)
```

3.346 $\int (b\cos(c+dx))^{4/3} (A + B\cos(c+dx) + C\cos^2(c+$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{70bd\sqrt{\sin^2(c+dx)}} - \frac{3I}{}$$

```
[Out] 3/10*C*(b*cos(d*x+c))^(7/3)*sin(d*x+c)/b/d-3/70*(10*A+7*C)*(b*cos(d*x+c))^(
7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2
)^(1/2)-3/10*B*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^
2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3102, 2827, 2722}

$$-\frac{3(10A+7C)\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{70bd\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} + \frac{3C\sin(c+dx)(b\cos(c+dx))^{7/3}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos
[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/3)*Hypergeo
metric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}+\frac{3\int(b\cos$$
$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}+\frac{B\int(b\cos$$
$$= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10bd}-\frac{3(10A+$$

Mathematica [A]
time = 0.25, size = 109, normalized size = 0.71

$$-\frac{3(b\cos(c+dx))^{7/3}\sin(c+dx)\left((10A+7C)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)-7C\sqrt{\sin^2(c+dx)}\right)}{70bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x]*((10*A + 7*C)*Hypergeometric2F1[1/2
, 7/6, 13/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3,
 8/3, Cos[c + d*x]^2] - 7*C*Sqrt[Sin[c + d*x]^2]))/(70*b*d*Sqrt[Sin[c + d*x
]^2])
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\cos(c+d\,x))^{4/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.347 $\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+d\right.$ $dx)\,dx$

Optimal. Leaf size=148

$$\frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7d}-\frac{3(7A+4C)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{28d\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

```
[Out] 3/7*C*(b*cos(d*x+c))^(4/3)*sin(d*x+c)/d-3/28*(7*A+4*C)*(b*cos(d*x+c))^(4/3)
*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
-3/7*B*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(7A+4C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{28d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{4/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x],x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c +
 d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])
/(28*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F
1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int \sqrt[3]{b\cos(c+dx)}\,(A+B\cos$$

$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx}{7d}$$

$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx}{7d}$$

$$= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx}{7d}$$

Mathematica [A]
time = 0.26, size = 109, normalized size = 0.74

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\left((7A+4C)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)-4C\sqrt{\sin^2(c+dx)}\right)\sin(2(c+dx))}{56d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*((7*A + 4*C)*Hypergeometric2F1[1/2, 2/3, 5/3,
Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c
+ d*x]^2] - 4*C*Sqrt[Sin[c + d*x]^2])*Sin[2*(c + d*x)])/(56*d*Sqrt[Sin[c +
d*x]^2])
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x), x)
```

3.348 $\int (b\cos(c+dx))^{4/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) dx$

Optimal. Leaf size=145

$$\frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{4d} - \frac{3b(4A+C)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos\ldots}{}$$

```
[Out] 3/4*b*C*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d-3/4*b*(4*A+C)*(b*cos(d*x+c))^(1/3
)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2
)-3/4*B*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3b(4A+C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}} + \frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^2,x]
```

```
[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c
 + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x
])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^2}$$

$$= \frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+d}{4d}$$

$$= \frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+d}{4d}$$

$$= \frac{3bC\sqrt[3]{b\cos(c+dx)}\,\sin(c+d}{4d}$$

Mathematica [A]
time = 0.23, size = 108, normalized size = 0.74

$$-\frac{3b^2\left((4A+C)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\sin(2(c+dx))}{8d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^2,x]
```

```
[Out] (-3*b^2*((4*A + C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*Cos
[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] - C*Sqrt[Sin[c +
 d*x]^2])*Sin[2*(c + d*x)])/(8*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2
])
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{4/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^2, x)
```

3.349 $\int (b\cos(c+dx))^{4/3} (A + B\cos(c+dx) + C\cos^2(c+dx)) \sec^3(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}-\frac{3bB\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}+\frac{3(A-2C)(b\cos(c+dx))^{4/3}\ldots}{8d\ldots}$$

```
[Out] 3/2*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)-3*b*B*(b*cos(d*x+c))^(1/3)*hype
rgeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)+3/8*
(A-2*C)*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d
*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}+\frac{3(A-2C)\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^3,x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) - (3*b*B*(b*Cos[c + d*x
])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
Sqrt[Sin[c + d*x]^2]) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)+C\,c}{(b\cos(c+dx))^5}$$

$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}+\frac{3}{2}\int \frac{2b}{}$$

$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}+\left(b^2B\right)$$

$$= \frac{3Ab^2\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}}-\frac{3bB\sqrt[3]{}}{}$$

Mathematica [**A**]
time = 0.25, size = 117, normalized size = 0.81

$$-\frac{3b^2\csc(c+dx)\left(-2A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)+\cos(c+dx)\left(4B\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+C\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2
] + Cos[c + d*x]*(4*B*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{4/3}\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^3, x)
```

3.350 $\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$

Optimal. Leaf size=152

$$\frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}+\frac{3b^2B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}-\frac{3b(2A+5C)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)+3/2*b^2*B*hypergeom([-1/3, 1/2]
,[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
-3/5*b*(2*A+5*C)*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)
^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}}-\frac{3b(2A+5C)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}}+\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c +
d*x]^4,x]
```

```
[Out] (3*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) + (3*b^2*B*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(
2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hyperg
eometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c +
d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)+C\,\mathrm{c}}{(b\cos(c+dx))^{8}}$$

$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{1}{5}(3b)$$

$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \left(b^3B\right)$$

$$= \frac{3Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}} + \frac{3b^2B\,{}_2}{2d}$$

Mathematica [A]
time = 0.22, size = 124, normalized size = 0.82

$$-\frac{3(b\cos(c+dx))^{4/3}\csc(c+dx)\left(-2A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5\cos(c+dx)\left(-B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)+2C\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)\right)\sec^3(c+dx)\sqrt{\sin^2(c+dx)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^4,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-5/6, 1/2,
1/6, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(-(B*Hypergeometric2F1[-1/3, 1/2, 2/3
, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[
c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*sec(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*se
c(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{4/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c
+ d*x)^4, x)
```

3.351 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{88b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B(}{}$$

```
[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^3/d-3/88*(11*A+8*C)*(b*cos(d*x+c))
^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c
)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*
x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{88b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^4d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*C
os[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c +
 d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hyper
geometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^4*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{5/3}\left(A+B\cos(c+dx)+C\cos\right.}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} + \frac{3\int (b\cos(c}{}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} + \frac{B\int (b\cos(c}{b}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^3d} - \frac{3(11A+8C}{}$$

Mathematica [A]
time = 0.29, size = 114, normalized size = 0.74

$$-\frac{3\cos^3(c+dx)\sin(c+dx)\left((11A+8C)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)+8B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)-8C\sqrt{\sin^2(c+dx)}\right)}{88d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^3*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, C
os[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*d*(b*Cos[c + d*x])^(1/3)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(dx+c)\right)\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{\left(b\cos(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3)/b, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4850 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3), x)
```

3.352 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^2/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3),x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2\right.}{b}$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} + \frac{3\int (b\cos(c+}{}$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} + \frac{B\int (b\cos(c-}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^2d} - \frac{3(8A+5C)(}{}$$

Mathematica [A]
time = 0.19, size = 109, normalized size = 0.71

$$-\frac{3(b\cos(c+dx))^{5/3}\sin(c+dx)\left((8A+5C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)-5C\sqrt{\sin^2(c+dx)}\right)}{40b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2,
 5/6, 11/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3,
7/3, Cos[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*b^2*d*Sqrt[Sin[c + d*
x]^2])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3065 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(1/3), x)
```

3.353 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

```
[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/
3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(
1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*s
in(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3102, 2827, 2722}

$$-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c
 + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x
])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometr
ic2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{3\int \frac{\frac{1}{3}b(5A+2C)+\frac{5}{3}bB\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,c}{5b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{B\int(b\cos(c+dx))^{2/3}\,dx}{b}+$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}-\frac{3(5A+2C)(b\cos(c+dx))}{10b}$$

Mathematica [A]
time = 0.14, size = 108, normalized size = 0.70

$$-\frac{3\left((5A+2C)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)-2C\sqrt{\sin^2(c+dx)}\right)\sin(2(c+dx))}{20d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*Cos
[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Sin[
c + d*x]^2])*Sin[2*(c + d*x)])/(20*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*
x]^2])
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b*
cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(1/3), x)
```

3.354 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}}{5b^2d\sqrt{}}$$

```
[Out] 3*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom(
[1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/5*(2*A
-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(1/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)
*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b*d*Sqrt
[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1
/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx\\
&= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}+\frac{3\int \frac{\frac{b^2B}{3}-\frac{1}{3}b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b^2}\\
&= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}+B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx-\\
&= \frac{3A\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\right.}{2bd\sqrt{\sin}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 779 vs. 2(149) = 298.
time = 6.32, size = 779, normalized size = 5.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(1/3),x]
```

```
[Out] (Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-3*(-4*A + C + C*Co
s[2*c])*Csc[c]*Sec[c])/(2*d) + (6*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/((b*C
os[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (2*
B*Cos[c + d*x]^(4/3)*Cos[d*x - ArcTan[Cot[c]]]*Hypergeometric2F1[1/2, 2/3,
3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sin
[d*x - ArcTan[Cot[c]]])/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*
```

```
x] + C*Cos[2*c + 2*d*x])*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x
 - ArcTan[Cot[c]]]^2)^(1/3)) + (4*A*Cos[c + d*x]^(4/3)*Csc[c]*(B + C*Cos[c
+ d*x] + A*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C +
2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (2*C*Cos[c + d*x]^(4/3)*Csc[c]*(B
 + C*Cos[c + d*x] + A*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*
(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)} + C\cos^2{(c + dx)})\sec{(c + dx)}}{\sqrt[3]{b\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3)
,x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)/(b*cos(c + d
*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos{(c + d\,x)}^2 + B\,\cos{(c + d\,x)} + A}{\cos{(c + d\,x)}\,\left(b\,\cos{(c + d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(1/3)), x)
```

3.355 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=145

$$\frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],c
os(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(A+
4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8bd\sqrt{\sin^2(c+dx)}}+\frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[
-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqr
t[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[
1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3\int \frac{\frac{4b^2B}{3}+\frac{1}{3}b^2(A+4C)\cos(c+d}{(b\cos(c+dx))^{4/3}}}{4b}$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + (bB)\int \frac{1}{(b\cos(c+dx))^{4/}}$$

$$= \frac{3Ab\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c\right.}{d\sqrt[3]{b\cos(c+dx)}\,\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 699 vs. 2(145) = 290.
time = 6.36, size = 699, normalized size = 4.82

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((6*B*Csc[c]*Sec[c]
)/d + (3*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(2*d) + (3*Sec[c]*Sec[c + d*x]*(
A*Sin[c] + 4*B*Sin[d*x]))/(2*d)))/((b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Co
s[c + d*x] + C*Cos[2*c + 2*d*x])) - (A*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[
Cot[c]]]*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C +
 B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x - ArcTan[Cot[c]]])/(2*d*(b*Cos[
```

```
c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*C
os[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)) - (2*
C*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[Cot[c]]]*Hypergeometric2F1[1/2, 2/3,
3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*S
in[d*x - ArcTan[Cot[c]]])/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c +
d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d
*x - ArcTan[Cot[c]]]^2)^(1/3)) + (4*B*Cos[c + d*x]^(7/3)*Csc[c]*(C + B*Sec[
c + d*x] + A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*(2*A +
C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(dx + c) + C(\cos^2(dx + c)))(\sec^2(dx + c))}{(b\cos(dx + c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx) + C\cos^2(c + dx))\sec^2(c + dx)}{\sqrt[3]{b\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1
/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c + d*x)**2/(b*cos(c
+ d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos(c + d\,x)^2 + B\,\cos(c + d\,x) + A}{\cos(c + d\,x)^2\,(b\,\cos(c + d\,x))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(1/3)), x)
```

3.356 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3bB\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}+\frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\,\mathrm{s}}{7d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx}}$$

```
[Out] 3/7*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*b*B*hypergeom([-2/3, 1/2],[
1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)+3
/7*(4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(
d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3(4A+7C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b*B*Hypergeometric
2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/
3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x
]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^3\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3}{7}\int \frac{\frac{7b^2B}{3}+\frac{1}{3}b^2(4A+7C\\ &= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))\\ &= \frac{3Ab^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3bB\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c}{4d(b\cos(c+dx))^{4/3}}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 118, normalized size = 0.79

$$\frac{3b^2\csc(c+dx)\left(4A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7\cos(c+dx)\left(B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+4C\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)\right)\sqrt{\sin^2(c+dx)}}{28d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(dx + c) + C(\cos^2(dx + c)))(\sec^3(dx + c))}{(b\cos(dx + c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1
/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^3*(b*cos(c + d*x)
)^(1/3)), x)
```

3.357 $\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d}-\frac{3(11A+8C)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{88b^4d\sqrt{\sin^2(c+dx)}}-\frac{3B(}{}$$

[Out] 3/11*C*(b*cos(d*x+c))^(8/3)*sin(d*x+c)/b^4/d-3/88*(11*A+8*C)*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)-3/11*B*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin(d*x+c)/b^5/d/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(11A+8C)\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{88b^4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^5d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{8/3}}{11b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^4*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^5*d*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int (b\cos(c+dx))^{5/3}\left(A+B\cos(c+dx)+C\cos\right.}{b^3}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} + \frac{3\int(b\cos(c\cdot}{}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} + \frac{B\int(b\cos(c}{b}$$

$$= \frac{3C(b\cos(c+dx))^{8/3}\sin(c+dx)}{11b^4d} - \frac{3(11A+8C}{}$$

Mathematica [A]
time = 0.38, size = 114, normalized size = 0.74

$$-\frac{3\cos^4(c+dx)\sin(c+dx)\left((11A+8C)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)+8B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)-8C\sqrt{\sin^2(c+dx)}\right)}{88d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^4*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, C
os[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*d*(b*Cos[c + d*x])^(4/3)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^3(dx+c)\right)\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(4/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^(2/3)/b^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.358 $\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}-\frac{3(8A+5C)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{40b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

```
[Out] 3/8*C*(b*cos(d*x+c))^(5/3)*sin(d*x+c)/b^3/d-3/40*(8*A+5*C)*(b*cos(d*x+c))^(
5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)
^2)^(1/2)-3/8*B*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^
2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^4d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{5/3}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos
[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c +
d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hyperge
ometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d*Sqrt[Sin[c
 + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos\right.}{b^2}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}+\frac{3\int (b\cos(c\right.}{}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}+\frac{B\int (b\cos(c}{b}\\ &= \frac{3C(b\cos(c+dx))^{5/3}\sin(c+dx)}{8b^3d}-\frac{3(8A+5C)}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 114, normalized size = 0.74

$$-\frac{3\cos^3(c+dx)\sin(c+dx)\left((8A+5C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)-5C\sqrt{\sin^2(c+dx)}\right)}{40d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^3*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2, 5/6, 11
/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos
[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*d*(b*Cos[c + d*x])^(4/3)*Sqrt
[Sin[c + d*x]^2])
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(dx+c)\right)\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.359 $\int \frac{\cos(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=154

$$\frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d}-\frac{3(5A+2C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B(b$$

[Out] 3/5*C*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/b^2/d-3/10*(5*A+2*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/5*B*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)

Rubi [A]

time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}}{5b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)\,(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{3\int \frac{\frac{1}{3}b(5A+2C)}{\sqrt[3]{b\cos}}}{}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} + \frac{B\int(b\cos(c}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{2/3}\sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(}{}$$

Mathematica [A]
time = 0.24, size = 111, normalized size = 0.72

$$-\frac{3\left((5A+2C)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)-2C\sqrt{\sin^2(c+dx)}\right)\sin(2(c+dx))}{20bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
 d*x])^(4/3),x]
```

```
[Out] (-3*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*Cos
[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Sin[
c + d*x]^2])*Sin[2*(c + d*x)])/(20*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
d*x]^2])
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\cos (dx+c) (A+B\cos (dx+c)+C(\cos^2 (dx+c)))}{(b\cos (dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^
2*cos(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(4/3),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x))
^(4/3), x)
```

3.360 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=152

$$\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}+\frac{3(2A-C)(b\cos(c+dx))^{5/3}}{5b^3d}$$

```
[Out] 3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeo
m([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)+3/5*
(2*A-C)*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3100, 2827, 2722}

$$\frac{3(2A-C)\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/
3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*
Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2
F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]
^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}+\frac{3\int \frac{\frac{b^2B}{3}-\frac{1}{3}b^2(2A-C)\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b^3}\\ &= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}+\frac{B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}-\frac{(2A-C)\int (\\ &= \frac{3A\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c-\right.}{2b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 115, normalized size = 0.76

$$-\frac{3\cot(c+dx)\left(-10A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)+\cos(c+dx)\left(5B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2C\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)\right)\sqrt{\sin^2(c+dx)}}{10d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
[Out] (-3*Cot[c + d*x]*(-10*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
 Cos[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2])/(10*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^
2*cos(d*x + c)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3),x)
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(b*cos(c + d*x))^(4/3), x)
```

3.361 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=147

$$\frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3(A+4C)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)+3*B*hypergeom([-1/6, 1/2],[5/6],cos
(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/8*(A+
4*C)*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3100, 2827, 2722}

$$-\frac{3(A+4C)\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}+\frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])
^(4/3),x]
```

```
[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1
/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqr
t[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[
1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3\int \frac{\frac{4b^2B}{3}+\frac{1}{3}b^2(A+4C)\cos(c+dx}{(b\cos(c+dx))^{4/3}}}{4b^2}$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + B\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$$

$$= \frac{3A\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}} + \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+\right.}{bd\sqrt[3]{b\cos(c+dx)}\,\sqrt{}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 703 vs. 2(147) = 294.
time = 6.33, size = 703, normalized size = 4.78

$$\frac{\frac{\cos^3(c+dx)(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{6B\csc(c)\sec(c)}{d}+\frac{3A\sec(c)\sec^2(c+dx)\sin(dx)}{2d}+\frac{3\sec(c)\sec(c+dx)(A\sin(c)+4B\sin(dx))}{2d}\right)}{\sqrt[3]{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))} - \frac{A\cos^{\frac{4}{3}}(c+dx)\cos(dx-\text{ArcTan}(\cot(c)))\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\cos^2(dx-\text{ArcTan}(\cot(c)))\right)(C+B\sec(c+dx)+A\sec^2(c+dx))\sin(dx-\text{ArcTan}(\cot(c)))}{2d\sqrt[3]{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))\sqrt[3]{\cos(c)\cos(dx)-\sin(c)\sin(dx)}\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}} - \frac{2C\cos^{\frac{4}{3}}(c+dx)\cos(dx-\text{ArcTan}(\cot(c)))\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\cos^2(dx-\text{ArcTan}(\cot(c)))\right)(C+B\sec(c+dx)+A\sec^2(c+dx))\sin(dx-\text{ArcTan}(\cot(c)))}{d\sqrt[3]{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))\sqrt[3]{\cos(c)\cos(dx)-\sin(c)\sin(dx)}\sqrt[3]{\sin^2(dx-\text{ArcTan}(\cot(c)))}} + \frac{4B\cos^{\frac{4}{3}}(c+dx)\csc(c)(C+B\sec(c+dx)+A\sec^2(c+dx))\left(\frac{{}_2F_1(\ldots)\sin(dx+\text{ArcTan}(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt{1+\cos(dx+\text{ArcTan}(\tan(c)))}\sqrt[3]{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}} - \frac{\ldots}{\sqrt[3]{\cos(c)\cos(dx+\text{ArcTan}(\tan(c)))\sqrt{1+\tan^2(c)}}}\right)}{d\sqrt[3]{b\cos(c+dx)}(2A+C+2B\cos(c+dx)+C\cos(2c+2dx))}}{b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
 d*x])^(4/3),x]
```

```
[Out] ((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((6*B*Csc[c]*Sec[c
])/d + (3*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(2*d) + (3*Sec[c]*Sec[c + d*x]*
(A*Sin[c] + 4*B*Sin[d*x]))/(2*d)))/((b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*C
os[c + d*x] + C*Cos[2*c + 2*d*x])) - (A*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan
[Cot[c]]]*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C
+ B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x - ArcTan[Cot[c]]])/(2*d*(b*Cos
```

```
[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*
Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)) - (2
*C*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[Cot[c]]]*Hypergeometric2F1[1/2, 2/3,
 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*
Sin[d*x - ArcTan[Cot[c]]])/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c +
 d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[
d*x - ArcTan[Cot[c]]]^2)^(1/3)) + (4*B*Cos[c + d*x]^(7/3)*Csc[c]*(C + B*Sec
[c + d*x] + A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*(2*A +
 C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/b
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(dx+c)+C(\cos^2(dx+c)))\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3)
,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
 c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)*(b*cos(c + d*x))^
(4/3)), x)
```

3.362 $\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=149

$$\frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}+\frac{3(4A+7C)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)+3/4*B*hypergeom([-2/3, 1/2],[1/3]
,cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)+3/7*(
4*A+7*C)*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*
x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3100, 2827, 2722}

$$\frac{3(4A+7C)\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*B*Hypergeometric2F1[
-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*S
qrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos
[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^
2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^2\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx \\
&= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3\int \frac{\frac{7b^2B}{3}+\frac{1}{3}b^2(4A+7C)\cos(c+}{(b\cos(c+dx))^{7/3}}}{7b} \\
&= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+(bB)\int \frac{1}{(b\cos(c+dx))^{7/}} \\
&= \frac{3Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}}+\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+\right.}{4d(b\cos(c+dx))^{4/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 118, normalized size = 0.79

$$
\frac{3b^2\cot(c+dx)\left(4A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7\cos(c+dx)\left(B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+4C\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)\right)\sqrt{\sin^2(c+dx)}}{28d(b\cos(c+dx))^{10/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$
\int \frac{\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)\left(\sec^2\left(dx+c\right)\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec
(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(4/3)),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(cos(c + d*x)^2*(b*cos(c + d*x)
)^(4/3)), x)
```

3.363 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A + B\cos(c+dx) + C\right.$

Optimal. Leaf size=232

$$\frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{d(10+3m)} - \frac{3b(C(7+3m)+A(10+3m))\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+d}}{d(7+3m)(10+3}$$

```
[Out] 3*b*C*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(10+3*m)-3*b*(C*(7
+3*m)+A*(10+3*m))*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6
+1/2*m],[13/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+51*m+70)/(sin(d*x+c)
^2)^(1/2)-3*b*B*cos(d*x+c)^(3+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 5/3+1
/2*m],[8/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(10+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 222, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{3b\left(\frac{A}{3m+7}+\frac{C}{3m+10}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}} - \frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+10);\frac{1}{6}(3m+16);\cos^2(c+dx)\right)}{d(3m+10)\sqrt{\sin^2(c+dx)}} + \frac{3bC\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)}{d(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3
*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Co
s[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c
+ d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}}{}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c}}{d(10+3m}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c}}{d(10+3m}$$

$$= \frac{3bC\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c}}{d(10+3m}$$

Mathematica [A]
time = 0.71, size = 169, normalized size = 0.73

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\sin(c+dx)\left(B(7+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3}+\frac{m}{2};\frac{8}{3}+\frac{m}{2};\cos^2(c+dx)\right)+(C(7+3m)+A(10+3m))\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)-C(7+3m)\sqrt{\sin^2(c+dx)}\right)}{d(7+3m)(10+3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x]*(B*(7 + 3*m)*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] +
(C*(7 + 3*m) + A*(10 + 3*m))*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)
```

```
/6, Cos[c + d*x]^2] - C*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]))/(d*(7 + 3*m)*(10 +
 3*m)*Sqrt[Sin[c + d*x]^2])
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(dx+c\right)\right) \left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}} \left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*co
s(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*co
s(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.364 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+\right.$

Optimal. Leaf size=229

$$\frac{3C\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)}{d(8+3m)}-\frac{3(C(5+3m)+A(8+3m))\cos^{1+m}(c+dx)(b\cos(c+dx))}{d(5+3m)(8+3}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*sin(d*x+c)/d/(8+3*m)-3*(C*(5+3*m)
+A*(8+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m
],[11/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+39*m+40)/(sin(d*x+c)^2)^(1
/2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 4/3+1/2*m],[7
/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(8+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{3\left(\frac{A}{3m+5}+\frac{C}{3m+8}\right)\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+8);\frac{1}{6}(3m+14);\cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)}{d(3m+8)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(2/3)*Hypergeometric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c}{}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+}{d(8+3m)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+}{d(8+3m)}$$

$$= \frac{3C\cos^{1+m}(c+dx)(b\cos(c+}{d(8+3m)}$$

Mathematica [A]
time = 0.46, size = 166, normalized size = 0.72

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\sin(c+dx)\left(\left(C(5+3m)+A(8+3m)\right){}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)+(5+3m)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(8+3m);\frac{7}{3}+\frac{m}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(5+3m)(8+3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x]*((C*(5 + 3*m)
+ A*(8 + 3*m))*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*
x]^2] + (5 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 +
 m/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(5 + 3*m)*(8 + 3*m)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (\cos^m (dx + c)) (b \cos (dx + c))^{\frac{2}{3}} \left(A + B \cos (dx + c) + C \left(\cos^2 (dx + c)\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*co
s(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.365 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=229

$$\frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\sin(c+dx)}{d(7+3m)}-\frac{3(C(4+3m)+A(7+3m))\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}}{d(4+3m)(7+3m}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*sin(d*x+c)/d/(7+3*m)-3*(C*(4+3*m)
+A*(7+3*m))*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m
],[5/3+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(9*m^2+33*m+28)/(sin(d*x+c)^2)^(1/
2)-3*B*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13
/6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{3\left(\frac{A}{3m+4}+\frac{C}{3m+7}\right)\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}+\frac{3C\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)}{d(3m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)
) - (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+}}{d(7+3m)}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+}}{d(7+3m)}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+}}{d(7+3m)}$$

Mathematica [A]
time = 0.44, size = 166, normalized size = 0.72

$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\sin(c+dx)\left(\left(C(4+3m)+A(7+3m)\right){}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\cos^2(c+dx)\right)+(4+3m)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(4+3m)(7+3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x]*((C*(4 + 3*m)
+ A*(7 + 3*m))*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^
2] + (4 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*
```

```
m)/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(4 + 3*m)*(7 + 3*m)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(dx+c\right)\right) \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}} \left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos\left(c+dx\right)} \left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)\cos^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*c
os(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*co
s(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.366 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=229

$$\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(2+3m)+A(5+3m))\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\mathrm{co}\right.}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(2+3*m)
+A*(5+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x
+c)^2)*sin(d*x+c)/d/(9*m^2+21*m+10)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/
2)-3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 5/6+1/2*m],[11/6+1/2*m],cos(d*x+c)^
2)*sin(d*x+c)/d/(5+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{3\left(\frac{A}{3m+2}+\frac{C}{3m+5}\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}+\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(1/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)
) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1
/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d
*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric
2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3
*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\left(A+B\cos(\right.}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(3\sqrt[3]{\cos(c+\right.}{}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+\right.}{}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3\left(\frac{A}{2+3m}+\frac{C}{5+}\right.}{}$$

Mathematica [A]
time = 0.47, size = 166, normalized size = 0.72

$$-\frac{3\cos^{1+m}(c+dx)\sin(c+dx)\left(\left(C(2+3m)+A(5+3m)\right){}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)+(2+3m)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C*(2 + 3*m) + A*(5 + 3*m))*Hypergeo
metric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + (2 + 3*m)*(B*Cos
```

```
[c + d*x]*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]
 - C*Sqrt[Sin[c + d*x]^2])))/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*
Sqrt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(\cos^{m}(dx+c))\,(A+B\cos(dx+c)+C(\cos^{2}(dx+c)))}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx)+C\cos^{2}(c+dx))\cos^{m}(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1
/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(1/3), x)
```

3.367 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=227

$$\frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}}-\frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos\right.}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)-3*(C+3*C*m+A
*(4+3*m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c
)^2)*sin(d*x+c)/d/(9*m^2+15*m+4)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-
3*B*cos(d*x+c)^(2+m)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(4+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{3(A(3m+4)+3Cm+C)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}+\frac{3C\sin(c+dx)\cos^{m+1}(c+dx)}{d(3m+4)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(2/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)
) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 +
 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 +
 m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
 + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\left(A+B\cos(c\right.}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} + \frac{\left(3\cos^{\frac{2}{3}}(c+\right.}{}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} + \frac{\left(B\cos^{\frac{2}{3}}(c+\right.}{(b}$$

$$= \frac{3C\cos^{1+m}(c+dx)\sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm}{}$$

Mathematica [A]
time = 0.44, size = 164, normalized size = 0.72

$$-\frac{3\cos^{1+m}(c+dx)\sin(c+dx)\left(\left(C+3Cm+A(4+3m)\right){}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)+(1+3m)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C + 3*C*m + A*(4 + 3*m))*Hypergeome
tric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + (1 + 3*m)*(B*Cos[c
 + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] - C*
Sqrt[Sin[c + d*x]^2])))/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m (dx + c)) (A + B \cos (dx + c) + C(\cos^2 (dx + c)))}{(b \cos (dx + c))^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \cos (c + dx) + C \cos^2 (c + dx)) \cos^m (c + dx)}{(b \cos (c + dx))^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2
/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^m\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(2/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(2/3), x)
```

3.368 $\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=235

$$\frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}}-\frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2\right.}{bd(1-3m)(2+3m)\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*C*cos(d*x+c)^m*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)-3*(C*(1-3*m)-A
*(2+3*m))*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2
)*sin(d*x+c)/b/d/(-9*m^2-3*m+2)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3
*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$\frac{3\left(\frac{A}{1-3m}-\frac{C}{3m+2}\right)\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}+\frac{3C\sin(c+dx)\cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x
])^(4/3),x]
```

```
[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) +
(3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 +
3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1
/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2
, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b
*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)\left(A+B\cos(\right.}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(3\sqrt[3]{\cos(c+dx}\right.}{}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+d}\right.}{b\sqrt[3]{}}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{3\left(\frac{A}{1-3m}-\frac{C}{2+3m}\right.}{}$$

Mathematica [A]
time = 0.59, size = 166, normalized size = 0.71

$$-\frac{3\cos^{1+m}(c+dx)\sin(c+dx)\left(\left(C(-1+3m)+A(2+3m)\right){}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)+(-1+3m)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(-1+3m)(2+3m)(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
 + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C*(-1 + 3*m) + A*(2 + 3*m))*Hyperge
ometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + (-1 + 3*m)*(B*
Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^
```

```
2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/
3)*Sqrt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(\cos^m(dx+c))\,(A+B\cos(dx+c)+C(\cos^2(dx+c)))}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos
(d*x + c)^m/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx)+C\cos^2(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4
/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c
+ d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x
 + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^m\,\left(C\,\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3),x)
```

```
[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/(b*cos(c + d*x
))^(4/3), x)
```

3.369 $\int (a\cos(c+dx))^m(b\cos(c+dx))^n\,(A+B\cos(c+dx)+$

Optimal. Leaf size=227

$$\frac{C(a\cos(c+dx))^{1+m}(b\cos(c+dx))^n\sin(c+dx)}{ad(2+m+n)}-\frac{(C(1+m+n)+A(2+m+n))(a\cos(c+dx))^{1+m}(b\cos(}{ad(1+m+n)}$$

```
[Out] C*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*sin(d*x+c)/a/d/(2+m+n)-(C*(1+m+n)+A
*(2+m+n))*(a*cos(d*x+c))^(1+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2+1/2*m+1
/2*n],[3/2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a/d/(1+m+n)/(2+m+n)/(sin(d
*x+c)^2)^(1/2)-B*(a*cos(d*x+c))^(2+m)*(b*cos(d*x+c))^n*hypergeom([1/2, 1+1/
2*m+1/2*n],[2+1/2*m+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/a^2/d/(2+m+n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{B\sin(c+dx)(a\cos(c+dx))^{m+2}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+2);\frac{1}{2}(m+n+4);\cos^2(c+dx)\right)}{a^2d(m+n+2)\sqrt{\sin^2(c+dx)}}-\frac{(A(m+n+2)+C(m+n+1))\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{ad(m+n+1)(m+n+2)\sqrt{\sin^2(c+dx)}}+\frac{C\sin(c+dx)(a\cos(c+dx))^{m+1}(b\cos(c+dx))^n}{ad(m+n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m +
n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c +
d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2]) - (B*(a*
Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)
/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Si
n[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a\cos(c+dx))^m (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left((a\cos(c+dx))^{-n}(b\cos(c-\right.$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(}{ad(2+m\cdot}$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(}{ad(2+m\cdot}$$

$$= \frac{C(a\cos(c+dx))^{1+m}(b\cos(}{ad(2+m\cdot}$$

Mathematica [A]
time = 0.28, size = 161, normalized size = 0.71

$$-\frac{\cos(c+dx)(a\cos(c+dx))^m(b\cos(c+dx))^n\sin(c+dx)\left(\left(C(1+m+n)+A(2+m+n)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\cos^2(c+dx)\right)+(1+m+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(2+m+n);\frac{1}{2}(4+m+n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(1+m+n)(2+m+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(1 +
 m + n) + A*(2 + m + n))*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/
2, Cos[c + d*x]^2] + (1 + m + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (2
```

```
+ m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(
1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(dx + c\right)\right)^{m} \left(b \cos \left(dx + c\right)\right)^{n} \left(A + B \cos \left(dx + c\right) + C\left(\cos^{2}\left(dx + c\right)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos
(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos(
d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(c + dx\right)\right)^{m} \left(b \cos \left(c + dx\right)\right)^{n} \left(A + B \cos \left(c + dx\right) + C \cos^{2}\left(c + dx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
```

```
[Out] Integral((a*cos(c + d*x))**m*(b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*co
s(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos
(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a\,\cos(c+d\,x))^m\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int((a*cos(c + d*x))^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.370 $\int \cos^2(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\right.$

Optimal. Leaf size=187

$$\frac{C(b\cos(c+dx))^{3+n}\sin(c+dx)}{b^3d(4+n)}-\frac{(C(3+n)+A(4+n))(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin}{b^3d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(3+n)*sin(d*x+c)/b^3/d/(4+n)-(C*(3+n)+A*(4+n))*(b*cos(d*x+
c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b
^3/d/(3+n)/(4+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(4+n)*hypergeom([1/2
, 2+1/2*n],[3+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(4+n)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{(A(n+4)+C(n+3))\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)(n+4)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+6}{2};\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}+\frac{C\sin(c+dx)(b\cos(c+dx))^{n+3}}{b^3d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^
2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A
*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]
^2]) - (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{\int (b\cos(c+dx))^{2+n} \left(A+B\cos\right.}{b}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx}{b^3d(4+n)}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx}{b^3d(4+n)}$$

$$= \frac{C(b\cos(c+dx))^{3+n}\sin(c+dx}{b^3d(4+n)}$$

Mathematica [A]
time = 0.52, size = 144, normalized size = 0.77

$$-\frac{\cos^3(c+dx)(b\cos(c+dx))^n\sin(c+dx)\left(\left(C(3+n)+A(4+n)\right){}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)+(3+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4+n}{2};\frac{6+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(3+n)(4+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c +
 d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(3 + n) + A*(4 + n))*
Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + (3 + n)*(B*C
os[c + d*x]*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] -
C*Sqrt[Sin[c + d*x]^2])))/(d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^n\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*
x + c))^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^
2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^
2), x)
```

3.371 $\int \cos(c+dx)(b\cos(c+dx))^n \left(A + B\cos(c+dx) + C \mathrm{c}\right.$

Optimal. Leaf size=187

$$\frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)} - \frac{(C(2+n)+A(3+n))(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin}{b^2d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(3+n)-(C*(2+n)+A*(3+n))*(b*cos(d*x+
c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d
/(2+n)/(3+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/
2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1
/2)
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{(A(n+3)+C(n+2))\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)(n+3)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}} + \frac{C\sin(c+dx)(b\cos(c+dx))^{n+2}}{b^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)
,x]
```

```
[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A
*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]
^2]) - (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \frac{\int (b\cos(c+dx))^{1+n}\,(A+B\cos(}{b}\\ &= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}\\ &= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}\\ &= \frac{C(b\cos(c+dx))^{2+n}\sin(c+dx)}{b^2d(3+n)}\end{aligned}$$

Mathematica [**A**]
time = 0.36, size = 144, normalized size = 0.77

$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n\sin(c+dx)\left(\left(C(2+n)+A(3+n)\right){}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)+(2+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(2+n)(3+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(2 + n) + A*(3 + n))*
Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + (2 + n)*(B*C
os[c + d*x]*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] -
C*Sqrt[Sin[c + d*x]^2])))/(d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c+d\,x)\ (b\,\cos (c+d\,x))^n\ \left(C\cos (c+d\,x)^2+B\,\cos (c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2)
,x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2)
, x)
```

3.372 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$

Optimal. Leaf size=187

$$\frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)} - \frac{(C(1+n)+A(2+n))(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin}{bd(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] C*(b*cos(d*x+c))^(1+n)*sin(d*x+c)/b/d/(2+n)-(C*(1+n)+A*(2+n))*(b*cos(d*x+c)
)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b/d
/(1+n)/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+
1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3102, 2827, 2722}

$$-\frac{(A(n+2)+C(n+1))\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}} - \frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}} + \frac{C\sin(c+dx)(b\cos(c+dx))^{n+1}}{bd(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(
2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])
 - (B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx &= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)}+\frac{\int (b\cos(c\cdots}{\cdots}\\ &= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)}+\frac{B\int (b\cos\cdots}{\cdots}\\ &= \frac{C(b\cos(c+dx))^{1+n}\sin(c+dx)}{bd(2+n)}-\frac{\left(A+\frac{C(\cdots}{2\cdots}\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 142, normalized size = 0.76

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\sin(c+dx)\left((C(1+n)+A(2+n))\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)+(1+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(1+n)(2+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(1 + n) + A*(2 + n))*Hy
pergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] + (1 + n)*(B*Cos
[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] - C*
Sqrt[Sin[c + d*x]^2])))/(d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\cos(c+d\,x))^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.373 $\int (b\cos(c+dx))^n\,(A+B\cos(c+dx)+C\cos^2(c+dx$ … $dx)\,dx$

Optimal. Leaf size=170

$$\frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)}-\frac{(A+An+Cn)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn(1+n)\sqrt{\sin^2(c+dx)}}-B\ldots$$

```
[Out] C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)-(A*n+C*n+A)*(b*cos(d*x+c))^n*hypergeo
m([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/n/(1+n)/(sin(d*x+c)^2)^
(1/2)-B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x
+c)^2)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{(An+A+Cn)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}+\frac{C\sin(c+dx)(b\cos(c+dx))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
,x]
```

```
[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c
 + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d
*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(1 + n)*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*
(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{-1+n}\left(A+B\cos\right.$$

$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} + \frac{\int}{}$$

$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} + B$$

$$= \frac{C(b\cos(c+dx))^n\sin(c+dx)}{d(1+n)} - \frac{(}{}$$

Mathematica [A]
time = 0.22, size = 127, normalized size = 0.75

$$-\frac{(b\cos(c+dx))^n\sin(c+dx)\left((A+An+Cn)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)+n\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{dn(1+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x],x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Sin[c + d*x]*((A + A*n + C*n)*Hypergeometric2F1[1/2,
n/2, (2 + n)/2, Cos[c + d*x]^2] + n*(B*Cos[c + d*x]*Hypergeometric2F1[1/2,
(1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*n*(1 +
 n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)*sec(c
 + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, al
gorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x), x)
```

3.374 $\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx\right.$ $\left. dx\right)dx$

Optimal. Leaf size=173

$$\frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx)}{dn}-\frac{b(C(1-n)-An)(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+\right.}{d(1-n)n\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*C*(b*cos(d*x+c))^(-1+n)*sin(d*x+c)/d/n-b*(C*(1-n)-A*n)*(b*cos(d*x+c))^(-1
+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n
)/n/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$-\frac{b(C(1-n)-An)\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}+\frac{bC\sin(c+dx)(b\cos(c+dx))^{n-1}}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^2,x]
```

```
[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(
b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c
+ d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*
x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^2(c+dx)\,dx = b^2\int (b\cos(c+dx))^{-2+n}\left(A+B\right.$$

$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx}{dn}$$

$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx}{dn}$$

$$= \frac{bC(b\cos(c+dx))^{-1+n}\sin(c+dx}{dn}$$

Mathematica [A]
time = 0.32, size = 131, normalized size = 0.76

$$\frac{(b\cos(c+dx))^n\left((C-An-Cn)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)-(-1+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)\tan(c+dx)}{d(-1+n)n\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] ((b*Cos[c + d*x])^n*((C - A*n - C*n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1
+ n)/2, Cos[c + d*x]^2] - (-1 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, n
/2, (2 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d*x])/(d*
(-1 + n)*n*Sqrt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^2, x)
```

3.375 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $\left. dx\right) dx$

Optimal. Leaf size=194

$$-\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+dx)}{d(1-n)}+\frac{b^2(A(1-n)+C(2-n))(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\text{co}\right.}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^2*C*(b*cos(d*x+c))^(-2+n)*sin(d*x+c)/d/(1-n)+b^2*(A*(1-n)+C*(2-n))*(b*co
s(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/d/(n^2-3*n+2)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(-1+n)*hypergeom([1/
2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^
(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$\frac{b^2(A(1-n)+C(2-n))\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2C\sin(c+dx)(b\cos(c+dx))^{n-2}}{d(1-n)}+\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^3,x]
```

```
[Out] -((b^2*C*(b*Cos[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(1 - n))) + (b^2*(A*(1
- n) + C*(2 - n))*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)
/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*(2 - n)*Sqrt[Sin[c + d*x]
^2]) + (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1
 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^3(c+dx)\,dx = b^3\int (b\cos(c+dx))^{-3+n}\left(A+B\cos\right.$$

$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+}{d(1-n)}$$

$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+}{d(1-n)}$$

$$= -\frac{b^2C(b\cos(c+dx))^{-2+n}\sin(c+}{d(1-n)}$$

Mathematica [A]
time = 0.51, size = 137, normalized size = 0.71

$$-\frac{b(b\cos(c+dx))^{-1+n}\left(\left(C(-2+n)+A(-1+n)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)+(-2+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)\tan(c+dx)}{d(-2+n)(-1+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
 + d*x]^3,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*((C*(-2 + n) + A*(-1 + n))*Hypergeometric2F1
[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] + (-2 + n)*(B*Cos[c + d*x]*Hypergeom
etric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^
2]))*Tan[c + d*x])/(d*(-2 + n)*(-1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\left(\sec^{3}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^3, x)
```

3.376 $\int (b\cos(c+dx))^n \left(A + B\cos(c+dx) + C\cos^2(c+dx\right.$ $\left.dx\right) dx$

Optimal. Leaf size=196

$$-\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+dx)}{d(2-n)}+\frac{b^3(A(2-n)+C(3-n))(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1\right.}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^3*C*(b*cos(d*x+c))^(-3+n)*sin(d*x+c)/d/(2-n)+b^3*(A*(2-n)+C*(3-n))*(b*co
s(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+c)^2)*sin
(d*x+c)/d/(n^2-5*n+6)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(-2+n)*hype
rgeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^
2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3102, 2827, 2722}

$$\frac{b^3(A(2-n)+C(3-n))\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^3C\sin(c+dx)(b\cos(c+dx))^{n-3}}{d(2-n)}+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]
^4,x]
```

```
[Out] -((b^3*C*(b*Cos[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(2 - n))) + (b^3*(A*(2
- n) + C*(3 - n))*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)
/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*(3 - n)*Sqrt[Sin[c
 + d*x]^2]) + (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (b\cos(c+dx))^n \left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\sec^4(c+dx)\,dx = b^4\int (b\cos(c+dx))^{-4+n}\left(A+B\,c\right.$$
$$= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+}{d(2-n)}$$
$$= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+}{d(2-n)}$$
$$= -\frac{b^3C(b\cos(c+dx))^{-3+n}\sin(c+}{d(2-n)}$$

Mathematica [A]
time = 0.36, size = 142, normalized size = 0.72

$$-\frac{(b\cos(c+dx))^n\sec^2(c+dx)\left(\left(C(-3+n)+A(-2+n)\right){}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos^2(c+dx)\right)+(-3+n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)\tan(c+dx)}{d(-3+n)(-2+n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Sec[c + d*x]^2*((C*(-3 + n) + A*(-2 + n))*Hypergeomet
ric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2] + (-3 + n)*(B*Cos[c + d
*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c
+ d*x]^2]))*Tan[c + d*x])/(d*(-3 + n)*(-2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sec^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x
 + c)^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*
x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^4, x)
```

3.377 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,(A+B\cos(c+dx)+C$

Optimal. Leaf size=223

$$\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(7+2n)}-\frac{2(C(5+2n)+A(7+2n))\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\big(}{d(5+2n)(7+2n)\sqrt{\sin}}$$

```
[Out] 2*C*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(7+2*n)-2*(C*(5+2*n)+A*(
7+2*n))*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+24*n+35)/(sin(d*x+c)^2)^(1/2)-2*B*c
os(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n],co
s(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{2\left(\frac{A}{2n+5}+\frac{C}{2n+7}\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2
*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeom
etric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
7 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(7+2n)}$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(7+2n)}$$

$$=\frac{2C\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n}{d(7+2n)}$$

Mathematica [A]
time = 0.57, size = 164, normalized size = 0.74

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)\left(\left(C(5+2n)+A(7+2n)\right){}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)+(5+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(5+2n)(7+2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(5 + 2*n) + A*(7
 + 2*n))*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] +
 (5 + 2*n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4
, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(5 + 2*n)*(7 + 2*n)*Sqrt[S
in[c + d*x]^2])
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="maxima")
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="fricas")
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x
+ c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)*
*2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.378 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,(A+B\cos(c+dx)$

Optimal. Leaf size=223

$$\frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)}{d(5+2n)}-\frac{2(C(3+2n)+A(5+2n))\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\big(}{d(3+2n)(5+2n)\sqrt{\sin}}$$

```
[Out] 2*C*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*sin(d*x+c)/d/(5+2*n)-2*(C*(3+2*n)+A*(
5+2*n))*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1
/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(4*n^2+16*n+15)/(sin(d*x+c)^2)^(1/2)-2*B*c
os(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n],cos
(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{2\left(\frac{A}{2n+3}+\frac{C}{2n+5}\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n}{d(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2),x]
```

```
[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2
*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*S
qrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeom
etric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5
 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))\right.$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx)}{d(5+2n)}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx)}{d(5+2n)}$$

$$= \frac{2C\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx)}{d(5+2n)}$$

Mathematica [A]
time = 0.47, size = 164, normalized size = 0.74

$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\sin(c+dx)\left((C(3+2n)+A(5+2n))\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)+(3+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(3+2n)(5+2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos
[c + d*x]^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(3 + 2*n) + A*(5
 + 2*n))*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] +
 (3 + 2*n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4,
 Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(3 + 2*n)*(5 + 2*n)*Sqrt[Si
n[c + d*x]^2])
```

Maple [**F**]
time = 0.29, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\left(\sqrt{\cos}\,(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(c
os(d*x + c)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(co
s(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(c
os(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(b\,\cos(c+d\,x)\right)^n\,\left(C\cos(c+d\,x)^2+B\,\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d
*x)^2), x)
```

3.379 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=221

$$\frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx)}{d(3+2n)}-\frac{2(C+2Cn+A(3+2n))\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F$$

[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(3+2*n)-2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*(C+2*C*n+A*(3+2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(4*n^2+8*n+3)/(sin(d*x+c)^2)^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$-\frac{2(A(2n+3)+2Cn+C)\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n}{d(2n+3)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c$$

$$= \frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx}{d(3+2n)}$$

$$= \frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx}{d(3+2n)}$$

$$= \frac{2C\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx}{d(3+2n)}$$

Mathematica [A]
time = 0.42, size = 162, normalized size = 0.73

$$-\frac{2\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\sin(c+dx)\left((C+2Cn+A(3+2n))\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)+(1+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(1+2n)(3+2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C + 2*C*n + A*(3 +
 2*n))*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + (
```

```
1 + 2*n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, C
os[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[
c + d*x]^2])
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A + B\cos(dx+c) + C(\cos^2(dx+c)))}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(c
os(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(co
s(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(c
os(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n\,\left(C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A\right)}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(1/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(1/2), x)
```

3.380 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=217

$$\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}}+\frac{2(A-C(1-2n)+2An)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\text{co}\right.}{d\left(1-4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1+2*n)/cos(d*x+c)^(1/2)+2*(A-C*(1-2*n)+2
*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2
)*sin(d*x+c)/d/(-4*n^2+1)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(
d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*sin(d*x+c)*c
os(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$\frac{2(2An+A-C(1-2n))\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d\left(1-4n^2\right)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}+\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(2n+1)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(3/2),x]
```

```
[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2
*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 +
2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[
c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^
n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{(2\cos^{-n}(c}{}$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \left(B\cos^{-n}(c\right.$$

$$= \frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\cos(c+dx)}} + \frac{2(A-C(1}{}$$

Mathematica [A]
time = 0.49, size = 157, normalized size = 0.72

$$-\frac{2(b\cos(c+dx))^n\sin(c+dx)\left(\left(A+2An+C(-1+2n)\right){}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)+(-1+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d\left(-1+4n^2\right)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Sin[c + d*x]*((A + 2*A*n + C*(-1 + 2*n))*Hypergeomet
ric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + (-1 + 2*n)*(B*Cos[
c + d*x]*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] -
 C*Sqrt[Sin[c + d*x]^2])))/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c +
d*x]^2])
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cos\left(dx+c\right)\right)^{n}\left(A+B\cos\left(dx+c\right)+C\left(\cos^{2}\left(dx+c\right)\right)\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\cos\left(c+dx\right)\right)^{n}\left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3
/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x) + C*cos(c + d*x)**2)/cos(c
 + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n\,\left(C\,{\cos\left(c+d\,x\right)}^2+B\,\cos\left(c+d\,x\right)+A\right)}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(3/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(3/2), x)
```

3.381 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}+\frac{2(A+C(3-2n)-2An)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);c\right.}{d(1-2n)(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(3/2)+2*(A+C*(3-2*n)-
2*A*n)*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(4*n^2-8*n+3)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*
cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*sin(d*x
+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$\frac{2(-2An+A+C(3-2n))\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(1-2n)(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(5/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Cos[c + d*x]^(3/2)) + (
2*(A + C*(3 - 2*n) - 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 +
 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*
Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeo
metric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*
(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} - \frac{(2\cos^{-n}(}{}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \left(B\cos^{-n}(\right.$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(1-2n)\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(\frac{C}{1-2n}+\right.}{}$$

Mathematica [A]
time = 0.45, size = 163, normalized size = 0.74

$$\frac{2(b\cos(c+dx))^n\sin(c+dx)\left(\left(A+C(3-2n)-2An\right){}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)-(-3+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(-3+2n)(-1+2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(5/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Sin[c + d*x]*((A + C*(3 - 2*n) - 2*A*n)*Hypergeometri
c2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] - (-3 + 2*n)*(B*Cos[c
+ d*x]*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] -
C*Sqrt[Sin[c + d*x]^2])))/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2)*Sqrt[
Sin[c + d*x]^2])
```

Maple [**F**]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A+B\cos(dx+c)+C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\cos(c+dx))^n\,\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(5/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(5/2), x)
```

3.382 $\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}+\frac{2(A(3-2n)+C(5-2n))(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n\right.}{d(3-2n)(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx}}$$

```
[Out] -2*C*(b*cos(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(5/2)+2*(A*(3-2*n)+C*
(5-2*n))*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(4*n^2-16*n+15)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*
B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3102, 2827, 2722}

$$\frac{2\left(\frac{A}{5-2n}+\frac{C}{3-2n}\right)\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2C\sin(c+dx)(b\cos(c+dx))^n}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*
x]^(7/2),x]
```

```
[Out] (-2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(5/2)) + (
2*(C/(3 - 2*n) + A/(5 - 2*n))*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5
 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2
)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c +
 d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} - \frac{(2\cos^{-n}(}{}$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \left(B\cos^{-n}(\right.$$

$$= -\frac{2C(b\cos(c+dx))^n\sin(c+dx)}{d(3-2n)\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(\frac{C}{3-2n}+\right.}{}$$

Mathematica [A]
time = 0.46, size = 164, normalized size = 0.74

$$-\frac{2(b\cos(c+dx))^n\sin(c+dx)\left((C(-5+2n)+A(-3+2n))\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)+(-5+2n)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)-C\sqrt{\sin^2(c+dx)}\right)\right)}{d(-5+2n)(-3+2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[
c + d*x]^(7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(-5 + 2*n) + A*(-3 + 2*n))*Hypergeo
metric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + (-5 + 2*n)*(B*
Cos[c + d*x]*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]
^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2)
*Sqrt[Sin[c + d*x]^2])
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n (A + B\cos(dx+c) + C(\cos^2(dx+c)))}{\cos(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x
 + c)^(7/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7
/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*
x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\cos(c+dx))^n \left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(7/2),x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x) + C*cos(c + d*x)^2))/cos(c + d*
x)^(7/2), x)
```

3.383 $\int (a+a\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e\right.$

Optimal. Leaf size=183

$$-\frac{(C-B(2+m))(a+a\cos(e+fx))^m\sin(e+fx)}{f(1+m)(2+m)}+\frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\frac{2^{\frac{1}{2}+m}(Bm(2+$$

```
[Out] -(C-B*(2+m))*(a+a*cos(f*x+e))^m*sin(f*x+e)/f/(1+m)/(2+m)+C*(a+a*cos(f*x+e))
^(1+m)*sin(f*x+e)/a/f/(2+m)+2^(1/2+m)*(B*m*(2+m)+C*(m^2+m+1)+A*(m^2+3*m+2))
*(1+cos(f*x+e))^(-1/2-m)*(a+a*cos(f*x+e))^m*hypergeom([1/2, 1/2-m],[3/2],1/
2-1/2*cos(f*x+e))*sin(f*x+e)/f/(m^2+3*m+2)
```

Rubi [A]
time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3102, 2830, 2731, 2730}

$$\frac{2^{m+\frac{1}{2}}(A(m^2+3m+2)+Bm(m+2)+C(m^2+m+1))\sin(e+fx)(\cos(e+fx)+1)^{-m-\frac{1}{2}}(a\cos(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx))\right)}{f(m+1)(m+2)}-\frac{(C-B(m+2))\sin(e+fx)(a\cos(e+fx)+a)^m}{f(m+1)(m+2)}+\frac{C\sin(e+fx)(a\cos(e+fx)+a)^{m+1}}{af(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m))
) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 +
 m)*(B*m*(2 + m) + C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^
(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 -
 Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(e+fx))^m\left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx &= \frac{C(a+a\cos(e+fx))^{1+m}\sin(e+fx)}{af(2+m)}+\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m}{f(1+m)(2+m)}\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m}{f(1+m)(2+m)}\\ &= -\frac{(C-B(2+m))(a+a\cos(e+fx))^m}{f(1+m)(2+m)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.80, size = 376, normalized size = 2.05

$$\frac{i4^{-1-m}e^{ifmx}\left(1+e^{i(e+fx)}\right)^{-2m}\left(e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\right)^{2m}\cos^{-2m}\left(\frac{1}{2}(e+fx)\right)(a(1+\cos(e+fx)))^m\left(\frac{Ce^{-i(2e+f(2+m)x)}\,{}_2F_1\left(-2-m,-2m;-1-m;-e^{i(e+fx)}\right)}{2+m}+\frac{2Be^{-i(e+f(1+m)x)}\,{}_2F_1\left(-1-m,-2m;-m;-e^{i(e+fx)}\right)}{1+m}+\frac{2Be^{i(e-f(-1+m)x)}\,{}_2F_1\left(1-m,-2m;2-m;-e^{i(e+fx)}\right)}{-1+m}+\frac{Ce^{2ie-if(-2+m)x}\,{}_2F_1\left(2-m,-2m;3-m;-e^{i(e+fx)}\right)}{-2+m}+\frac{4Ae^{-ifmx}\,{}_2F_1\left(-2m,-m;1-m;-e^{i(e+fx)}\right)}{m}+\frac{2Ce^{-ifmx}\,{}_2F_1\left(-2m,-m;1-m;-e^{i(e+fx)}\right)}{m}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*E^(I*f*m*x)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)
*(a*(1 + Cos[e + f*x]))^m*((C*Hypergeometric2F1[-2 - m, -2*m, -1 - m, -E^(I
*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*B*Hypergeometric2F1[
-1 - m, -2*m, -m, -E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*x))*(1 + m)) + (2
*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E^(I*(e
+ f*x))])/(-1 + m) + (C*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F1[2 -
m, -2*m, 3 - m, -E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1[-2*m,
-m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m) + (2*C*Hypergeometric2F1[-2*m
, -m, 1 - m, -E^(I*(e + f*x))])/(E^(I*f*m*x)*m)))/((1 + E^(I*(e + f*x)))^(2
*m)*f*Cos[(e + f*x)/2]^(2*m))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int (a + a\cos(fx + e))^m \left(A + B\cos(fx + e) + C\left(\cos^2(fx + e)\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\cos(e + fx) + 1))^m \left(A + B\cos(e + fx) + C\cos^2(e + fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Integral((a*(cos(e + f*x) + 1))**m*(A + B*cos(e + f*x) + C*cos(e + f*x)**2)
, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos \left (e+f\,x\right))^m\,\left (C\,{\cos \left (e+f\,x\right)}^2+B\,\cos \left (e+f\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

```
[Out] int((a + a*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

3.384 $\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c\right.$

Optimal. Leaf size=144

$$\frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{40d}+\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad}+\frac{(40A+16B+19C)(\cdots}{}$$

```
[Out] 3/40*(8*B-3*C)*(a+a*cos(d*x+c))^(2/3)*sin(d*x+c)/d+3/8*C*(a+a*cos(d*x+c))^(
5/3)*sin(d*x+c)/a/d+1/20*(40*A+16*B+19*C)*(a+a*cos(d*x+c))^(2/3)*hypergeom(
[-1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(
7/6)
```

Rubi [**A**]
time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3102, 2830, 2731, 2730}

$$\frac{(40A+16B+19C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{10\ 2^{5/6}d(\cos(c+dx)+1)^{7/6}}+\frac{3(8B-3C)\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{40d}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{5/3}}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*(8*B - 3*C)*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a +
a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 16*B + 19*C)*(a + a*
Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]
*Sin[c + d*x])/(10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(a+a\cos(c+dx))^{5/3}\sin(c+dx)}{8ad}+$$

$$=\frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c}{40d}$$

$$=\frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c}{40d}$$

$$=\frac{3(8B-3C)(a+a\cos(c+dx))^{2/3}\sin(c}{40d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.95, size = 137, normalized size = 0.95

$$\frac{3(a(1+\cos(c+dx)))^{2/3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-2i(40A+16B+19C)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-e^{i(c+dx)}\right)(1+\cos(c+dx)+i\sin(c+dx))^{2/3}+2(40A+32B+28C+2(8B+7C)\cos(c+dx)+5C\cos(2(c+dx)))\sin(c+dx)\right)}{320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (3*(a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*((-2*I)*(40*A + 16*B + 1
9*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x))]*(1 + Cos[c + d*x] +
 I*Sin[c + d*x])^(2/3) + 2*(40*A + 32*B + 28*C + 2*(8*B + 7*C)*Cos[c + d*x]
 + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(320*d)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (a+a\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (a+a\,\cos \left (c+d\,x\right)\right)}^{2/3}\,\left (C\,{\cos \left (c+d\,x\right)}^2+B\,\cos \left (c+d\,x\right)+A\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.385 $\int \sqrt[3]{a+a\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2($

Optimal. Leaf size=144

$$\frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{28d}+\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+\frac{(28A+7B+13C)\sqrt[3]{}}{}$$

```
[Out] 3/28*(7*B-3*C)*(a+a*cos(d*x+c))^(1/3)*sin(d*x+c)/d+3/7*C*(a+a*cos(d*x+c))^(
4/3)*sin(d*x+c)/a/d+1/28*(28*A+7*B+13*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([
1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/d/(1+cos(d*x+c))^(5/
6)
```

Rubi [A]

time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3102, 2830, 2731, 2730}

$$\frac{(28A+7B+13C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{14\sqrt[6]{2}\,d(\cos(c+dx)+1)^{5/6}}+\frac{3(7B-3C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{28d}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{4/3}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*(7*B - 3*C)*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a +
a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 7*B + 13*C)*(a + a*C
os[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*S
in[c + d*x])/(14*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+a\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx=\frac{3C(a+a\cos(c+dx))^{4/3}\sin(c+dx)}{7ad}+$$

$$=\frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+}{28d}$$

$$=\frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+}{28d}$$

$$=\frac{3(7B-3C)\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+}{28d}$$

Mathematica [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+a\cos(c+dx)}\,\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
), x]
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (a+a\cos(dx+c))^{\frac{1}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)+C\cos^{2}\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)
**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\cos(c+d\,x))^{1/3}\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.386 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=144

$$\frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}}+\frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad}+\frac{(10A-5B+7C)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c\right.}{5\ 2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\ \sqrt[3]{a+a}}$$

```
[Out] 3/10*(5*B-3*C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+3/5*C*(a+a*cos(d*x+c))^(
2/3)*sin(d*x+c)/a/d+1/10*(10*A-5*B+7*C)*hypergeom([1/2, 5/6],[3/2],1/2-1/2*
cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3
)
```

Rubi [A]
time = 0.12, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3102, 2830, 2731, 2730}

$$\frac{(10A-5B+7C)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{5\ 2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\ \sqrt[3]{a\cos(c+dx)+a}}+\frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a\cos(c+dx)+a}}+\frac{3C\sin(c+dx)(a\cos(c+dx)+a)^{2/3}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*(5*B - 3*C)*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a +
a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A - 5*B + 7*C)*Hypergeom
etric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1
 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5ad} + \frac{3\int \frac{\frac{1}{3}a(5A+2C)+\frac{1}{3}a(5B-3C)}{\sqrt[3]{a+a\cos(c+}}}{5a}$$

$$= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx}{5ad}$$

$$= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx}{5ad}$$

$$= \frac{3(5B-3C)\sin(c+dx)}{10d\sqrt[3]{a+a\cos(c+dx)}} + \frac{3C(a+a\cos(c+dx))^{2/3}\sin(c+dx}{5ad}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.66, size = 105, normalized size = 0.73

$$\frac{-3i(10A-5B+7C)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-e^{i(c+dx)}\right)(1+\cos(c+dx)+i\sin(c+dx))^{2/3}+3(5B-C+2C\cos(c+dx))\sin(c+dx)}{10d\sqrt[3]{a(1+\cos(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3
),x]
```

```
[Out] ((-3*I)*(10*A - 5*B + 7*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x)
)]*(1 + Cos[c + d*x] + I*Sin[c + d*x])^(2/3) + 3*(5*B - C + 2*C*Cos[c + d*x
])*Sin[c + d*x])/(10*d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(a+a\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}}{\sqrt[3]{a\left(\cos{\left(c + dx \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(
1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(1/3), x)
```

3.387 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=144

$$\frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}}+\frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad}-\frac{(4A-8B+7C)\sqrt[3]{a+a\cos(c+dx)}\,{}_2F_1}{2\sqrt[6]{2}\,ad(1+\mathrm{co}}$$

```
[Out] 3*(A-B+C)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)+3/4*C*(a+a*cos(d*x+c))^(1/3)*
sin(d*x+c)/a/d-1/4*(4*A-8*B+7*C)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2
],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(5/6)/a/d/(1+cos(d*x+c))^(5/6)
```

Rubi [A]

time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3102, 2829, 2731, 2730}

$$-\frac{(4A-8B+7C)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}\,ad(\cos(c+dx)+1)^{5/6}}+\frac{3(A-B+C)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}+\frac{3C\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*C
os[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A - 8*B + 7*C)*(a + a*Cos[c
+ d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c
+ d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx)}{4ad} + \frac{3\int \frac{\frac{1}{3}a(4A+C)+\frac{1}{3}a(4B-3C)\operatorname{co}}{(a+a\cos(c+dx))^{2/3}}}{4a}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx}{4ad}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx}{4ad}$$

$$= \frac{3(A-B+C)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} + \frac{3C\sqrt[3]{a+a\cos(c+dx)}\,\sin(c+dx}{4ad}$$

Mathematica [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3
),x]
```

```
[Out] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3
), x]
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)}{\left(a+a\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}}{\left(a\left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a*(cos(c + d*x) + 1))**(
2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + a*cos(c + d*x))^(2/3), x)
```

3.388 $\int (a+b\cos(c+dx))^{2/3} \left(A + B\cos(c+dx) + C\cos^2(c\right.$

Optimal. Leaf size=290

$$\frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd}+\frac{(a+b)(8bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\,\left(\frac{a+b\cos(c+dx}{a+b}\right.}$$

```
[Out] 3/8*C*(a+b*cos(d*x+c))^(5/3)*sin(d*x+c)/b/d+1/8*(a+b)*(8*B*b-3*C*a)*AppellF
1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(2/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*
x+c))^(1/2)+1/8*(8*A*b^2-8*B*a*b+3*C*a^2+5*C*b^2)*AppellF1(1/2,-2/3,1/2,3/2
,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+
c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sin(c+dx)\left(3a^2C-8abB+8Ab^2+5b^2C\right)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{(a+b)(8bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{4\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{5/3}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((8*A*b^2 - 8*
a*b*B + 3*a^2*C + 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])
/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x]
)/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2
/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{2/3}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} +$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} +$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} -$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} +$$

$$= \frac{3C(a+b\cos(c+dx))^{5/3}\sin(c+dx)}{8bd} +$$

Mathematica [A]
time = 3.63, size = 296, normalized size = 1.02

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(20(-a^2+b^2)(8bB-3aC)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+4(40Ab^2+16abB-6a^2C+25b^2C)\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-20b^2(8bB+2aC+5bC\cos(c+dx))\sin^2(c+dx)\right)}{800b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(20*(-a^2 + b^2)*(8*b*B - 3*a*C
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
 d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos
[c + d*x]))/(a - b))] + 4*(40*A*b^2 + 16*a*b*B - 6*a^2*C + 25*b^2*C)*Appell
F1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(
a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x])
)/(-a + b)]*(a + b*Cos[c + d*x]) - 20*b^2*(8*b*B + 2*a*C + 5*b*C*Cos[c + d*
x])*Sin[c + d*x]^2))/(800*b^3*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (a+b\cos(dx+c))^{\frac{2}{3}}\left(A+B\cos(dx+c)+C\left(\cos^2(dx+c)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cos(c+dx))^{2/3}\left(C\cos(c+dx)^2+B\cos(c+dx)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.389 $\int \sqrt[3]{a+b\cos(c+dx)}\,(A+B\cos(c+dx)+C\cos^2(c$

Optimal. Leaf size=290

$$\frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd}+\frac{\sqrt{2}\,(a+b)(7bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx}{a+b}\right.}{7b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+}{a+b}}}$$

```
[Out] 3/7*C*(a+b*cos(d*x+c))^(4/3)*sin(d*x+c)/b/d+1/7*(a+b)*(7*B*b-3*C*a)*AppellF
1(1/2,-4/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+
c))^(1/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*
x+c))^(1/2)+1/7*(7*A*b^2-7*B*a*b+3*C*a^2+4*C*b^2)*AppellF1(1/2,-1/3,1/2,3/2
,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+
c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\sin(c+dx)\left(3a^2C-7abB+7Ab^2+4b^2C\right)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}(a+b)(7bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{7b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{4/3}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7
*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - C
os[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(7*A*
b^2 - 7*a*b*B + 3*a^2*C + 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c
 + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[
c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1
/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+b\cos(c+dx)}\left(A+B\cos(c+dx)+C\cos^2(c+dx)\right)\,dx = \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \cdots$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \cdots$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} - \cdots$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \cdots$$

$$= \frac{3C(a+b\cos(c+dx))^{4/3}\sin(c+dx)}{7bd} + \cdots$$

Mathematica [A]
time = 3.76, size = 294, normalized size = 1.01

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\csc(c+dx)\left(4(-a^2+b^2)(7bB-3aC)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{-\frac{b(1+\cos(c+dx))}{a-b}}+(28Ab^2+7abB-3a^2C+16b^2C)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-4b^2(7bB+aC+4bC\cos(c+dx))\sin^2(c+dx)\right)}{112b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (28*A*b^2 + 7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4
/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a +
b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*Cos[c + d*x])*Sin
[c + d*x]^2))/(112*b^3*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\left(A+B\cos\left(dx+c\right)+C\left(\cos^2\left(dx+c\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a + b\cos{(c + dx)}}\left(A + B\cos{(c + dx)} + C\cos^{2}{(c + dx)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)**
2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cos(c+d\,x))^{1/3}\left(C\cos(c+d\,x)^2+B\cos(c+d\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x) + C*cos(c + d*x)^2), x)
```

3.390 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=287

$$\frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd}+\frac{\sqrt{2}\,(5bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a}{5b^2d\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/}}$$

```
[Out] 3/5*C*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)/b/d+1/5*(5*B*b-3*C*a)*AppellF1(1/2,
-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2
/3)*sin(d*x+c)*2^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^
(1/2)+1/5*(5*A*b^2-5*B*a*b+3*C*a^2+2*C*b^2)*AppellF1(1/2,1/3,1/2,3/2,b*(1-c
os(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x
+c)*2^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\sin(c+dx)\left(3a^2C-5abB+5Ab^2+2b^2C\right)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}(5bB-3aC)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{5b^2d\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{3C\sin(c+dx)(a+b\cos(c+dx))^{2/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(5*A*b^2 - 5*
a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/
2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin
[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx) + C\cos^2(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{3\int \frac{\frac{1}{3}b(5A+2C)+\frac{1}{3}(5bB-3a}{\sqrt[3]{a+b\cos(c-}}}{5b}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{(5bB-3aC)\int(a+b\,c}{5b^2}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)\sin(c+}{5b^2d\sqrt{}}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} - \frac{((5bB-3aC)(a+b\,c}{5b^2d}$$

$$= \frac{3C(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{5bd} + \frac{\sqrt{2}\,(5bB-3aC)F_1\Big(\frac{1}{2}}{}$$

Mathematica [A]
time = 2.61, size = 268, normalized size = 0.93

$$-\frac{3(a+b\cos(c+dx))^{2/3}\csc(c+dx)\left(5(5Ab^2-5abB+3a^2C+2b^2C)\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}+2(5bB-3aC)F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-10b^2C\sin^2(c+dx)\right)}{50b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3
),x]

[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 - 5*a*b*B + 3*a^2*C
 + 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a +
 b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*
(1 + Cos[c + d*x]))/(-a + b)] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*
Cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(dx+c) + C(\cos^2(dx+c))}{(a+b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}}{\sqrt[3]{a + b\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(1/
3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(1/3), x)
```

3.391 $\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=286

$$\frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd}+\frac{(4bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\,c}}{2\sqrt{2}\,b^2d\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 3/4*C*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)/b/d+1/4*(4*B*b-3*C*a)*AppellF1(1/2,
-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1
/3)*sin(d*x+c)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/3)*2^(1/2)/(1+cos(d*x+c))^
(1/2)+1/4*(4*A*b^2-4*B*a*b+3*C*a^2+C*b^2)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos
(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c
)/b^2/d/(a+b*cos(d*x+c))^(2/3)*2^(1/2)/(1+cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sin(c+dx)\left(3a^2C-4abB+4Ab^2+b^2C\right)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\,(a+b\cos(c+dx))^{2/3}}+\frac{(4bB-3aC)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{2\sqrt{2}\,b^2d\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{3C\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*Ap
pellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a
 + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + C
os[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((4*A*b^2 - 4*a*b*B +
3*a^2*C + b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 -
 Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])
/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{3\int \frac{\frac{1}{3}b(4A+C)+\frac{1}{3}(4bB-3aC)\,\mathrm{co}}{(a+b\cos(c+dx))^{2/3}}}{4b}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{(4bB-3aC)\int\sqrt[3]{a+b\,\mathrm{c}}}{4b^2}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{((4bB-3aC)\sin(c+dx}{4b^2d\sqrt{1-}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} - \frac{\left((4bB-3aC)\sqrt[3]{a+b\,\mathrm{co}}\right.}{4b^2d\sqrt{}}$$

$$= \frac{3C\sqrt[3]{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd} + \frac{(4bB-3aC)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3}\right.}{2}$$

Mathematica [A]
time = 2.61, size = 266, normalized size = 0.93

$$-\frac{3\sqrt[3]{a+b\cos(c+dx)}\,\csc(c+dx)\left(4(4Ab^2-4abB+3a^2C+b^2C)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}+(4bB-3aC)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}(a+b\cos(c+dx))-4b^2C\sin^2(c+dx)\right)}{16b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3
),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 - 4*a*b*B + 3*a^2*C
 + b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b
*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1
 + Cos[c + d*x]))/(-a + b)] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3,
(a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[
c + d*x]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(dx+c)+C(\cos^2(dx+c))}{(a+b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3)
, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{\left(c + dx \right)} + C\cos^{2}{\left(c + dx \right)}}{\left(a + b\cos{\left(c + dx \right)}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)/(a + b*cos(c + d*x))**(2/
3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\cos\left(c+d\,x\right)^2+B\,\cos\left(c+d\,x\right)+A}{\left(a+b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + B*cos(c + d*x) + C*cos(c + d*x)^2)/(a + b*cos(c + d*x))^(2/3), x)
```

3.392 $\int (a+b\cos(e+fx))^m\,(A+(A+C)\cos(e+fx)+C\,\mathrm{c}$

Optimal. Leaf size=215

$$\frac{4\sqrt{2}\,CF_1\left(\frac{1}{2};-\frac{3}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}\sin(e+f}{f\sqrt{1+\cos(e+fx)}}$$

```
[Out] 4*C*AppellF1(1/2,-m,-3/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+
b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+2*(A-C)*AppellF1(1/2,-m,-1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2
*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+e)*2^(1/2)/f/(((a+b*cos(f*x+e))/(a+
b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3096, 2834, 144, 143, 2863}

$$\frac{2\sqrt{2}\,(A-C)\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};-\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}+\frac{4\sqrt{2}\,C\sin(e+fx)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};-\frac{3}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (4*Sqrt[2]*C*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos
[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e
 + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (2*Sqrt[2]*(A - C)*AppellF1[1/
2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a
+ b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e +
 f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
```

```
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 3096

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = (A-C)\int (1+\cos(e+fx))(a \ldots$$

$$= -\frac{(C\sin(e+fx))\text{Subst}\left(\int \frac{(1+x)\ldots}{\sqrt{\ldots}}\right.}{f\sqrt{1-\cos(e+fx)}\ldots}$$

$$= -\frac{\left(C(a+b\cos(e+fx))^m\left(-\frac{a+\ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{4\sqrt{2}\,CF_1\left(\frac{1}{2};-\frac{3}{2},-m;\frac{3}{2};\frac{1}{2}(1-\ldots\right.}{\ldots}$$

Mathematica [F]
time = 3.67, size = 0, normalized size = 0.00

$$\int (a+b\cos(e+fx))^m \left(A+(A+C)\cos(e+fx)+C\cos^2(e+fx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]
^2),x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]
^2), x]
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int (a+b\cos(fx+e))^m \left(A+(A+C)\cos(fx+e)+C\left(\cos^2(fx+e)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a
)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a)
^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorit
hm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a
)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\cos\left(e+f\,x\right))^m\,\left(C\,{\cos\left(e+f\,x\right)}^2+\left(A+C\right)\,\cos\left(e+f\,x\right)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)),x)
```

```
[Out] int((a + b*cos(e + f*x))^m*(A + C*cos(e + f*x)^2 + cos(e + f*x)*(A + C)), x
)
```

3.393 $\int (a+b\cos(e+fx))^m \left(A + B\cos(e+fx) + C\cos^2(e\right.$

Optimal. Leaf size=303

$$\frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)} - \frac{\sqrt{2}\,(a+b)(aC-bB(2+m))F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)\right.}{b^2f(2+m)\sqrt{1\ldots}}$$

```
[Out] C*(a+b*cos(f*x+e))^(1+m)*sin(f*x+e)/b/f/(2+m)-(a+b)*(a*C-b*B*(2+m))*AppellF
1(1/2,-1-m,1/2,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+
e))^m*sin(f*x+e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*
x+e))^(1/2)+(a^2*C+b^2*C*(1+m)+A*b^2*(2+m)-a*b*B*(2+m))*AppellF1(1/2,-m,1/2
,3/2,b*(1-cos(f*x+e))/(a+b),1/2-1/2*cos(f*x+e))*(a+b*cos(f*x+e))^m*sin(f*x+
e)*2^(1/2)/b^2/f/(2+m)/(((a+b*cos(f*x+e))/(a+b))^m)/(1+cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\sin(e+fx)\left(a^2C-abB(m+2)+Ab^2(m+2)+b^2C(m+1)\right)(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}} - \frac{\sqrt{2}(a+b)\sin(e+fx)(aC-bB(m+2))(a+b\cos(e+fx))^m\left(\frac{a+b\cos(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\cos(e+fx)),\frac{b(1-\cos(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\cos(e+fx)+1}} + \frac{C\sin(e+fx)(a+b\cos(e+fx))^{m+1}}{bf(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
 b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) + A*b^2*(2 + m) - a*b*B*(2 + m))*AppellF1[1/2,
1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*
Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b
*Cos[e + f*x])/(a + b))^m)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^m \left(A+B\cos(e+fx)+C\cos^2(e+fx)\right)\,dx = \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$

$$= \frac{C(a+b\cos(e+fx))^{1+m}\sin(e+fx)}{bf(2+m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16142 vs. $2(303) = 606$.
time = 27.18, size = 16142, normalized size = 53.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^m \left(A+B\cos\left(fx+e\right)+C\left(\cos^2\left(fx+e\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cos(e+f\,x))^m\,\left(C\cos(e+f\,x)^2+B\,\cos(e+f\,x)+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2),x)
```

```
[Out] int((a + b*cos(e + f*x))^m*(A + B*cos(e + f*x) + C*cos(e + f*x)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```